About it how to space-time was eliminated from the General Relativity

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Abstract: *The m(GR) theory is the new theory of gravitation, where the space-time (with metric tensor $g_{\mu\nu}$) was eliminated and replaced with the medium (with the effective mass density tensor $\rho_{\mu\nu}$). It is a new paradigm in the research of the gravitational phenomena.*

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Introduction

General Relativity (GR) is a theory which a since about 100 years describes the gravitational phenomena as geometric properties of the space-time. Although GR is widely accepted as a fundamental theory of gravitation for the many physicists still this is not a perfect theory.

In GR the space-time plays a very important role. The space-time continuum is a mathematical model that joins three-dimensional space and one dimension time into a single idea, the four-dimensional space-time. Under influence outer gravitational field the space-time is curved. The gravitational wave is the ripple in the curvature of the space-time that propagates as a wave.

We suppose that there is an alternative description of gravitational phenomena. The arena, where gravitational phenomena take place is *the medium*. It is a material medium with a density, embedded in the Minkowski space-time. Under influence outer gravitational field *the medium* is changes and deformed (is curved) but the space-time plays role only of the passive background and does not change. The space-time is in certain sense only scaffold.

The *m(GR)* theory is the new theory of gravitation, where the space-time (with metric tensor $g_{\mu\nu}$) was eliminated and replaced with *the medium* (with the effective mass density tensor $\rho_{\mu\nu}$). The gravitational wave is the ripple in the deformation (in the curvature) of *the medium* that propagate as a wave [1].

Postulates

The *m(GR)* theory based on the following postulates:

1. The homogeneous, isotropic and independent of the time *the bare medium* with the bare mass density ρ^{bare}, embedded in the Minkowski space-time, is defined by the bare mass density tensor $\rho_{\mu\nu}^{bare}$, where: $\rho_{\mu\nu}^{bare} = \rho^{bare} \cdot \eta_{\mu\nu} = \mathrm{diag}(-\rho^{bare}, \rho^{bare}, \rho^{bare}, \rho^{bare})$, $\eta_{\mu\nu}$ is the Minkowski tensor, μ, ν = 0, 1, 2, 3.
2. Under the influence of the outer gravitational field *the bare medium* changes and becomes *medium* with the effective mass density tensor $\rho_{\mu\nu}$. This medium is characterized by a deformation (by a curvature).
3. The metric of the *medium* is defined by the formula $ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho^{bare}} dx^\mu dx^\nu$.

4. The metric $ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho^{bare}} dx^\mu dx^\nu$ has the same properties as the metric $ds^2(\rho_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu$, if we assume that $g_{\mu\nu} = \frac{\rho_{\mu\nu}}{\rho^{bare}}$.
5. The deformation (the curvature) of *the medium* depends on the stress–energy tensor $T_{\mu\nu}$.
6. In absence of the outer gravitational field, i.e. $\rho_{\mu\nu} = \rho_{\mu\nu}^{bare}$ metric $ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho^{bare}} dx^\mu dx^\nu$ has form of the Minkowski metric $ds^2(\rho_{\mu\nu}^{bare}) = \eta_{\mu\nu} dx^\mu dx^\nu$.

The equations of motion in curved *medium*

Let us consider the Lagrangian function

$$L = \rho_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \tag{1}$$

The equation of motion expressed by $\rho_{\mu\nu}$ has the form

$$\frac{dp_\gamma}{d\tau} - \frac{1}{2} \frac{\partial \rho_{\mu\nu}}{\partial x^\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \tag{2}$$

where: $p_\gamma = \rho_{\gamma\nu} \frac{dx^\nu}{d\tau}$ is the density of the four-momentum. If the four-gradient of the effective mass density tensor vanish i.e. $\frac{\partial \rho_{\mu\nu}}{\partial x^\gamma} = 0$ then $\frac{dp_\gamma}{d\tau} = 0$ and finally $p_\gamma = const$. Equation (2) has the also different equivalent form

$$\rho_{\gamma\nu} \frac{d^2 x^\nu}{d\tau^2} + \Gamma_{\gamma\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \tag{3}$$

where: $\Gamma_{\gamma\mu\nu} \equiv \frac{1}{2} \left(\frac{\partial \rho_{\gamma\mu}}{\partial x^\nu} + \frac{\partial \rho_{\gamma\nu}}{\partial x^\mu} - \frac{\partial \rho_{\mu\nu}}{\partial x^\gamma} \right)$ is the Christoffel symbols of the first kind.

The equations of motion in the weak gravitational field

How are these equations connected with Newton's equations of motion? In the weak gravitational field we can decompose $\rho_{\mu\nu}$ in to following simple form $\rho_{\mu\nu} = \rho_{\mu\nu}^{bare} + \overset{*}{\rho}_{\mu\nu}$, where $\overset{*}{\rho}_{\mu\nu} << 1$ is a small perturbation. Now we assume that: the velocity of the material particles be very small compared to the speed of light, the gravitational field varies so little with the time that the derivatives of the $\overset{*}{\rho}_{\gamma\mu}$ by x_4 may be neglected and we additionally assume also $\overset{*}{\rho}_{\gamma\nu} \frac{d^2 x^\nu}{d\tau^2} \approx 0$. Finally

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial}{\partial x^i} \left(\frac{1}{2} c^2 \frac{\overset{*}{\rho}_{00}}{\rho^{bare}} \right) \tag{4}$$

where i = 1, 2, 3. Equation (4) has the also different equivalent form, ($c = const$)

$$\rho^{bare} \frac{d^2 x^i}{dt^2} = -\frac{1}{2} c^2 \frac{\partial \overset{*}{\rho}_{00}}{\partial x^i} \tag{5}$$

Equation (5) is equivalent to Newton's second law in *m(GR)* theory. We can see that this equation is different than the classical Newton's equation for the gravity and the principle of equivalence does not make sense.

If $\frac{\partial \overset{*}{\rho}_{00}}{\partial x^i} = 0$ then $\rho^{bare} \frac{d^2 x^i}{dt^2} = 0$ ($\rho^{bare} \neq 0$). We can see that *the bare medium* can mimic the inertial frame of reference. The new quality of the understanding, kept in the Mach's spirit, was reached.

If we assume in the equation (4) that $\frac{1}{2} c^2 \frac{\overset{*}{\rho}_{00}}{\rho^{bare}} = V$, then we get the simple relationship between $\frac{\overset{*}{\rho}_{00}}{\rho^{bare}}$ and the gravitational potential V and the equation of motion has well know form

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial V}{\partial x^i} \tag{6}$$

In the equation (6) the principle of equivalence is satisfy.

The field equation

The Einstein – Hilbert action for the *m(GR)* theory has form

$$S = \int \left(\frac{c^4}{16\pi G} R(\rho_{\mu\nu}) + L_m(\rho_{\mu\nu}) \right) \sqrt{-\det\left(\frac{\rho_{\mu\nu}}{\rho^{bare}} \right)} \cdot d^4 x \tag{7}$$

where: $R(\rho_{\mu\nu})$ is the Ricci scalar expressed by the $\rho_{\mu\nu}$, G is the Newton's gravitational constant, c is the speed of light in the *medium*, $L_m(\rho_{\mu\nu})$ describing any matter fields, $\det(\rho_{\mu\nu})$ is the determinant of the effective mass density tensor.

The Einstein field equation for the *m(GR)* theory expressed by the $\rho_{\mu\nu}$ has the form

$$R_{\mu\nu}(\rho_{\mu\nu}) - \frac{1}{2} \frac{\rho_{\mu\nu}}{\rho^{bare}} \cdot R(\rho_{\mu\nu}) = \frac{8\pi G}{c^4} \cdot T_{\mu\nu}(\rho_{\mu\nu}) \tag{8}$$

The left side of the equation (8) represents the deformation of *the medium* (the curvature of *the medium*) expressed by the $\rho_{\mu\nu}$. The right side of the equation (8) represents the distribution of the matter and energy expressed by the $T_{\mu\nu}(\rho_{\mu\nu})$.

In the weak gravitational field approximation the field equation has the form of the Poisson's equation

$$\nabla^2 \left(\frac{\overset{*}{\rho}_{00}}{\rho^{bare}} \right) = \frac{8\pi G}{c^2} \rho \tag{9}$$

The *m(GR)* vs. GR

The *m(GR)* theory satisfied classical tests of the GR but their the physical interpretation is different, e.g. under the influence of the gravitational field only physical properties of the

- rods was changed, but not the space properties,
- clocks was changed, but not the time properties.

The *m(GR)* theory proposes a new look for the mass density, which is now the tensor, the propagation and detection of the gravitational waves. Theory predicts the mass anisotropy [2].

Table below includes comparison of both theories.

GR	***m(GR)***
Space-time with metric tensor $g_{\mu\nu}$	*Medium with effective mass density tensor* $\rho_{\mu\nu}$
Metric $ds^2(g_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu$	*Metric* $ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho^{bare}} dx^\mu dx^\nu$
Equation of motion $\frac{d^2 x^\gamma}{d\tau^2} + \Gamma^\gamma_{\mu\nu}(g_{\mu\nu}) \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$ where: $\Gamma^\gamma_{\mu\nu}(g_{\mu\nu})$ is the Christoffel symbols of the second kind expressed by $g_{\mu\nu}$	*Equation of motion* $\rho_{\gamma\nu}\frac{d^2 x^\nu}{d\tau^2} + \Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}) \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$ where: $\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu})$ the Christoffel symbols of the first kind expressed by $\rho_{\mu\nu}$
Equation of field $R_{\mu\nu}(g_{\mu\nu}) - \frac{1}{2} g_{\mu\nu} \cdot R(g_{\mu\nu}) = \frac{8\pi G}{c^4} \cdot T_{\mu\nu}(g_{\mu\nu})$	*Equation of field* $R_{\mu\nu}(\rho_{\mu\nu}) - \frac{1}{2}\frac{\rho_{\mu\nu}}{\rho^{bare}} \cdot R(\rho_{\mu\nu}) = \frac{8\pi G}{c^4} \cdot T_{\mu\nu}(\rho_{\mu\nu})$

Conclusion

In this paper it was proposed a new approach to gravitation. The *m(GR)* theory is the new theory of gravitation, where the space-time (with metric tensor $g_{\mu\nu}$) was eliminated and replaced with *the medium* (with the effective mass density tensor $\rho_{\mu\nu}$).

It is a new paradigm in the research of the gravitational phenomena, which opens a new way for research the missing mass in the Universe.

References

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