Comment on the ASTG-model

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Abstract: More recently, G. G. Nyambuya published an article entitled *"Azimuthally symmetric theory of gravitation – II. On the perihelion precession of solar planetary orbits"* [MNRAS **451** (3) 3034-3043 (2015)], which considered by the author as an improved version of the *"Azimuthally Symmetric Theory of Gravitation (I) – On the Perihelion Precession of Planetary Orbits"* [MNRAS **403** (3) 1381-1391 (2010)]. This comment proves that the so-called ASTG-model is physico-mathematically incorrect. Consequently, the said model cannot possibly be a (new) gravity theory.

1. Introduction

In his paper "*Azimuthally symmetric theory of gravitation – II. On the perihelion precession of solar planetary orbits*" [1], G. G. Nyambuya proposed Symmetric Theory of Gravitation (ASTG-model) as a gravity theory, which is exclusively based on the misunderstanding of Poisson's gravitational potential equation. In that paper [1], the author wrongly called this equation: Poisson-Laplace equation. And in the beginning of the introduction, he wrote « *At first glance, this theory appears as nothing more than the mundane azimuthally symmetric solutions of the well known Poisson-Laplace equation, namely:*

$$\nabla^2\Phi = 4\pi G\rho, \tag{1}$$

(...) *The ASTG-model is a "seemingly non-relativistic classical theory" where spin is not only taken into account but takes center stage in the theory, especially when the spin is significantly high. This is not the case with classical theories of gravity hence this very development makes it a new theory of gravitation.*

As will become clear in the present reading, the ASTG-model is surely a new classical theory of gravitation which makes the seemingly ambitious hypothesis that the spin of a gravitating mass has a significant and decisive role to play in the emergent gravitational field of the spinning mass. The ASTG-model is based on the solutions $\Phi = \Phi(r,\theta)$ *of* (*1*)*, i.e.*:

$$\Phi(r,\theta) = -\frac{GM_{\mathrm{star}}}{r}\left[1+\sum_{\ell=1}^{\infty}\lambda_\ell\left(\frac{GM_{\mathrm{star}}}{rc^2}\right)^{\ell}P_\ell(\sin\theta)\right], \tag{2}$$

where $P_\ell(\sin\theta)$ *are the Legendre polynomials written in terms of* $\sin\theta$*, (...)»*

Since all the ASTG-model is exclusively based on the expression (2), thus let us focus our attention on Eq.(1).

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1.1. Historical background

Historically, the French mathematician, physicist and astronomer Pierre-Simon Laplace (1749 – 1827) discovered his equation

$$\nabla^2 U = 0, \quad U \equiv U(r), \quad r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}, \tag{i}$$

in 1782 and published it in 1784 under the title "*Théorie du movement et de la figure elliptique des planètes*". Eq.(i) was formulated in the context of potential theory. Eq.(i) is valid only when the test-body of mass m is outside the gravitational source of mass M and radius R, that is, when $r > R$.

However, the French mathematician Siméon-Denis Poisson (1781–1840), who was one of the founders of mathematical physics, formulated his equation

$$\nabla^2 U = 4\pi G\rho, \rho \equiv \rho(r), \tag{ii}$$

within the framework of potential theory and published it in 1813 (in the Bulletin de la Société Philomatique, pp. 388-392). Eq.(ii) is exclusively valid when the test-body is inside the gravitational source, that is, when $r < R$.

In his original article (in French), Poisson never considered his equation (ii) as a correction and/or a generalisation of Laplace's Eq.(i). In modern language, Poisson said in his paper that Laplace's Eq.(i), is applicable only if the material body is outside the gravitational source, and when the body is inside the source we should take into consideration the mass density $\rho \equiv \rho(r)$ inside the radius $r < R$ where Laplace's Eq.(i) should be replaced with his Eq.(ii).

1.2. Physico-mathematical background

In the pure framework of classical gravitational physics, the fundamental solutions of Eqs.(i) and (ii) are

$$U \equiv U(r) = \begin{cases} -\dfrac{GM}{r}, & r > R \\ \frac{2}{3}\pi G\rho\left(r^2 - 3R^2\right), & r < R \end{cases}, \tag{iii}$$

where G, ρ, M, R and r are, respectively, the gravitational constant, density, mass of gravitational source (which is supposed to be spherically symmetric), radius of source, and the relative distance between the centre of the source and the centre of test-body of mass m $(m < M)$.

Thus, according to (iii), the gravitational potential function $U \equiv U(r)$ is at the same time a fundamental solution for Eqs.(i) and (ii) for the cases $r > R$ and $r < R$, respectively. Curiously, it seems until now, that Poisson's Eq.(ii) has not been correctly understood because many specialised textbooks and research articles erroneously claimed that Eq.(ii) may be reduced to Eq.(i) when $\rho = 0$.

However, the mass density ρ in Eq.(ii) plays the role of a gravitational source and in Eq.(i) this role is played by the mass M which is contained in the expression of U for the case $r > R$. Therefore, physically, Eq.(ii) cannot reduce to Eq.(i) since the mass density itself gives rise to the gravitational field *via* the gravitational potential U and once again this implies that the mass density must play the role as a gravitational source, which is why ρ is explicitly present in the expression of U for the case $r < R$.

Therefore, according to (iii), when $\rho = 0 \Rightarrow U = 0$ and Eq.(ii) becomes an identity of the form $0 = 0$. For this reason Poisson himself did not consider his equation as a correction and/or a generalisation of Laplace's Eq.(i). From all that, we arrive at the following result: it is absolutely unphysical to suppose that Poisson's Eq.(ii) may be reduced to Laplace's Eq.(i) –in the case of empty space– *i.e.*, when $\rho = 0$ because, I repeat, the mass density $\rho \equiv \rho(r)$ in Poisson's Eq.(ii) plays the role of a gravitational source. That's why, in his original work, Poisson used the expressions like *external* points and *internal* points to mean $r > R$ and $r < R$, respectively. In this sense, the expression (iii) of $U \equiv U(r)$ for the cases $r < R$ and $r > R$ is called gravitational potential inside and outside the source (sphere).

2. Fallacy of ASTG-model

In order to show the fallacy of ASTG-model, let us return to Eqs.(i) and (ii). Since the gravitational source is supposed to be spherically symmetric, thus we can rewrite these two equations as follows:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) = 0, \quad r > R \tag{iv}$$

and

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right) = 4\pi G\rho, \quad r < R. \tag{v}$$

Now, physico-mathematically, it is so easy to prove that the (gravitational) potential function (iii) is really a fundamental solution to (iv) for the case $r > R$ and also is a fundamental solution to (v) for the case $r < R$. All that may be performed via a direct derivation/differentiation and substitution. Hence, contrary to the author's claim, the expression (2) in Ref.[1] cannot be a solution to Eq.(1) because of the presence of the mass density ρ in (1) and it is absence in (2).

In the subsection 5.1 entitled "Brief Exposition of Proposed Gravitomagnetic Model" [1], the author, wrote: «*In the case of dynamic gravitational fields, i.e., non-static time-dependent gravitational fields, the Poisson-Laplace equation (1) upon whose shoulders the ASTG-model stands; this equation (1) is not Lorentz invariant and apart from this, it is obvious and clear that this equation will fail to describe* *a time-dependent gravitational field as it is not in its natural form equipped to do this. In order to make it Lorentz invariant, one can add a time dependent term, in which event the resulting equation is the four Poisson-Laplace equation i.e.:*

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 4\pi G\rho \ . \tag{23}$$

In the reading Nyambuya (2015b), equation (23) has been solved for five gravitational potentials and this has been done in the context of the gravitomagnetic theory given in Nyambuya (2015a).»

as we can remark it from the above passage, in order to make Poisson's equation invariant under Lorentz transformations, the author simply added a time-dependent term $-\partial^2\Phi/c^2\partial t^2$ to Eq.(1) without any explicit physical information about the gravitational potential function Φ since it should contain the time as an essential parameter to measure the dynamic evolution of Φ itself, again in this sense the author failed to inform us on the main property of Φ, which now should be of the form $\Phi \equiv \Phi(\mathbf{r},t)$. Physico-mathematically, in order that $\Phi \equiv \Phi(\mathbf{r},t)$ to be a fundamental solution to Eq.(23), Φ should be a retarded gravitational potential, *i.e.*, the potential $\Phi(\mathbf{r},t)$ at $\mathbf{r}$, at time t, is the summation on contributions at $\mathbf{r}'$, at the retarded time $\tau = t - \|\mathbf{r}-\mathbf{r}'\| c^{-1}$. Explicitly, we should have

$$\Phi \equiv \Phi(\mathbf{r},t) = -G\int \frac{\rho(\mathbf{r}', \tau)}{\|\mathbf{r}-\mathbf{r}'\|} d^3\mathbf{r}' \ . \tag{vi}$$

Again, as we can remark it, the explicit presence of the mass density $\rho \equiv \rho(\mathbf{r}', \tau)$ in Eq.(23) and in its fundamental solution (vi) signifies that the mass density itself plays the role of the gravitational source. Like before, that is, if we put $\rho = 0$ this implies $\Phi(\mathbf{r},t) = 0$ and Eq.(23) becomes an identity of the form $0 = 0$. Therefore, it is unphysical to set $\rho = 0$ and justifying this choice *via* the ill-argument of empty space. However, instead of this unphysical argument, we must follow Poisson's original statement, namely, the use of $r > R$ and/or $r < R$ as criterion.

All these above considerations have been ignored or neglected by the author in the paper under discussion and also in [2015a: Journal of Modern of Physics **5** (4),1–10; 2015b: Journal of Modern of Physics **5** (10),1–35] and in their extension. For example, in his first paper entitled *"Azimuthally Symmetric Theory of Gravitation (I) – On the Perihelion Precession of Planetary Orbits"* published in [MNRAS **403** (3) 1381-1391 (2010)], and in the introduction, the author wrote: «*Einstein's GTR explains the perihelion shift of planetary orbits as a result of the curvature of space-time around the Sun. It does not take into account the spin of the Sun and at the same time it assumes all the planets lay on the same plane. The assumption that the planets lay on the same plane is in the GTR solution only taken as a first order approximation – in reality, planets do not lay on the same plane. In this reading we set forth what we believe is a new paradigm; we have coined this paradigm the Azimuthally Symmetric Theory of Gravitation (ASTG) and this is derived from Poisson's well accepted equation for empty space – namely* $\nabla^2\Phi = 0$*.»*

The above passage reflects more conclusively the author's fatal error since he reduced the Poisson's equation to Laplace's equation *via* the ill-argument of empty space. And by putting $\Phi = \Phi(r,\theta)$, he wrote:*« (...) Now, putting all the things together, the most general solution is given*:

$$\Phi(r,\theta) = -\sum_{\ell=0}^{\infty}\left[\lambda_\ell c^2\left(\frac{GM_{\text{star}}}{rc^2}\right)^{\ell+1} P_\ell(\cos\theta)\right] . \tag{11}$$

which is a linear combination of all the solutions for ℓ. In the case of ordinary bodies such as the Sun, the higher orders terms [i.e. $\ell > 1$: *of the term* $(GM/rc^2)^{\ell+1}$*], will be small and in these cases, the gravitational field will tend to Newton's gravitational theory. Equation* (11) *is the embodiment of our ASTG and from this, we shall show that one is able to explain the precession of the perihelion of planetary orbits.»*

In spite of its incorrect derivation, the expression of the so-called general solution (11) raises many questions, the most important one is the following: since the alleged ASTG-model is developed in the framework of the classical gravitational physics, therefore what is the physical reason behind the presence of the light speed squared c^2 in (2) & (11)? And in the same paper under consideration, the author wrote: *«About the λ-parameters, it should be mentioned that this property that the λ's are dynamic parameters assumed to be related to the gravitating body in question is the novelty of the ASTG-model.»*

But the author failed to give us a physico-mathematical expression that should illustrate the supposed dynamicity of the *λ-parameters* because the gravitating body is characterised by the mean orbital angular velocity $\omega_{\text{orb}} = 2\pi/T$ where T is the mean orbital period or equivalently the *λ-parameters* should be depended on the mean orbital velocity $v_{\text{orb}} = \omega_{\text{orb}}\, r$, therefore the *λ-parameters* should be in fact of the form $\lambda_\ell \equiv \lambda_\ell(\omega_{\text{orb}})$ or equivalently $\lambda_\ell \equiv \lambda_\ell(v_{\text{orb}})$. Furthermore, in (2) & (11), the Legendre polynomials have not the same expression, however, historically and physico-mathematically, the Legendre polynomials were first introduced in 1782 by the French mathematician Adrien-Marie Legendre (1752–1833) as the coefficients in the expansion of the Newtonian potential $\Phi(r,\theta) = \sum_{\ell=0}^{\infty}\left[A_\ell r^\ell + B_\ell r^{-(\ell+1)}\right]P_\ell(\cos\theta)$, since in general $\cos\theta \neq \sin\theta$ therefore the correct Legendre polynomials are of the form $P_\ell(\cos\theta)$ instead of $P_\ell(\sin\theta)$. Moreover, let us show another contradiction concerning the so-called solutions (2) & (11). According to the author, we have $\lambda_{\ell=0} = 1$ and by taking into account the well-known propriety of the Legendre polynomials, *i.e.*, $P_{\ell=0}(\cos\theta) = 1$ the so-called general solution (11) of $\nabla^2\Phi(r,\theta) = 0$ may be expressed as

$$\Phi(r,\theta) = -\frac{GM}{r}\left[1 + \sum_{\ell=1}^{\infty}\lambda_\ell\left(\frac{GM}{rc^2}\right)^{\ell} P_\ell(\cos\theta)\right] , \tag{11.1}$$

which is very similar to the alleged solution (2) of $\nabla^2\Phi = 4\pi G\rho$. Hence, the author incorrectly found the same solution to two different partial differential equations. Finally, the author proposed the expression *"Azimuthally Symmetric Theory of Gravitation"* as a name for his theory, but there is no connection between this name and the physico-mathematical formalism of the so-called ASTG-model. In general, the adverb *"Azimuthally"* means: in a manner with respect to the azimuthal angle, however, the θ-parameter is not explicitly considered by the author as an azimuthal angle and the exact (physical) role of θ stayed unknown. Below, is some clarifications concerning the notion of azimuthal angle in physics and mathematics. Firstly, the use of symbols and the order of the coordinates differs between sources. In one system (spherical coordinates) frequently encountered in physics (r,θ,φ) gives the radial distance r, polar angle θ, and azimuthal angle φ, as illustrated in Figure 1:

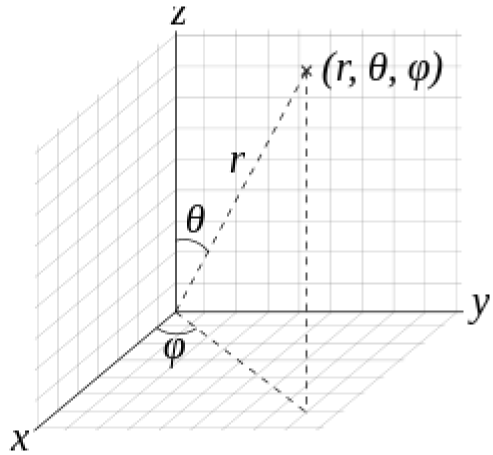

Fig.1

whereas in another system used in many mathematics books (r,φ,θ) gives the radial distance r, azimuthal angle φ, and polar angle θ. In both systems ρ is often used instead of r, as exemplified in Figure 2:

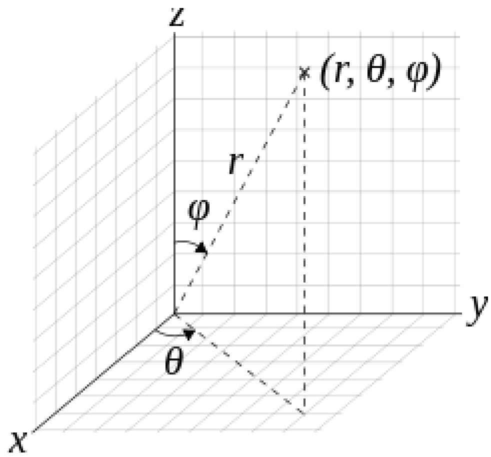

Fig.2

It is worthwhile to note that other conventions are also used, so great care needs to be taken to check which one is being used.

3. Conclusion

In this comment, we have scrutinised *"Azimuthally symmetric theory of gravitation – II. On the perihelion precession of solar planetary orbits"* [1] and proved that this paper is physico-mathematically incorrect because it is exclusively based on the misunderstanding of the Poisson's gravitational potential equation. Consequently, the so-called ASTG-model and its extension cannot be considered as an intellectual and scientific contribution to the science in general and to the gravitational physics in particular. Nevertheless, in order that the alleged ASTG-model to be viable theory, it should have its proper physico-mathematical formalism, for instance, ASTG-model should be characterised by a certain fundamental ASRG-equations in which the couple (r,θ) plays the role of dynamical variables and the presence of light speed in vacuum should be well justified.

Reference

[1] G. G. Nyambuya, MNRAS **451** (3) 3034-3043 (2015)