CONTRA CANTOR PRO OCCAM

- PROPER CONSTRUCTIVISM WITH ABSTRACTION

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Abstract

> Context • In the philosophy of mathematics there is the distinction between *platonism* (realism), *formalism*, and *constructivism*. There seems to be no distinguishing or decisive experiment to determine which approach is best according to non-trivial and self-evident criteria. As an alternative approach it is suggested here that philosophy finds a sounding board in the *didactics of mathematics* rather than mathematics itself. Philosophers can go astray when they don't realise the distinction between mathematics (possibly pure modeling) and the didactics of mathematics (an empirical science). The approach also requires that the didactics of mathematics is cleansed of its current errors. Mathematicians are trained for abstract thought but in class they meet with real world students. Traditional mathematicians resolve their cognitive dissonance by relying on tradition. That tradition however is not targetted at didactic clarity and empirical relevance with respect to psychology. The mathematical curriculum is a mess. Mathematical education requires a (constructivist) re-engineering. Better mathematical concepts will also be crucial in other areas, such as e.g. brain research. **> Problem** • Aristotle distinguished between potential and actual infinite, Cantor proposed the transfinites, and Occam would want to reject those transfinites if they aren't really necessary. My book *"A Logic of Exceptions"* already refuted 'the' general proof of Cantor's Conjecture on the power set, so that the latter holds only for finite sets but not for 'any' set. There still remains Cantor's diagonal argument on the real numbers. **> Results** • There is a *bijection by abstraction* between $\mathbb{N}$ and $\mathbb{R}$. Potential and actual infinity are two faces of the same coin. Potential infinity associates with counting, actual infinity with the continuum, but they would be 'equally large'. The notion of a limit in $\mathbb{R}$ cannot be defined independently from the construction of $\mathbb{R}$ itself. Occam's razor eliminates Cantor's transfinites. **> Constructivist content** • Constructive steps S_1, ..., S_5 are identified while S_6 gives non-constructivism (possibly the transfinites). Here S_3 gives potential infinity and S_4 actual infinity. The latter is taken as 'proper constructivism with abstraction'. The confusions about S_6 derive rather from logic than from infinity.

Introduction & Conclusion

This paper has been edited and included in Colignatus (2015).

The original results of the paper of 2013-03-26 still stand, albeit that one might argue that an edit always affects what the full results properly are. For the latter reason it is useful to state explicitly that this paper & title has become a legacy version, that is superseded by Colignatus (2015).

Rererences

Colignatus, Th. (2015), *Foundations of Mathematics. A Neoclassical Approach to Infinity",* published by mijnbestseller.nl, http://thomascool.eu/Papers/FMNAI/Index.html

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Thomas Colignatus is the name in science of Thomas Cool, econometrician (Groningen 1982) and teacher of mathematics (Leiden 2008).

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