# Depicting of electric fields

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**Abstract**
Examples are presented that geometrical images of generated electromagnetic fields are emitted by the geometrical images of the electromagnetic fields, which are the sources

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## 1. Problem

As is well known, electric field can be written as the sum of two terms, $\mathbf{E} = \underset{l}{\mathbf{E}} + \underset{t}{\mathbf{E}}$, where $\underset{l}{\mathbf{E}}$ is the longitudinal or irrotational field and $\underset{t}{\mathbf{E}}$ is the transverse or solenoidal field. A charge density $\rho$ *generates* the irrotational electric *vector* field $\underset{l}{\mathbf{E}}$; $\rho$ is the *source* of the field:

$$\rho = \mathrm{div}\,\underset{l}{\mathbf{E}} = \partial_i \underset{l}{E}{}^i . \tag{1}$$

Accordingly, the charge density $\rho$ emits the field lines of vectors $\underset{l}{\mathbf{E}}$ (see Fig. 1a[1]).

At the same time, the solenoidal *vector* field $\underset{t}{\mathbf{E}}$ has no source

$$\mathrm{div}\,\underset{t}{\mathbf{E}} = \partial_i \underset{t}{E}{}^i = 0 . \tag{2}$$

Accordingly, the field lines of vectors $\underset{t}{\mathbf{E}}$ are closed (see Fig. 1b[2]).

The derivative of magnetic field, $\dot{\mathbf{B}}$, *generates* the solenoidal electric field $\underset{t}{\mathbf{E}}$; $\dot{\mathbf{B}}$ is the source of the field:

$$-\dot{\mathbf{B}} = \mathrm{curl}\,\underset{t}{\mathbf{E}} . \tag{3}$$

But, as you see, the source, $\dot{\mathbf{B}}$, does not emit field lines of the solenoidal field $\underset{t}{\mathbf{E}}$ as well as $\rho$ emits field lines of the irrotational field $\underset{l}{\mathbf{E}}$. Why? What is the cause of this different relationship between field lines and the sources for irrotational and solenoidal fields?

## 2. Solution

The point is $\dot{\mathbf{B}}$ generates the solenoidal electric field $\underset{t}{\mathbf{E}}$ as a *covector* field $\underset{t}{E}{}_i$, not as a vector field. The solenoidal field $\underset{t}{\mathbf{E}}$ in eq. (3) is a covector field. $\dot{\mathbf{B}}$ is the source of a *covector* field, and $\dot{\mathbf{B}}$ itself is a covariant bivector. Really,

$$-\dot{\mathbf{B}} = \mathrm{curl}\,\underset{t}{\mathbf{E}}, \text{ i.e. } -\dot{B}_{ik} = \partial_i \underset{t}{E}{}_k - \partial_k \underset{t}{E}{}_i \tag{3}$$

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[1] Figure 2.5 from [1] is used here, but its sense is modified
[2] This figure is from [2]

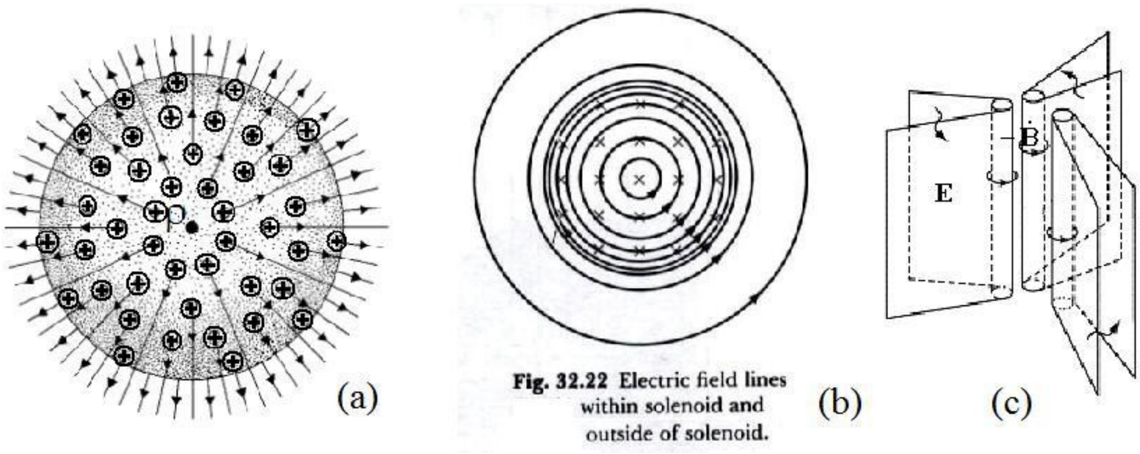



**Fig.1** Generating of electric fields.
(a) Field lines of vectors $\underset{l}{\mathbf{E}}$ emerge from the charge density $\rho$, which is a source of vector field $\underset{l}{\mathbf{E}}$.
(b) The derivative, $\dot{\mathbf{B}}$, is a source of the solenoidal field $\underset{t}{\mathbf{E}}$.
(c) Field bisurfaces of covector field $\underset{t}{\mathbf{E}}$ emerge from the field tubes of the vector density $-\dot{\mathbf{B}}$.

But covector fields are depicted not by field lines. Covector fields are depicted by bisurfaces. In the case of (3), the field bisurfaces **emerge** from the field tubes, which represent the derivative of magnetic field, $\dot{\mathbf{B}}$, as is shown in Fig. 1c[3].

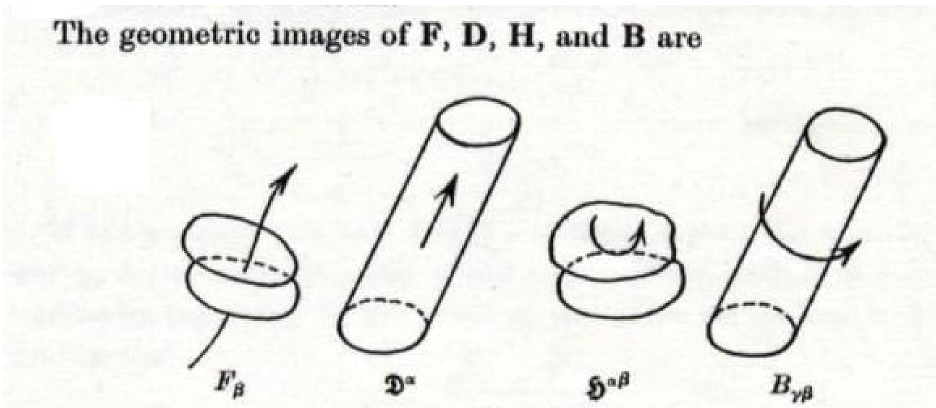
If we interested in the force, which the solenoidal field exerts on a charge $q$, we must use a *vector* $\underset{t}{E}^{k}$, which may be obtained by raising the index with the metric tensor $g^{ki}$:

$$F^k = q\underset{t}{E}^{k} = q\underset{t}{E}_{i} g^{ik}, \tag{4}$$

but the solenoidal electric field $\underset{t}{\mathbf{E}}$ is generated as a *covector* field $\underset{t}{E}_{i}$.

## 3. Geometrical quantities

It is important to recognize that the electromagnetism involves geometrical quantities of two different types [4]. These are: covariant (antisymmetric) tensors, e.g. $\mathbf{E} = E_i$, $\mathbf{B} = B_{ik}$, which are named exterior differential forms or simply forms, and contravariant (antisymmetric) tensor *densities*, e.g. $\rho$, $\mathbf{E} = E^i$, $\mathbf{B} = B^{ik}$ (see Fig. 2[4]), but here we ignore density nature of vector $\mathbf{E} = E^i$.

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[3] This figure is from [3], p. 7.
[4] This is figure 23 from [4].

**Fig. 2.** Schouten's $F_\beta$ and $B_{\gamma\beta}$ depict covector $E_i$ and covariant bivector $B_{ik}$, respectively. Covector is represented by two parallel plane elements equipped with an outer orientation, and covariant bivector is represented by a cylinder with an outer orientation.. Schouten's $\mathfrak{D}^\alpha$ depicts vector density $E^i$, which is represented by a cylinder with an inner orientation.

So, according to Fig. 1c, the field tubes of the covariant bivector $-\dot{\mathbf{B}}$ emit field biplanes of covector $\mathbf{E}$. Their orientations are consistent.

## References


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[2] Ohanian H C 1985 *Physics* (W. W. Norton, N. Y).
[3] Khrapko R I 2011 V*isible representation of exterior differential forms and pseudo forms. Electromagnetism in terms of sources and generation of fields. Наглядное представление дифференциальных форм и псевдоформ. Электромагнетизм в терминах источников и порождений полей.* (Saarbrucken: Lambert). http://khrapkori.wmsite.ru/ftpgetfile.php?id=105&module=files
[4] Schouten J A 1951 *Tensor Analysis for Physicists* (Oxford: Clarendon).