Title: The Structure of Space and the Nature of Elementary Particles

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Abstract:

Can General Relativity and Quantum Mechanics emerge from a model where the Universe is represented as a four dimensional elastic media comprised of energy density and particles are travelling wave and standing wave solutions to a second order hyperbolic partial differential equation that provides shear wave, compression wave, and surface wave solutions to the differential equation that evolve in time? Quantum Mechanics and General Relativity are the two principle theories of modern physics and both work extremely well in its realm of use. However, the theories appear to be incompatible. String Theory and Quantum Loop Gravity have been proposed as means of unifying Quantum Mechanics and General Relativity, but neither has been successful at recreating the results of both theories or of providing new predictions. The model exactly reproduces the geodesic paths of General Relativity, explains how forces work, provides a framework for Quantum Mechanics, and makes useful predictions.

The Structure of Space

In our normal day to day experience, we observe space-time to be comprised of three space dimensions and one time dimension. We also observe objects in space, where these objects comprise massless gauge bosons and massive particles. Additionally, every object in space is comprised of energy ‘E’, where massive objects have both rest energy (also known as internal energy) and momentum energy (also known as kinetic energy).[1, 2] Accordingly, each object in space has an energy density ‘P_E’ associated with that object for each point in space, where the energy density at each point X in space is approximately equal to the object’s energy divided by the distance squared ‘$E/r(X)^2$’ from the object to the point X. The total energy density at a point in space is equal to the sum of the energy densities for all objects at that point in space and the energy density changes as a function of time depending upon how the distribution of energy changes as a function of time, where the magnitudes of energy, distance, and time are also dependent upon the observer’s reference frame.

$$P_E(X, t) \approx E/r(X, t)^2. \qquad (1).$$

$$P_{E(tot)}(X, t) = \sum P_E(X, t) \qquad (2).$$

Accordingly, each point of space has a total energy density associated with that point, where the total energy density can change as a function of time. We can consider energy density to provide a fourth space dimension such that space is considered to comprise a four dimensional shell with the three traditional space dimensions being the surface of the shell and energy density providing the thickness of the Shell. It is likely, but not necessary, that the shell is the four dimensional analog to a sphere, where the sphere would have a radius and the thickness of the shell would be measured parallel to the radius and the space dimensions would be perpendicular to the radius.

Accordingly, the Universe can be thought of as being analogous to a four dimensional balloon, where the magnitude of energy density is equivalent to the thickness of the rubber of the balloon, such that the thickness of the 4D elastic shell that comprises the Universe at a point in space is the sum of the energy densities of all particles at that point in space.[3, 4] However, the four dimensional elastic shell of energy density does not have an absolute reference frame, since it comprises energy density from all objects in the Universe, where changes in energy density propagate through the universe at the speed of light.

The metric tensor is a is a 4X4 matrix G that is used to determine distance in Special Relativity and General Relativity and that provides the inner product.[2] In Special Relativity, only the diagonal elements of the metric tensor are non-zero, where the diagonal elements are 1, 1, 1, 1 and where the first element is contravariant such that it provides a negative one in the inner product. However, in General Relativity, all 16 elements of the metric tensor can be non-zero, unless limiting restrictions are placed on the mass distribution or on the intensity of the gravity field.[1-4]

The value of the matrix elements of the metric tensor in General Relativity are determined by the energy density in the Universe in accordance with the appropriate solution to the equations of General Relativity. [1-4] For analysis purposes, simplified mass distributions are often used. In weak gravity fields, the metric tenser can be approximated by applying a metric tensor having non-zero elements only along the diagonal.[3, 4] Further, by representing each of the diagonal elements of the tensor as the infinite sum of analytic basis functions, you can diagonalize the matrix by reducing the matrix to a matrix that has only non-zero elements on the diagonal, which is done by applying the appropriate transforms.[2] For the remainder of this paper we will assume that the metric tensor has been diagonalized.

The G_{00} element of the General Relativity metric tensor is the element in the upper left hand corner of the matrix and it is contravariant such that it has a negative sign applied to it to provide the inner product. The G_{00} element of the metric tensor is a function of the energy density, where the G_{00} element equals 1 at a location infinitely far from a source of energy and where it has a value of zero at the surface of a black hole. The weak field approximation for the G_{00} element of the metric tensor is just the first two elements of the infinite sum that defines the G_{00} element for all cases, where the weak field approximation for a symmetric mass distribution is given by the following equation.

$G_{00}(r) = 1 + - 2GM/rc^2$.[3, 4] (3)

'G' is the gravitational constant; 'M' is the mass; 'r' is the distance from X_r to X_0; and 'c' is the speed of light. The error of the approximation can be made as small as desired by adding additional terms to the infinite sum that equals the G_{00} element.

The other three diagonal elements of the metric tensor describe the asymmetry of metric tensor and the asymmetry of the corresponding inner products, where the asymmetry is caused by the asymmetry of the energy distribution in space. Accordingly, the G_{11}, G_{22}, and G_{33} elements of the metric tensor correspond to the gradient of the energy density, where the G_{11} element corresponds to the x dimension, the G_{22} term corresponds to the y dimension, and the G_{33} term corresponds to the z dimension. If the energy density distribution is asymmetrical, the values of the elements G_{11}, G_{22}, and G_{33} will each be different from 1. However, the sum of G_{11}, G_{22}, and G_{33} at any point on the surface of will always be 3. [3, 4]

Additionally, constant values of the G_{00} element of the metric tensor provide 3D contours of the G_{00} element on the 3D surface of space in the same way that constant values of atmospheric pressure or constant values of elevation can provide contours of constant atmospheric pressure or of constant elevation, which be drawn on a globe or a map to provide 2D circle like structures that have tangent directions and perpendicular directions. The direction along the 3D surface of space that is perpendicular (normal) to a contour of the G_{00} element of the metric tensor at a point on the surface of space is the direction ($\sqrt{G_{11}}$, $\sqrt{G_{22}}$, $\sqrt{G_{33}}$) for that point on the surface.

We can consider space-time to be a four dimensional shell where the thickness of the shell varies with time based upon how the energy distribution in the Universe varies with time as viewed by the observer. Accordingly, contours of equal magnitude of the thickness of the Universe are the same contours as the contours of equal value of the G_{00} element, where the thickness of the Universe increases as the magnitude of $1/\sqrt{G_{00}}$ element of the metric tensor increases. The thickness of the Universe will evolve in time as the G_{00} element correspondingly evolves in time.

Additionally, the solutions to the General Relativity equations require that the Universe is either expanding or contracting in time.[1, 2] The G_{00} element of the metric tensor and the thickness of the Universe will vary as a function of time depending upon the evolution of energy in time and depending upon on how the Universe is expanding or contracting in time. The expansion of the

Universe would decrease the energy density (thickness) and increase the magnitude of the G_{00} element for each point in space, while contraction of the Universe would increase the energy density of the shell and decrease the magnitude of the G_{00} element.

For objects that have the same velocity as the observer, the rate of time experienced by the object at a point in space is proportional to the $\sqrt{G_{00}}$ element of the General Relativity metric tensor at the point.[3, 4] Accordingly, the observed rate of time experienced by an object at a point in space decreases as the total energy density increases at that point in space, where the thickness of space at a point in space is proportional to the total energy density at that point. Since space is expanding or contracting in time, we can consider time to correspond to the radius of the 4D sphere analog that comprises the Universe, where the radius of the 4D sphere associated with areas of the 3D surface of space having high energy density would increase or decrease at a lower rate than the areas of the surface having lower energy density since the areas having higher energy density have lower rates of time associated with them.

The four dimensional space-time of traditional relativity equates to the 3D surface of our four D shell plus the radius of our 4D sphere, where then rate of time at a point on the 3D surface equates the rate of change of the radius of the 4D sphere. Accordingly, all objects in space are moving through the time dimension as postulated by traditional General relativity, where an objects motion through 4D space time is a combination of its motion across the 3D surface plus its motion in time, which is its motion parallel to the radius. If there was no energy density at a point in space, then the radius would increase or decrease at its maximum rate, which is the rate of time in a Minkowski space. If there is energy density, an object’s velocity through the time dimension will decrease and its velocity through the space dimension will correspondingly increase. However, if no force is applied to an object, it will follow a geodesic path through the curved space time that is the shortest path through space-time provided by the total distance across the 3 surface dimensions and the time dimension parallel to the radius.[2, 3, 4] Thus, a path through the surface of the 4D shell is the same curved path through space-time described in traditional General relativity, since the 3D surface is moving in time.

In Special Relativity, an object S can be described by two state vectors, the energy momentum vector **P** and the position vector **X,** where **S** = (**P; X**), where the Special Relativity inner products of **P** and **X** with themselves is shown below:

$$\mathbf{P} \cdot \mathbf{P} = -E_{rest}^2/C^2 + P_x^2 + P_y^2 + P_z^2 = E_{total}^2/c^2 \qquad (4);$$

$$\mathbf{X} \cdot \mathbf{X} = -(CT)^2 + x^2 + y^2 + z^2 = D^2 \qquad (5).$$

'E_{rest}' is the rest energy of the object, 'c' is the speed of light, 'P_x' is the momentum energy in the x direction, 'E_{total}' is the total energy including its rest energy and its momentum energy, 'T' is the rate of time in the reference frame. 'D' is the distance of the object S located at (T, x, y, z) from a reference point $\mathbf{X}_0$ = (0, 0, 0, 0).[2] The inner products of the state vectors will not change for an object provided that the G_{00} element of the metric tensor is constant.[2, 3, 4, 6, 7] However, if G_{00} element changes along an objects path, then the Special Relativity inner products of the state vectors with themselves will change.

However, the diagonalized version of the General Relativity metric tensor provides an inner product for the state vectors that remains constant, where the infinite sums of the G_{00} element is contravariant and the infinite sums for the infinite sums of analytic basis functions for the G_{11}, G_{22}, and G_{33} elements are covariant. Accordingly, the diagonalized version of the General Relativity metric tensor provides a constant inner product for the state vectors in General Relativity provided that no force has been applied to the objects. This allows Lorentz transforms to be used in General Relativity without changing the General Relativity inner products of the state vectors. The diagonalized General Relativity metric tensor provides the following inner products:

$$\mathbf{P} \cdot \mathbf{P} = - G_{00}E_{rest}^2/C^2 + G_{11}P_x^2 + G_{22}P_y^2 + G_{33}P_z^2 = E_{total}^2/C^2 \qquad (6);$$

$$\mathbf{X} \cdot \mathbf{X} = - G_{00}(CT)^2 + G_{11}x^2 + G_{22}y^2 + G_{33}z^2 = D^2 \qquad (7).$$

'E_{rest}' is the rest energy of the object in the observer's reference frame, 'c' is the speed of light in the observer's reference frame that is the well known constant, 'E_{total}' is the total energy including its rest energy and its momentum energy. 'D' is the distance of the object S located at (T, x, y, z) from a reference point $\mathbf{X}_0$ = (0, 0, 0, 0).[2, 3, 4] By looking at equations 6 and 7, it can be seen that momentum energy must increase to counter the decrease in rest energy as an object moves lower in a gravity well and that space distance must increase to counter the decrease in time distance as the object moves deeper in a gravity well. Hence, equations 6 and 7 describe the curved space of General relativity and they describe how energy, momentum, time, and distance vary for objects moving the four dimensional shell that describes space in time. However, the expansion or contraction of the Universe in time will change E_{total}^2/C^2 and D^2., which needs to be considered when observing distant objects like other galaxies.

The above equations also show that an object observed to be traveling through a high density area of space will be observed to have a lower rest energy, increased momentum energy, a reduced rate of time, and an increased rate of speed through space. Accordingly, the curved space-time described by the solutions to the General relativity equations can be seen to be a consequence of the difference in the magnitudes of energy density at different points in space that alter the diagonalized elements of the metric tensor. Hence, the magnitude of energy density at a location in space determines the relative extent to which an object is moving through time or space at that point as was shown by Einstein when he developed the General theory of Relativity.

The Nature of Elementary Particles

As explained above, space provides a four dimensional elastic medium that evolves in time. Three dimensional elastic mediums, such as the Earth's crust, are known to have compression waves (P waves), shear waves (S waves) and surface waves (Rayleigh waves and Love waves), where those waves are time dependent solutions to a second order hyperbolic partial differential equation.[2, 5] Accordingly, similar waves should exist in the four dimensional elastic medium that comprises space. Indeed, elementary particles are travelling wave and standing wave solutions to a second order hyperbolic partial differential equation, which is the same differential equation that applies to seismic waves in the Earth.

Photons are shear waves that oscillate in two of the normal space dimensions and travel in the third space direction. The differential equations and the wave function solutions and the wave properties for a photon can be seen to be exactly the same as those of a shear wave in the Earth. An electron is shown to be a standing wave comprising two photons, where the standing wave of the electron affects the space surrounding the electron and causes some of that space to become anisotropic (having different velocities in different directions).[2, 5] The location of an electron is at its central wave packet, which is where a photon would most likely interact with the standing wave. All gauge bosons are traveling waves, like photons. All massive particles are standing waves or linear combinations of standing waves, like electrons. The standing waves provide the particles with a rest energy that corresponds to all of the energy of the particle (standing wave) when it is at rest. Kinetic energy (momentum energy) is additional energy that the particle has in addition to its rest energy and it causes the standing wave as measured from the observer's wave to move with respect to the observer's standing wave and will correspond to the group velocity of the particle.

A group of seismic waves will split off a travelling wave, if the seismic wave encounters anisotropic space.[5] Similarly, when a charged massive particle (standing wave) encounters anisotropic space, the anisotropic space of the charged particle interacts with the anisotropic space and splits off a photon. This occurs when two electrons interact and exchange photons (traveling waves).

Savickas and Hilo have separately demonstrated that the speed of light in a vacuum is proportional to the square root of the G_{00} element of the metric tensor.[6, 7, 8] Hilo generalizes the Special Relativity gamma factor such that it can be applied in the presence of gravity in accordance with General Relativity, where the generalized gamma factor is given by the following equation.

$Y_{SpecialGeneral} = 1/ \sqrt{(G_{00} - (v^2/c^2))}$.[7, 8] (8)

Since the generalized gamma factor approaches infinity as v approaches $\sqrt{(G_{00})}c$, the speed of light in a vacuum must be proportional to $\sqrt{(G_{00})}$.

A gravity well is a volume of space where the total energy density increases as you approach the center of the volume. Accordingly, light will refract towards the normal in accordance with Snell's law when it travels into a gravity well, since the G_{00} element of the metric tensor gets smaller for greater energy density.[2, 9]

$N = c/v = c/\sqrt{(G_{00})}c = 1/\sqrt{(G_{00})}$ (9).[2,6,7,8]

$N_1 Sin(Ø_1) = N_2 Sin(Ø_2)$ (10)[2,9]

$Ø_2 = Ø_1(invsin(\sqrt{(G_{00}(X_2)}/ \sqrt{(G_{00}(X_1)})))$. (11)[2,6,7,8,9]

'N' is the index of refraction in a vacuum; 'c' is the speed of light in Minkowski space measured at the rate of time in Minkowski space, where the speed of light will always be measured at c for light traveling in the same G_{00} level as the observer; 'Ø' is the angle with respect to the normal, where the normal is provided by the gradient of energy density; 'X' is the location of the object on the 3D surface of space at the appropriate time.

The refraction of a standing wave is determined by the refraction of travelling waves it is comprised of, since the phase velocity of the standing waves is the speed of traveling waves of which they are comprised (the speed light).[2] Accordingly, all massive objects will refract at exactly the same angle 'Ø' as light for the same location in space-time in accordance with equation 11. The direction of the normal at a point in space to which the angle Ø is referenced is determined by the G11 element, the G22 element, and the G33 element of the metric tensor at that point, since those matrix elements represent the gradient of energy density, which is perpendicular to the contour of the G_{00} element of the metric tensor and which is perpendicular to the contour of energy density at that point. On the Earth, the normal is approximately perpendicular to the surface of the Earth.

Although the phase velocity of massive objects decreases as the gauge boson velocity decrease as the objects go deeper into a gravity well, the group velocity of massive objects (standing waves) typically increases as the particles go deeper into a gravity well. When a massive particle goes deeper into a gravity well (an area of high energy density), the particle's rest energy decreases proportional to the decrease in $\sqrt{(G_{00})}$, which means that the particle needs less energy to exist as a standing wave when it is deeper in the gravity well then when it is higher up in the gravity well.[3, 4] The kinetic energy of the massive object must increase by the amount that the rest energy decreases as the massive object goes deeper into the gravity well to conserve energy as observed from any specific reference frame.[3, 4] Accordingly, the velocity of the massive object must increase proportional to the change in the in G_{00}. However, photons will get slower when they go deeper into a gravity well, since the G_{00} element of the metric tensor gets closer to zero.[7, 9] Accordingly, the gamma factor will eventually cause the speed of massive particles to decrease as they get deeper in a gravity well, since the standing wave velocity can never exceed the travelling wave velocity

Gravity is provided by the refraction of the standing waves and traveling waves, since refraction changes the direction of travel across the 3D surface of both the traveling waves and the standing waves in the same way, since the change in the G_{00} element, and since the G11 element, the G22 element, and the G33 element of the metric tensor determine the normal, which is perpendicular to the tangent of energy density.

However, the $\sqrt{(G_{00})}$ does not provide an index of refraction in a vacuum that has sharp edges like the typical boundary between transparent condensed matter. Accordingly, gravity is provided by a gradually reducing index of refraction, since G_{00} is a continuous function. However, Maxwell has shown that a gradually reducing index of refraction can provide circular or elliptical orbits.[9] Accordingly, the refraction of all objects in a vacuum provides gravity identical to traditional General Relativity, since the paths of the particles are determined by all of the information contained in the metric tensor that represents the specific solution to the General Relativity equations for the applicable mass distribution in time. Since both the refraction path and traditional GR geodesic path are provided by the identical information using the same solution to the General Relativity equations, both paths must be identical.

Standing waves made of compression traveling waves or shear traveling waves can reproduce all of the observable characteristics of massive elementary particles, such as spin and charge and quantum behavior. Gluons and photons are traveling shear waves, while quarks and electrons are standing waves made of two traveling shear waves. A photon involves oscillation between two space dimensions that correspond to the B and E fields, while all versions of quarks and gluons involves oscillation in a space dimension and the energy density dimension, including the quarks that make protons and neutrons.

Charged particles are standing waves that affect the distribution of energy density in space and which cause space to have different velocity in different directions, where space having different velocities in different directions is known as "anisotropic space." Forces are a consequence of wave splitting or wave combining in anisotropic space, which is a phenomenon observed in 3D seismic waves.[5] The anisotropic space caused by two charged separate charged particles causes the charged particles to exchange photons when the anisotropic space caused by two distinct charged particles (charged standing waves) intersects.

Neutrinos are compression waves and can have velocities greater than C, since compression waves always travel faster than shear waves in elastic media.[5] In the 3D elastic medium of the of the Earth, the travelling P waves travel about 1.7 times faster than the travelling S waves, where the 1.7 ratio applies for all media.[5] However, the velocity of neutrino gauge bosons is also proportional to G_{00} element of the metric tensor. Accordingly, the refraction of neutrinos and neutrino gauge bosons will be the same as the refraction of photons and electrons, which is provided by Snell's law as shown in equations 9-11.

Anti-matter is a massive particle having a traveling wave component 180 degree out of phase with a traveling wave component of the corresponding matter particle, such that the combination of the two standing waves' out of phase component travelling waves would free the remaining traveling wave from each particle after the two out of phase traveling waves cancelled each other out.

Discusion

The observed characteristics of elementary particles can all be represented by the characteristics of travelling waves or of standing waves. Indeed, we know that photon waves can have spin and angular momentum and the additional degrees of freedom provided by shear and compression waves can represent all known elementary particle characteristics, where some of the more exotic particles could correspond to surface waves and their linear combination with shear and compression waves.[2, 6, 7] Energy between compression waves and shear waves and Surface waves can couple amongst them as determined by the differential equations, but the combinations are not always stable. [5] Further, we can expect energy density waves to have quantum behavior because they exist as eigenvectors (Eigen wave functions) of the system and thus only have specific quantums of value and because the uncertainty relation will naturally hold and because the travelling and standing waves are objects that extend through space.[2,6] Accordingly, the probabilities of interactions of a photon with a massive particle will obey the statistical probabilities of Quantum Mechanics or Quantum Field theory.

In elastic media, a wave will split into two waves when it encounters anisotropic media, where anisotropic media is media where the wave velocity varies according to direction. [5] Anisotropic media can also provide wave combining. [5] Charged particles (charged standing waves) create anisotropic space, unless they are balanced out by an equal but opposite charge. Accordingly, neutral particles do not have anisotropic space except in the close vicinity of the charged particle. When the anisotropic space of two standing waves intersect, both standing waves emit traveling waves (i.e. photons or gluons) that are absorbed by the other standing wave. However, such photons likely couple directly form one particle (standing wave) to the other so that no free standing photon ever exists when a force is applied by a charge. When a photon interacts with the anisotropic space near a particle, the velocity difference between the particle’s component travelling wave and the incident travelling wave cause the waves to combine.

Although the four dimensional shell that comprises space does not have an absolute reference frame, frame dragging will be caused if a large mass, such as a black hole, is rotating, since the energy of the large mass over its radius squared will dominate the energy density in its local area. This frame dragging imposes a rotation into the local elastic shell near the large rotating mass. The rotation of the frame will likely cause a close orbiting star to acquire large additional velocity and kinetic energy as it goes through the frame dragging area. The relative amount of rotation of space (the energy density shell) will increase as you approach the boundary of the black hole and the rotation of the energy density shell will provide velocity and energy to the star (standing wave) in a manner similar to what would happen to a sound wave travelling through an ocean current.

When a massive particle (standing wave) is moved upwards in a gravity well, the rest energy and physical size of the standing wave varies in exactly the same way that is predicted for general relativity.[3] The increase in rest energy is represented by a greater peak to peak value of the standing wave and by a greater frequency of the component traveling waves and the decrease in physical size or radius is represented by a wave packet that is more tightly bound and has a shorter wavelength. However, the photon energy does not change when the photon moves up in the gravity well. Accordingly, Lev Okun’s statement "the phenomenon known as the red shift of a photon is the blue shift of an atom" applies to the standing wave solutions (atoms) and travelling wave solutions (photons) moving up in a gravity well.[3, 4]

Space is shown to exist as a four dimension elastic shell of energy density comprised of the energy densities of each object in the Universe. Time is shown to correspond to the radius of the 4D sphere analog. Elementary particles are travelling wave and standing wave solutions to a second order hyperbolic partial differential equation for a four dimensional elastic medium. This partial differential equation is the same equation as the partial differential equation for seismic waves in the Earth, except that it includes an additional dimension. Forces are created by anisotropic space, which causes a standing wave to emit travelling waves and to absorb travelling waves. General Relativity and Quantum mechanics emerge from this model. Gluons and photons are traveling shear waves, while quarks and electrons are standing waves made of two traveling shear waves and which also include a standing compression wave (neutrino). W bosons are traveling surface waves and Z bosons are standing waves comprised of W bosons. Gravity is a type of refraction, where the velocity of a traveling wave is proportional to the $\sqrt{G_{00}}$ element of the metric tensor, where the angle of an object with respect to the normal of a gravity well will be given by equation 11.

The proposed model unifies Quantum Mechanics and General Relativity, provides gravity identical to traditional General relativity, it explains how forces work, it shows how Quantum Mechanics emerges, and it allows numerical methods for solving differential equations to predict masses and radioactive decay and it will allow for easier calculations of orbits.

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