Fermat's Last Theorem: a simple proof

Mark K Malmros

A binomial substitution and expansion demonstrates generally, Fermat's Last Theorem, stated as: *No three positive integers X, Y, and Z can satisfy the equation* $Z^n = X^n + Y^n$ *for any integer value of n greater than two.*

By letting $b = Y - X$ and substituting $Y = (X + b)$ into the right side of the expression, we find

$$X^n + Y^n = X^n + (X + b)^n$$

Expanding[1] the binomial substitution for Y^n, the two X^n terms combine (add), doubling the first coefficient:

$$X^n + Y^n = 2X^n + \binom{n}{1} X^{n-1}b + \binom{n}{2} Xb^2 + ... + \binom{n}{n-1} Xb^{n-1} + \binom{n}{n} b^n$$

where the resulting coefficients are expressed by

$$\binom{n}{k} = \binom{n}{0} + \frac{n!}{k!(n-k)!}$$

This expanded and combined binomial for X and Y is incommensurate with <u>any and all</u> binomial pairs of integers that may be obtained from any integer root of any Z^n because for $Z^n = (X + b)^n$ the coefficients[2] are

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$\because$

$$Z^n = (X + b)^n \neq X^n + (X + b)^n = X^n + Y^n$$

for positive integers for Z, X, Y, n for n>2.

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[1]Equivalently $X^n + Y^n = X^n + \sum_{k=0}^{n} \binom{n}{k} X^k b^{n-k}$

[2]Assigning $Z = (X + b)$ after having assigned $Y = (X + b)$ may be confounding but having rearranged the $X^n + Y^n$ terms into a polynomial in X and b, it serves to point out the coefficients are incommensurate with any binomial expansion of any Z regardless of the binomial variables assigned.

$$\left[\frac{n!}{k!(n-k)!}\right]_{(Z^n)} \neq \left[\binom{n}{0} + \frac{n!}{k!(n-k)!}\right]_{(X^n+Y^n)}$$

We have expressed $X^n + Y^n$ as a binomial expansion of degree n and demonstrate it is incommensurate with any binomial expansion of any Z^n to the same degree for $n > 2$. Q.E.D.

For n=*2*, substituting the binomial $Y = X + b$ into the equation $Z^2 = X^2 + Y^2$ we obtain $Z^2 = X^2 + (X + b)^2$ which can be rearranged to:

$Z^2 - X^2 = (X + b)^2$, factoring

$(Z - X)(Z + X) = (X + b)(X + b)$

Alternatively, expand and rearrange :

$Z^2 - b^2 = 2X^2 + 2Xb$, factoring

$(Z - b)(Z + b) = 2X(X + b)$

for which there are infinitely many solutions (the Pythagorean Theorem). [3]

Chimney Point, VT
malmros@chimneypoint.com
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[3]Both forms of the equation $Z^2 = X^2 + (X + b)^2$ (after either subtracting b^2 or X^2 from each side of the expression and factoring) can be written as ratios useful for heuristically finding primitive Pythagorean triples. (*see for example* Richard Courant and Herbert Robbins (1941). What is Mathematics?: An Elementary Approach to Ideas and Methods. London: Oxford University Press. ISBN 0-19-502517-2. pgs 40-42 (Ian Stewart revision (1995)