The Dirac Equation
is a Special Case of
the Maxwell-Cassano Equations

For vector ψ_D The Klein-Gordon equation may be written [4]:

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\psi_D = \mathbf{0} \ .$$

Whenever $\psi_D \equiv \begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix}$ is a 2^M-dimensional vector, via a matrix differential operator factorization, it may be written (in the Dirac representation):

$$\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix} = \mathbf{0}$$

and, since these matrix operators are commutative:

$$\begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix} = \mathbf{0}$$

Where:

$$\boldsymbol{\sigma}\bullet\vec{\nabla} = \sum_{\nu=1}^{3}\boldsymbol{\sigma}^\nu\frac{\partial}{\partial x^\nu}$$

$$\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma}_0 \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{\sigma}_0 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_0 & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{\sigma}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}\partial_2 \Leftrightarrow \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_1 & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{\sigma}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma}_1 \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{\sigma}_1 & \mathbf{0} \end{pmatrix}\partial_3$$

$$\boldsymbol{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}_2$$

$$\boldsymbol{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \boldsymbol{\sigma}\bullet\vec{\nabla} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\partial_1 + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\partial_2 + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3$$

$$= \begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix}$$

$$\Rightarrow \left(\boldsymbol{\sigma}\bullet\vec{\nabla}\right)^2 = \begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix}\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} = (\partial_3^2 + \partial_1^2 + \partial_2^2)\mathbf{I}_2 = \nabla^2\mathbf{I}_2$$

Let:

$$\begin{pmatrix} \psi_D^A \\ \psi_D^B \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix}$$

$$\begin{pmatrix} \psi_D^C \\ \psi_D^D \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix}$$

then:

$$\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \psi_D^A \\ \psi_D^B \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} \mathrm{I}_2\left(-i\frac{\partial}{\partial t}+m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathrm{I}_2\left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \psi_D^C \\ \psi_D^D \end{pmatrix} = \mathbf{0}$$

conformability of the matrices requires that:

ϕ_D^A . ϕ_D^B ; ψ_D^A . ψ_D^B .ψ_D^C .ψ_D^D are all 2×1 matrices; so setting:

Let:

$\phi_D^A = \begin{pmatrix} \phi_D^0 \\ \phi_D^1 \end{pmatrix}$	$\phi_D^B = \begin{pmatrix} \phi_D^2 \\ \phi_D^3 \end{pmatrix}$

$\boldsymbol{\psi}_D^A = \begin{pmatrix} \psi_D^0 \\ \psi_D^1 \end{pmatrix}$	$\boldsymbol{\psi}_D^B = \begin{pmatrix} \psi_D^2 \\ \psi_D^3 \end{pmatrix}$	$\boldsymbol{\psi}_D^C = \begin{pmatrix} \psi_D^4 \\ \psi_D^5 \end{pmatrix}$	$\boldsymbol{\psi}_D^D = \begin{pmatrix} \psi_D^6 \\ \psi_D^7 \end{pmatrix}$

then:

$$\begin{pmatrix} \begin{pmatrix} \left(i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} \left(-i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \psi_D^0 \\ \psi_D^1 \end{pmatrix} \\ \begin{pmatrix} \psi_D^2 \\ \psi_D^3 \end{pmatrix} \end{pmatrix} = \mathbf{0}$$

$$= -\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\boldsymbol{\psi}_D \ .$$

$$\begin{pmatrix} \begin{pmatrix} \left(-i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} \left(i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \psi_D^4 \\ \psi_D^5 \end{pmatrix} \\ \begin{pmatrix} \psi_D^6 \\ \psi_D^7 \end{pmatrix} \end{pmatrix} = \mathbf{0}$$

$$= -\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\begin{pmatrix} \boldsymbol{\psi}_D^C \\ \boldsymbol{\psi}_D^D \end{pmatrix} \ .$$

For symmetry purposes, let:

$t \equiv ix^0$

then, combining into a single matrix equation,

Let:

$$\boldsymbol{\theta} = \begin{pmatrix} \begin{pmatrix} \theta_D^0 \\ \theta_D^1 \end{pmatrix} \\ \begin{pmatrix} \theta_D^2 \\ \theta_D^3 \end{pmatrix} \\ \begin{pmatrix} \theta_D^4 \\ \theta_D^5 \end{pmatrix} \\ \begin{pmatrix} \theta_D^6 \\ \theta_D^7 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \psi_D^0 \\ \psi_D^1 \end{pmatrix} \\ \begin{pmatrix} \psi_D^2 \\ \psi_D^3 \end{pmatrix} \\ \begin{pmatrix} \psi_D^4 \\ \psi_D^5 \end{pmatrix} \\ \begin{pmatrix} \psi_D^6 \\ \psi_D^7 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \theta_D^0 \\ \theta_D^1 \end{pmatrix} \\ \begin{pmatrix} \theta_D^2 \\ \theta_D^3 \end{pmatrix} \\ \begin{pmatrix} \theta_D^4 \\ \theta_D^5 \end{pmatrix} \\ \begin{pmatrix} \theta_D^6 \\ \theta_D^7 \end{pmatrix} \end{pmatrix} =$$

$$= -(\Box - m^2)\boldsymbol{\theta} = \mathbf{0} \ .$$

Now,

Just as there are a number of representations of th Dirac equation, there is more than one matrix operator factorization of the Maxwell-Cassano equations [3].
One such, is:

$$\begin{pmatrix} \mathbf{H}^1 \\ \mathbf{H}^2 \\ \mathbf{H}^3 \\ \mathbf{H}^4 \\ \mathbf{H}^5 \\ \mathbf{H}^6 \\ \mathbf{H}^7 \\ \mathbf{H}^8 \end{pmatrix} = \begin{pmatrix} (\partial_1 - m_1) & (\partial_2 - m_2) & (\partial_3 - m_3) & (\partial_4 - m_4) & 0 & 0 & 0 & 0 \\ -(\partial_2 + m_2) & (\partial_1 + m_1) & 0 & 0 & 0 & 0 & (\partial_4 - m_4) & -(\partial_3 - m_3) \\ (\partial_3 + m_3) & 0 & -(\partial_1 + m_1) & 0 & 0 & (\partial_4 - m_4) & 0 & -(\partial_2 - m_2) \\ (\partial_4 + m_4) & 0 & 0 & -(\partial_1 + m_1) & 0 & -(\partial_3 - m_3) & (\partial_2 - m_2) & 0 \\ 0 & 0 & 0 & 0 & (\partial_1 + m_1) & (\partial_2 + m_2) & (\partial_3 + m_3) & (\partial_4 + m_4) \\ 0 & 0 & (\partial_4 + m_4) & -(\partial_3 + m_3) & -(\partial_2 - m_2) & (\partial_1 - m_1) & 0 & 0 \\ 0 & (\partial_4 + m_4) & 0 & -(\partial_2 + m_2) & (\partial_3 - m_3) & 0 & -(\partial_1 - m_1) & 0 \\ 0 & -(\partial_3 + m_3) & (\partial_2 + m_2) & 0 & (\partial_4 - m_4) & 0 & 0 & -(\partial_1 - m_1) \end{pmatrix} \begin{pmatrix} h^1 \\ h^2 \\ h^3 \\ h^4 \\ h^5 \\ h^6 \\ h^7 \\ h^8 \end{pmatrix}$$

Then:

$$\begin{pmatrix} (\partial_1 + m_1) & -(\partial_2 - m_2) & (\partial_3 - m_3) & (\partial_4 - m_4) & 0 & 0 & 0 & 0 \\ (\partial_2 + m_2) & (\partial_1 - m_1) & 0 & 0 & 0 & 0 & (\partial_4 - m_4) & -(\partial_3 - m_3) \\ (\partial_3 + m_3) & 0 & -(\partial_1 - m_1) & 0 & 0 & (\partial_4 - m_4) & 0 & (\partial_2 - m_2) \\ (\partial_4 + m_4) & 0 & 0 & -(\partial_1 - m_1) & 0 & -(\partial_3 - m_3) & -(\partial_2 - m_2) & 0 \\ 0 & 0 & 0 & 0 & (\partial_1 - m_1) & -(\partial_2 + m_2) & (\partial_3 + m_3) & (\partial_4 + m_4) \\ 0 & 0 & (\partial_4 + m_4) & -(\partial_3 + m_3) & (\partial_2 - m_2) & (\partial_1 + m_1) & 0 & 0 \\ 0 & (\partial_4 + m_4) & 0 & (\partial_2 + m_2) & (\partial_3 - m_3) & 0 & -(\partial_1 + m_1) & 0 \\ 0 & -(\partial_3 + m_3) & -(\partial_2 + m_2) & 0 & (\partial_4 - m_4) & 0 & 0 & -(\partial_1 + m_1) \end{pmatrix} \begin{pmatrix} \mathbf{H}^1 \\ \mathbf{H}^2 \\ \mathbf{H}^3 \\ \mathbf{H}^4 \\ \mathbf{H}^5 \\ \mathbf{H}^6 \\ \mathbf{H}^7 \\ \mathbf{H}^8 \end{pmatrix} =$$

$$\begin{pmatrix} (\Box - |m|^2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\Box - |m|^2) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (\Box - |m|^2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (\Box - |m|^2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (\Box - |m|^2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\Box - |m|^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (\Box - |m|^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (\Box - |m|^2) \end{pmatrix} \begin{pmatrix} h^1 \\ h^2 \\ h^3 \\ h^4 \\ h^5 \\ h^6 \\ h^7 \\ h^8 \end{pmatrix}$$

Another matrix operator factorization of the Maxwell-Cassano equations may be compactly written,
as follows [1][3]:
From the definitions:

$$\mathbf{f} \equiv \mathbf{w}^{4;i} f^i \quad , \mathit{where}: \quad f^i \equiv \begin{pmatrix} f^i_+ \\ f^i_- \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f^i_+ \\ f^i_- \end{pmatrix} \quad , \quad f^i_+, f^i_- \in R$$

$$D^+_i \equiv (\partial_i + m_i) \quad , \quad D^-_i \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D^+_i & 0 \\ 0 & D^-_i \end{pmatrix} \quad , \quad D^{\Updownarrow}_i \equiv \begin{pmatrix} D^-_i & 0 \\ 0 & D^+_i \end{pmatrix} \quad ,$$

$$D^{\Leftrightarrow}_i \equiv \begin{pmatrix} 0 & D^-_i \\ D^+_i & 0 \end{pmatrix} \quad , \quad D^{\Leftrightarrow\Updownarrow}_i \equiv \begin{pmatrix} 0 & D^+_i \\ D^-_i & 0 \end{pmatrix}$$

$$\widehat{\mathbf{f}} \equiv \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

such a factorization may be

$$\begin{pmatrix} D_0 & D^{\Leftrightarrow}_3 & -D^{\Leftrightarrow}_2 & D_1 \\ -D^{\Leftrightarrow}_3 & D_0 & D^{\Leftrightarrow}_1 & D_2 \\ D^{\Leftrightarrow}_2 & -D^{\Leftrightarrow}_1 & D_0 & D_3 \\ D^{\Updownarrow}_1 & D^{\Updownarrow}_2 & D^{\Updownarrow}_3 & -D^{\Updownarrow}_0 \end{pmatrix} \begin{pmatrix} D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_3 & D^{\Leftrightarrow}_2 & D_1 \\ D^{\Leftrightarrow}_3 & D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_1 & D_2 \\ -D^{\Leftrightarrow}_2 & D^{\Leftrightarrow}_1 & D^{\Updownarrow}_0 & D_3 \\ D^{\Updownarrow}_1 & D^{\Updownarrow}_2 & D^{\Updownarrow}_3 & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

$$= \begin{pmatrix} -D_0 & D^{\Leftrightarrow}_3 & -D^{\Leftrightarrow}_2 & -D_1 \\ -D^{\Leftrightarrow}_3 & -D_0 & D^{\Leftrightarrow}_1 & -D_2 \\ D^{\Leftrightarrow}_2 & -D^{\Leftrightarrow}_1 & -D_0 & -D_3 \\ -D^{\Updownarrow}_1 & -D^{\Updownarrow}_2 & -D^{\Updownarrow}_3 & D^{\Updownarrow}_0 \end{pmatrix} \begin{pmatrix} -D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_3 & D^{\Leftrightarrow}_2 & -D_1 \\ D^{\Leftrightarrow}_3 & -D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_1 & -D_2 \\ -D^{\Leftrightarrow}_2 & D^{\Leftrightarrow}_1 & -D^{\Updownarrow}_0 & -D_3 \\ -D^{\Updownarrow}_1 & -D^{\Updownarrow}_2 & -D^{\Updownarrow}_3 & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$$

$$
= \begin{pmatrix} D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\ D_3^{\Leftrightarrow} & D_0 & -D_1^{\Leftrightarrow} & D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0 & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\Updownarrow} & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\ -D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & D_1^{\Leftrightarrow} & D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\ D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

$$
= \begin{pmatrix} (\Box - |m|^2) f^1 \\ (\Box - |m|^2) f^2 \\ (\Box - |m|^2) f^3 \\ (\Box - |m|^2) f^0 \end{pmatrix} = (\Box - |m|^2)\,\widehat{\mathbf{f}}
$$

For the stationary state the source/sink density term vanishes in the Maxwell-Cassano equations, which allows an equating of the Maxwell-Cassano equation & Dirac equation factorizations.
These may imply correlations between the Dirac equation and the Maxwell-Cassano equations as the correspondences/mappings:

$m \Leftrightarrow |m|$ & $-\theta_D^j \Leftrightarrow f^h$.

The Dirac equation may be expanded with the above notation into:

$$
\begin{aligned}
&(-\partial_0 + m)\theta_D^0 - i\partial_3\theta_D^2 - i(\partial_1 - i\partial_2)\theta_D^3 = 0\\
&(-\partial_0 + m)\theta_D^1 - i(\partial_1 + i\partial_2)\theta_D^2 + i\partial_3\theta_D^3 = 0\\
&i\partial_3\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^1 + (\partial_0 + m)\theta_D^2 = 0\\
&i(\partial_1 + i\partial_2)\theta_D^0 - i\partial_3\theta_D^1 + (\partial_0 + m)\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^4 + i\partial_3\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^7 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 - i\partial_3\theta_D^7 = 0\\
&-i\partial_3\theta_D^4 - i(\partial_1 - i\partial_2)\theta_D^5 + (-\partial_0 + m)\theta_D^6 = 0\\
&-i(\partial_1 + i\partial_2)\theta_D^4 + i\partial_3\theta_D^5 + (-\partial_0 + m)\theta_D^7 = 0
\end{aligned}
$$

or:

$$
\begin{aligned}
&(-\partial_0 + m)\theta_D^0 - i\partial_3\theta_D^2 - i(\partial_1 - i\partial_2)\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^4 + i\partial_3\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^7 = 0\\
&(-\partial_0 + m)\theta_D^1 - i(\partial_1 + i\partial_2)\theta_D^2 + i\partial_3\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 - i\partial_3\theta_D^7 = 0\\
&i\partial_3\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^1 + (\partial_0 + m)\theta_D^2 = 0\\
&-i\partial_3\theta_D^4 - i(\partial_1 - i\partial_2)\theta_D^5 + (-\partial_0 + m)\theta_D^6 = 0\\
&i(\partial_1 + i\partial_2)\theta_D^0 - i\partial_3\theta_D^1 + (\partial_0 + m)\theta_D^3 = 0\\
&-i(\partial_1 + i\partial_2)\theta_D^4 + i\partial_3\theta_D^5 + (-\partial_0 + m)\theta_D^7 = 0
\end{aligned}
$$

As [1] shows. the component pairs may be organized such that this organization exhibits the mass-generalization of Maxwell's equations, but organizing them while comparing them analogously to the Dirac equations yields:

$$
\begin{aligned}
&(-\partial_0 + m)\theta_D^0 - i\partial_3\theta_D^2 - i(\partial_1 - i\partial_2)\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^4 + i\partial_3\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^7 = 0\\
&(\partial_0 + m_0)Z^0 + (\partial_3 - m_3)Z^5 - (\partial_2 - m_2)Z^6 + (\partial_1 + m_1)Z^3 = J^0\\
&(\partial_0 - m_0)Z^4 + (\partial_3 + m_3)Z^1 - (\partial_2 + m_2)Z^2 + (\partial_1 - m_1)Z^7 = J^1\\
&(-\partial_0 + m)\theta_D^1 - i(\partial_1 + i\partial_2)\theta_D^2 + i\partial_3\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 - i\partial_3\theta_D^7 = 0\\
&(\partial_0 + m_0)Z^1 + (\partial_1 - m_1)Z^6 + (\partial_2 + m_2)Z^3 - (\partial_3 - m_3)Z^4 = J^2\\
&(\partial_0 - m_0)Z^5 + (\partial_1 + m_1)Z^2 + (\partial_2 - m_2)Z^7 - (\partial_3 + m_3)Z^0 = J^3\\
&i\partial_3\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^1 + (\partial_0 + m)\theta_D^2 = 0\\
&-i\partial_3\theta_D^4 - i(\partial_1 - i\partial_2)\theta_D^5 + (-\partial_0 + m)\theta_D^6 = 0\\
&(\partial_3 + m_3)Z^3 - (\partial_1 - m_1)Z^5 + (\partial_0 + m_0)Z^2 + (\partial_2 - m_2)Z^4 = J^4\\
&(\partial_3 + m_3)Z^7 - (\partial_1 + m_1)Z^1 + (\partial_0 - m_0)Z^6 + (\partial_2 + m_2)Z^0 = J^5\\
&i(\partial_1 + i\partial_2)\theta_D^0 - i\partial_3\theta_D^1 + (\partial_0 + m)\theta_D^3 = 0\\
&-i(\partial_1 + i\partial_2)\theta_D^4 + i\partial_3\theta_D^5 + (-\partial_0 + m)\theta_D^7 = 0\\
&(\partial_1 - m_1)Z^0 + (\partial_3 - m_3)Z^2 + (\partial_2 - m_2)Z^1 - (\partial_0 - m_0)Z^3 = J^6\\
&(\partial_1 + m_1)Z^4 + (\partial_3 + m_3)Z^6 + (\partial_2 + m_2)Z^5 - (\partial_0 + m_0)Z^7 = J^7
\end{aligned}
$$

So:

$$
\begin{aligned}
&(\partial_0 - m)\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^3 + i\partial_3\theta_D^2 = 0\\
&(\partial_0 + m)\theta_D^4 + i(\partial_1 - i\partial_2)\theta_D^7 + i\partial_3\theta_D^6 = 0\\
&(\partial_0 + m_0)Z^0 + (\partial_1 + m_1)Z^3 - (\partial_2 - m_2)Z^6 + (\partial_3 - m_3)Z^5 = J^0\\
&(\partial_0 - m_0)Z^4 + (\partial_1 - m_1)Z^7 - (\partial_2 + m_2)Z^2 + (\partial_3 + m_3)Z^1 = J^1\\
&(\partial_0 - m)\theta_D^1 + i(\partial_1 + i\partial_2)\theta_D^2 - i\partial_3\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 - i\partial_3\theta_D^7 = 0\\
&(\partial_0 - m_0)Z^5 + (\partial_1 + m_1)Z^2 + (\partial_2 - m_2)Z^7 - (\partial_3 + m_3)Z^0 = J^3\\
&(\partial_0 + m_0)Z^1 + (\partial_1 - m_1)Z^6 + (\partial_2 + m_2)Z^3 - (\partial_3 - m_3)Z^4 = J^2\\
&(\partial_0 + m)\theta_D^2 + i(\partial_1 - i\partial_2)\theta_D^1 + i\partial_3\theta_D^0 = 0\\
&(\partial_0 - m)\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^5 + i\partial_3\theta_D^4 = 0\\
&(\partial_0 + m_0)Z^2 - (\partial_1 - m_1)Z^5 + (\partial_2 - m_2)Z^4 + (\partial_3 + m_3)Z^3 = J^4\\
&(\partial_0 - m_0)Z^6 - (\partial_1 + m_1)Z^1 + (\partial_2 + m_2)Z^0 + (\partial_3 + m_3)Z^7 = J^5\\
&(\partial_0 + m)\theta_D^3 + i(\partial_1 + i\partial_2)\theta_D^0 - i\partial_3\theta_D^1 = 0
\end{aligned}
$$

$$(\partial_0 - m)\theta_D^7 + i(\partial_1 + i\partial_2)\theta_D^4 \quad - i\partial_3\theta_D^5 \quad = 0$$
$$(\partial_0 + m_0)Z^7 - (\partial_1 + m_1)Z^4 - (\partial_2 + m_2)Z^5 \quad - (\partial_3 + m_3)Z^6 = J^7$$
$$(\partial_0 - m_0)Z^3 - (\partial_1 - m_1)Z^0 - (\partial_2 - m_2)Z^1 \quad - (\partial_3 - m_3)Z^2 = J^6$$

Continuing the comparison with the Maxwell-Cassano equations in the special case:

$m_0 \to -m, m_1 = m_2 = m_3 = 0$:

$$(\partial_0 - m)\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^3 \quad + i\partial_3\theta_D^2 \quad = 0$$
$$(\partial_0 + m)\theta_D^4 + i(\partial_1 - i\partial_2)\theta_D^7 \quad + i\partial_3\theta_D^6 \quad = 0$$
$$(\partial_0 - m)Z^0 \quad + \partial_1 Z^3 \quad - \partial_2 Z^6 \quad + \partial_3 Z^5 \quad = J^0$$
$$(\partial_0 + m)Z^4 \quad + \partial_1 Z^7 \quad - \partial_2 Z^2 \quad + \partial_3 Z^1 \quad = J^1$$
$$(\partial_0 - m)\theta_D^1 \quad + i(\partial_1 + i\partial_2)\theta_D^2 \quad - i\partial_3\theta_D^3 \quad = 0$$
$$(\partial_0 + m)\theta_D^5 \quad + i(\partial_1 + i\partial_2)\theta_D^6 \quad - i\partial_3\theta_D^7 \quad = 0$$
$$(\partial_0 - m)Z^1 \ + \ \partial_1 Z^6 \ + \partial_2 Z^3 - \partial_3 Z^4 = J^2$$
$$(\partial_0 + m)Z^5 \ + \ \partial_1 Z^2 \ + \partial_2 Z^7 - \partial_3 Z^0 = J^3$$
$$(\partial_0 + m)\theta_D^2 + i(\partial_1 - i\partial_2)\theta_D^1 \quad + i\partial_3\theta_D^0 \quad = 0$$
$$(\partial_0 - m)\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^5 \quad + i\partial_3\theta_D^4 \quad = 0$$
$$(\partial_0 - m)Z^2 - \partial_1 Z^5 + \partial_2 Z^4 + \partial_3 Z^3 \quad = J^4$$
$$(\partial_0 + m)Z^6 - \partial_1 Z^1 + \partial_2 Z^0 + \partial_3 Z^7 \quad = J^5$$
$$(\partial_0 + m)\theta_D^3 + i(\partial_1 + i\partial_2)\theta_D^0 \quad - i\partial_3\theta_D^1 \quad = 0$$
$$(\partial_0 - m)\theta_D^7 + i(\partial_1 + i\partial_2)\theta_D^4 \quad - i\partial_3\theta_D^5 \quad = 0$$
$$(\partial_0 - m)Z^7 - \partial_1 Z^4 - \partial_2 Z^5 \quad - \partial_3 Z^6 \ = J^7$$
$$(\partial_0 + m)Z^3 - \partial_1 Z^0 - \partial_2 Z^1 \quad - \partial_3 Z^2 \ = J^6$$

So, extending the Dirac equation beyond the source/sink free case (so looking beyond just eigenvalues and eigenvectors); and writing in matrix form, and comparing:

$$\begin{pmatrix} (\partial_0 - m) & 0 & 0 & 0 & i\partial_3 & 0 & i(\partial_1 - i\partial_2) & 0 \\ 0 & (\partial_0 + m) & 0 & 0 & 0 & i\partial_3 & 0 & i(\partial_1 - i\partial_2) \\ 0 & 0 & (\partial_0 - m) & 0 & i(\partial_1 + i\partial_2) & 0 & -i\partial_3 & 0 \\ 0 & 0 & 0 & (\partial_0 + m) & 0 & i(\partial_1 + i\partial_2) & 0 & -i\partial_3 \\ i\partial_3 & 0 & i(\partial_1 - i\partial_2) & 0 & (\partial_0 + m) & 0 & 0 & 0 \\ 0 & i\partial_3 & 0 & i(\partial_1 - i\partial_2) & 0 & (\partial_0 - m) & 0 & 0 \\ i(\partial_1 + i\partial_2) & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0 + m) & 0 \\ 0 & i(\partial_1 + i\partial_2) & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0 - m) \end{pmatrix} \begin{pmatrix} \theta_D^0 \\ \theta_D^4 \\ \theta_D^1 \\ \theta_D^5 \\ \theta_D^2 \\ \theta_D^6 \\ \theta_D^3 \\ \theta_D^7 \end{pmatrix} = \begin{pmatrix} \Phi^0 \\ \Phi^4 \\ \Phi^1 \\ \Phi^5 \\ \Phi^2 \\ \Phi^6 \\ \Phi^3 \\ \Phi^7 \end{pmatrix}$$

$$\begin{pmatrix} (\partial_0 - m) & 0 & 0 & \partial_3 & 0 & -\partial_2 & \partial_1 & 0 \\ 0 & (\partial_0 + m) & \partial_3 & 0 & -\partial_2 & 0 & 0 & \partial_1 \\ 0 & -\partial_3 & (\partial_0 - m) & 0 & 0 & \partial_1 & \partial_2 & 0 \\ -\partial_3 & 0 & 0 & (\partial_0 + m) & \partial_1 & 0 & 0 & \partial_2 \\ 0 & \partial_2 & 0 & -\partial_1 & (\partial_0 - m) & 0 & \partial_3 & 0 \\ \partial_2 & 0 & -\partial_1 & 0 & 0 & (\partial_0 + m) & 0 & \partial_3 \\ -\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0 + m) & 0 \\ 0 & -\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0 - m) \end{pmatrix} \begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^2 \\ Z^6 \\ Z^3 \\ Z^7 \end{pmatrix} = \begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^2 \\ J^6 \\ J^3 \\ J^7 \end{pmatrix}$$

The matrix is equivalent (have the same solution set) under the elementary row operation of interchanging rows, so interchanging rows 4 & 5:

$$\begin{pmatrix} (\partial_0 - m) & 0 & 0 & \partial_3 & 0 & -\partial_2 & \partial_1 & 0 \\ 0 & (\partial_0 + m) & \partial_3 & 0 & -\partial_2 & 0 & 0 & \partial_1 \\ 0 & -\partial_3 & (\partial_0 - m) & 0 & 0 & \partial_1 & \partial_2 & 0 \\ -\partial_3 & 0 & 0 & (\partial_0 + m) & \partial_1 & 0 & 0 & \partial_2 \\ \partial_2 & 0 & -\partial_1 & 0 & 0 & (\partial_0 + m) & 0 & \partial_3 \\ 0 & \partial_2 & 0 & -\partial_1 & (\partial_0 - m) & 0 & \partial_3 & 0 \\ -\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0 + m) & 0 \\ 0 & -\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0 - m) \end{pmatrix} \begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^2 \\ Z^6 \\ Z^3 \\ Z^7 \end{pmatrix} = \begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^2 \\ J^6 \\ J^3 \\ J^7 \end{pmatrix}$$

To retain equality under the elementary row operation of interchanging columns 4 & 5 , the rows of vectors $\mathbf{Z}$ & $\mathbf{J}$ are interchanged:

$$\begin{pmatrix} (\partial_0 - m) & 0 & 0 & \partial_3 & -\partial_2 & 0 & \partial_1 & 0 \\ 0 & (\partial_0 + m) & \partial_3 & 0 & 0 & -\partial_2 & 0 & \partial_1 \\ 0 & -\partial_3 & (\partial_0 - m) & 0 & \partial_1 & 0 & \partial_2 & 0 \\ -\partial_3 & 0 & 0 & (\partial_0 + m) & 0 & \partial_1 & 0 & \partial_2 \\ \partial_2 & 0 & -\partial_1 & 0 & (\partial_0 + m) & 0 & 0 & \partial_3 \\ 0 & \partial_2 & 0 & -\partial_1 & 0 & (\partial_0 - m) & \partial_3 & 0 \\ -\partial_1 & 0 & -\partial_2 & 0 & 0 & -\partial_3 & (\partial_0 + m) & 0 \\ 0 & -\partial_1 & 0 & -\partial_2 & -\partial_3 & 0 & 0 & (\partial_0 - m) \end{pmatrix} \begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^6 \\ Z^2 \\ Z^3 \\ Z^7 \end{pmatrix} = \begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^6 \\ J^2 \\ J^3 \\ J^7 \end{pmatrix}$$

Now, consider each matrix as a paired sum:

$$
\begin{pmatrix}
(\partial_0 - m) & 0 & 0 & 0 & i\partial_3 & 0 & i\partial_1 & 0 \\
0 & (\partial_0 + m) & 0 & 0 & 0 & i\partial_3 & 0 & i\partial_1 \\
0 & 0 & (\partial_0 - m) & 0 & i\partial_1 & 0 & -i\partial_3 & 0 \\
0 & 0 & 0 & (\partial_0 + m) & 0 & i\partial_1 & 0 & -i\partial_3 \\
i\partial_3 & 0 & i\partial_1 & 0 & (\partial_0 + m) & 0 & 0 & 0 \\
0 & i\partial_3 & 0 & i\partial_1 & 0 & (\partial_0 - m) & 0 & 0 \\
i\partial_1 & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0 + m) & 0 \\
0 & i\partial_1 & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0 - m)
\end{pmatrix}
+
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \partial_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_2 \\
0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 \\
0 & 0 & \partial_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \partial_2 & 0 & 0 & 0 & 0 \\
-\partial_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\partial_2 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{pmatrix}
(\partial_0 - m) & 0 & 0 & 0 & -\partial_2 & 0 & \partial_1 & 0 \\
0 & (\partial_0 + m) & 0 & 0 & 0 & -\partial_2 & 0 & \partial_1 \\
0 & 0 & (\partial_0 - m) & 0 & \partial_1 & 0 & \partial_2 & 0 \\
0 & 0 & 0 & (\partial_0 + m) & 0 & \partial_1 & 0 & \partial_2 \\
\partial_2 & 0 & -\partial_1 & 0 & (\partial_0 + m) & 0 & 0 & 0 \\
0 & \partial_2 & 0 & -\partial_1 & 0 & (\partial_0 - m) & 0 & 0 \\
-\partial_1 & 0 & -\partial_2 & 0 & 0 & 0 & (\partial_0 + m) & 0 \\
0 & -\partial_1 & 0 & -\partial_2 & 0 & 0 & 0 & (\partial_0 - m)
\end{pmatrix}
+
\begin{pmatrix}
0 & 0 & 0 & \partial_3 & 0 & 0 & 0 & 0 \\
0 & 0 & \partial_3 & 0 & 0 & 0 & 0 & 0 \\
0 & -\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_3 \\
0 & 0 & 0 & 0 & 0 & 0 & \partial_3 & 0 \\
0 & 0 & 0 & 0 & 0 & -\partial_3 & 0 & 0 \\
0 & 0 & 0 & 0 & -\partial_3 & 0 & 0 & 0
\end{pmatrix}
$$

In this form the transformations are easy to see:

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \partial_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_2 \\
0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 \\
0 & 0 & \partial_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \partial_2 & 0 & 0 & 0 & 0 \\
-\partial_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\partial_2 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\Leftrightarrow
\begin{pmatrix}
0 & 0 & 0 & \partial_3 & 0 & 0 & 0 & 0 \\
0 & 0 & \partial_3 & 0 & 0 & 0 & 0 & 0 \\
0 & -\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_3 \\
0 & 0 & 0 & 0 & 0 & 0 & \partial_3 & 0 \\
0 & 0 & 0 & 0 & 0 & -\partial_3 & 0 & 0 \\
0 & 0 & 0 & 0 & -\partial_3 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}\partial_2
\Leftrightarrow
\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0
\end{pmatrix}\partial_3
$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}\partial_2
\Leftrightarrow
\begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0
\end{pmatrix}\partial_3
$$

$$
\begin{pmatrix}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma}^0 \\
\mathbf{0} & \mathbf{0} & -\boldsymbol{\sigma}^0 & \mathbf{0} \\
\mathbf{0} & \boldsymbol{\sigma}^0 & \mathbf{0} & \mathbf{0} \\
-\boldsymbol{\sigma}^0 & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{pmatrix}\partial_2
\Leftrightarrow
\begin{pmatrix}
\mathbf{0} & \boldsymbol{\sigma}^1 & \mathbf{0} & \mathbf{0} \\
-\boldsymbol{\sigma}^1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma}^1 \\
\mathbf{0} & \mathbf{0} & -\boldsymbol{\sigma}^1 & \mathbf{0}
\end{pmatrix}\partial_3
$$

$$
\begin{pmatrix}
\begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} & \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}^0 \\ -\boldsymbol{\sigma}^0 & \mathbf{0} \end{pmatrix} \\
\begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}^0 \\ -\boldsymbol{\sigma}^0 & \mathbf{0} \end{pmatrix} & \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}
\end{pmatrix}\partial_2
\Leftrightarrow
\begin{pmatrix}
\begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}^1 \\ -\boldsymbol{\sigma}^1 & \mathbf{0} \end{pmatrix} & \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \\
\begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} & \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}^1 \\ -\boldsymbol{\sigma}^1 & \mathbf{0} \end{pmatrix}
\end{pmatrix}\partial_3
$$

Let:

$$
\boldsymbol{\sigma}_2^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad , \quad
\boldsymbol{\sigma}_2^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad
\boldsymbol{\sigma}_2^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad
\boldsymbol{\sigma}_2^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ,
$$

$$\mathbf{0}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad \mathbf{0}_4 = \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix}$$

$$\boldsymbol{\sigma}_4^0 = \begin{pmatrix} \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \\ \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}_4^1 = \begin{pmatrix} \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}_4^2 = \begin{pmatrix} \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \end{pmatrix}$$

$$\boldsymbol{\sigma}_8^0 = \begin{pmatrix} \boldsymbol{\sigma}_4^0 & \mathbf{0}_4 \\ \mathbf{0}_4 & \boldsymbol{\sigma}_4^0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}_8^1 = \begin{pmatrix} \mathbf{0}_4 & \boldsymbol{\sigma}_4^0 \\ \boldsymbol{\sigma}_4^0 & \mathbf{0}_4 \end{pmatrix}$$

Using these definitions and block matrix multiplcation tis may be written compactly on a single line as:

$$\boldsymbol{\sigma}_2^0\boldsymbol{\sigma}_4^2\boldsymbol{\sigma}_8^1\partial_2 = \boldsymbol{\sigma}_2^1\boldsymbol{\sigma}_4^1\boldsymbol{\sigma}_8^0\partial_3$$

Although the form is different, the matrices are the same, so by the invertible matrix theorem each is invertible. Thus, these transformations are oneto.

From the first matrices on each side of the sum, the rest of the transformations are even more easily seen. The full set of transformations follow.

$(\partial_0 \pm m) \Leftrightarrow (\partial_0 \pm m) \Rightarrow \bar{x}^0 = x^0$
$-\partial_1 \Leftrightarrow i\partial_1 \qquad \Rightarrow \bar{x}^1 = -ix^1$
$\boldsymbol{\sigma}_2^0\boldsymbol{\sigma}_4^2\boldsymbol{\sigma}_8^1\partial_2 = \boldsymbol{\sigma}_2^1\boldsymbol{\sigma}_4^1\boldsymbol{\sigma}_8^0\partial_3 \Rightarrow \bar{x}^2 = (\boldsymbol{\sigma}_2^0\boldsymbol{\sigma}_4^2\boldsymbol{\sigma}_8^1)^{-1}(\boldsymbol{\sigma}_2^1\boldsymbol{\sigma}_4^1\boldsymbol{\sigma}_8^0)x^3$
$-\partial_2 \Leftrightarrow i\partial_3 \qquad \Rightarrow \bar{x}^3 = -i\bar{x}^2$
[Dirac (barred) on the left = Maxwell-Cassano on the right]

$\begin{pmatrix} \theta_D^0 \\ \theta_D^4 \\ \theta_D^1 \\ \theta_D^5 \\ \theta_D^2 \\ \theta_D^6 \\ \theta_D^3 \\ \theta_D^7 \end{pmatrix} = \begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^6 \\ Z^2 \\ Z^3 \\ Z^7 \end{pmatrix}$	$\begin{pmatrix} \Phi^0 \\ \Phi^4 \\ \Phi^1 \\ \Phi^5 \\ \Phi^2 \\ \Phi^6 \\ \Phi^3 \\ \Phi^7 \end{pmatrix} = \begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^6 \\ J^2 \\ J^3 \\ J^7 \end{pmatrix}$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \overline{\partial_2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \partial_3$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \partial_3$$

$$\Rightarrow \overline{\partial_2}\begin{pmatrix} \theta_D^0 \\ \theta_D^4 \\ \theta_D^1 \\ \theta_D^5 \\ \theta_D^2 \\ \theta_D^6 \\ \theta_D^3 \\ \theta_D^7 \end{pmatrix} = \begin{pmatrix} 0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \end{pmatrix} \partial_3 \begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^6 \\ Z^2 \\ Z^3 \\ Z^7 \end{pmatrix} = \partial_3 \begin{pmatrix} Z^2 \\ Z^6 \\ Z^7 \\ Z^3 \\ Z^4 \\ Z^0 \\ Z^5 \\ Z^1 \end{pmatrix}$$

Thus, if desired, transforming these parts of the equations may be accomplished using this, element-by-element, or the equivalent table:

$\overline{\partial_2}\theta_D^0 \leftrightarrow \partial_3 Z^2$	$\overline{\partial_2}\theta_D^1 \leftrightarrow \partial_3 Z^7$	$\overline{\partial_2}\theta_D^2 \leftrightarrow \partial_3 Z^4$	$\overline{\partial_2}\theta_D^3 \leftrightarrow \partial_3 Z^5$
$\overline{\partial_2}\theta_D^4 \leftrightarrow \partial_3 Z^6$	$\overline{\partial_2}\theta_D^5 \leftrightarrow \partial_3 Z^3$	$\overline{\partial_2}\theta_D^6 \leftrightarrow \partial_3 Z^0$	$\overline{\partial_2}\theta_D^7 \leftrightarrow \partial_3 Z^1$

- this may be simplified using: $\zeta(?, i) \equiv \begin{cases} ? & , \ i = 0 \\ \sim ?\ (\ NOT : ?\ )\ , & otherwise \end{cases}$

to: $\overline{\partial_2}\psi_?^i \leftrightarrow \partial_3 A_{\zeta(?,i)}^{m_0(1,0,i)}$

(where, clearly, $NOT : + = -$ and $NOT : - = +$)

The rest transform simply and directly as noted above.

This proves that the mass-generalized Maxwell's equations (Maxwell-Cassano equations) is a more general analysis of fundamental-elementary particle phenomena.
It further proves that the Lagrangian is far simpler than that consisting of the Glashow-Salam-Weinberg + fermion + Higgs + Yukawa kludge.

Also, it explains the group structure and architecture of the fermions [2].

It also proves that those with wealth to seek the truth choose not to do so, but with all deceivableness and unrighteousness in them they have not the love of the truth, but rather embrace strong delusion, that they profess a lie.

References
and further readings

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