OVER ONE HUNDRED AND FIFTY CONJECTURES ON PRIMES,
MANY BASED ON THE OBSERVATION OF FERMAT PSEUDOPRIMES

(COLLECTED PAPERS)

2014

INTRODUCTION

In two of my previous published books, "Two hundred conjectures and one hundred and fifty open problems on primes", respectively "Conjectures on primes and Fermat pseudoprimes, many based on Smarandache function", I already expressed my conviction that the study of Fermat pseudoprimes, fascinating numbers that seem to be a little bit more willing to let themselves ordered and understood than the prime numbers, can help a lot in understanding these latter.

This book brings together forty papers on prime numbers, many of them supporting the author's belief, expressed above, namely that new ordered patterns can be discovered in the "undisciplined" set of prime numbers, observing the ordered patterns in the set of Fermat pseudoprimes, especially in the set of Carmichael numbers, the absolute Fermat pseudoprimes, and in the set of Poulet (sometimes also called Sarrus) numbers, the relative Fermat pseudoprimes to base two. Few papers, which are not based on the observation of pseudoprimes, though apparently heterogenous, still have something in common: they are all directed toward the same goal, discovery of new patterns in the set of primes, using the same means, namely the old and reliable integers.

Part One of this book of collected papers contains over one hundred and fifty conjectures on primes and Part Two of this book brings together the articles regarding primes, submitted by the author to the preprint scientific database Vixra, representing the context of the conjectures listed in Part One.

SUMMARY

Part one. Over one hundred and fifty conjectures on primes

Conjecture 1: The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q and r are distinct primes, q ≥ 5 and r ≥ 5.

Conjecture 2: There exist an infinity of primes p that can be written as p = (q^2 + q + 1)/3, where q is also a prime.

Conjecture 3: The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q is a Poulet number and r a prime, r ≥ 5.

Conjecture 4: For any prime p, p ≥ 5, there exist an infinity of pairs of distinct primes [q, r] such that p = sqrt(3*q – r – 1).

Conjecture 5: For any prime p, p ≥ 5, there exist at least a pair of distinct primes [q, r] such that p = (q^2 + r + 1)/3.

Conjecture 6: For any prime p of the form p = 6*k + 1 there exist an infinity of pairs of distinct primes [q, r] such that p = 3*q – p^2 – 1.

Conjecture 7: For an infinity of odd positive integers m there is an infinite set of primes with the property that the sum of their digits is equal to m + 1.

Conjecture 8: For an infinity of primes p there is an infinite set of primes with the property that the sum of their digits is equal to p + 1.

Conjecture 9: There is an infinite number of values the sum of the digits of the numbers p + 1, where p is odd prime, may have.

Conjecture 10: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that the sum of its digits it’s equal to 14.

Conjecture 11: There is an infinity of primes with the property that the sum of their digits is equal to 14.

Conjecture 12: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 2.

Conjecture 13: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 5.

Conjecture 14: There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 8.

Conjecture 15: Let a_i be the sequence of the lesser of twin primes whose digital root is equal to 2, b_i be the sequence of the lesser of twin primes whose digital root is equal to 5 and c_i be the sequence of the lesser of twin primes whose digital root is equal to 8. Than:

: there exist an infinity of terms n of a_i for which the number of the terms of b_i smaller than n is equal to the number of the terms of c_i smaller than n;
: there exist an infinity of terms n of b_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of c_i smaller than n;
: there exist an infinity of terms n of c_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of b_i smaller than n.

Conjecture 16: There is an infinity of primes with the property that the sum of their digits is equal to 14.

Conjecture 17: If n and n + 6 are both primes (in other words if [n, n + 6] is a pair of sexy primes), where n ≥ 7, then the number m = n + 3 can be written at least in one way as m = p + q, where p and q are primes, q = p + 6*r and r is positive integer.

Conjecture 18: If p and p + p(n)# are both primes, where p > p(n)#, n ≥ 2 and p(n)# is a primorial number (which means the product of first n primes), then the number m = p + p(n)#/2 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + p(n)#*r and r is positive integer.

Conjecture 19: For any odd prime p there exist at least one prime q such that p + n! = q, where n is a positive integer, n < p.

Conjecture 20: For any odd prime p, p ≥ 5, there exist an infinity of primes q of the form q = (p + n!)/n^k, where n and k are positive integers and n ≥ p.

Conjecture 21: Any odd prime p can be written in an infinity of distinct ways like p = q – r + 1, where q and r are also primes; in other words, there exist an infinity of pairs of primes (q, r) such that q – r = p – 1, for any odd prime p (it can be seen that for p = 3 the conjecture states the same thing with the twin primes conjecture).

Conjecture 22: Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i – 1 and, where h and i are positive integers.

Conjecture 23: Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Conjecture 24: Any prime p of the form p = 6*k - 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Conjecture 25: There exist an infinity of pairs of primes (p, q), where p is of the form 6*k – 1 and q is of the form 6*h + 1, such that q – p + 1 = 3^n, for any n non-null positive integer (it can be seen that for n = 1 the conjecture states the same thing with the twin primes conjecture).

Conjecture 26: Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1.

Conjecture 27: Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i – 1.

Conjecture 28: For any r prime, r ≥ 5, there exist an infinity of pairs of primes (p, q) such that the numbers (q^2 – p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Conjecture 29: For any pair of primes (p, r), p ≥ 5, r ≥ 5, there exist an infinity of primes q such that the numbers (q^2 – p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Conjecture 30: For any p prime, p ≥ 5, there exist an infinity of primes q such that the numbers (q^2 – p^2 – 2*p)/2 and (q^2 – p^2 + 2*p)/2 are both primes.

Conjecture 31: If x, y and r are odd primes such that y = x + 2*r, where r ≥ 5, then there exist p and q also primes such that x = (q^2 – p^2 – 2*r)/2 and y = (q^2 – p^2 + 2*r)/2.

Conjecture 32: For any p prime, p ≥ 7, there exist a pair of smaller primes (q, r) such that the numbers x = (p^2 – q^2 – 2*r)/2 and y = (p^2 – q^2 + 2*r)/2 are both primes.

Conjecture 33: There exist an infinity of primes p such that 2*p^2 – 1 = q*r, where q and r are also primes.

Conjecture 34: If p is prime and 2*p^2 – 1 = q*r, where q and r are also primes, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)*(r + n), such that p + m, q + n and r + n are also primes.

Conjecture 35: If p is prime and 2*p^2 – 1 = q^2, where q is also prime, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)^2, such that p + m and q + n are also primes.

Conjecture 36: Any Poulet number of the form 10*n + 1 or 10*n + 9 can be written at least in one way as p*q + 10*k*h, where p and q are primes or powers of primes of the same form from the following four ones: 10*m + 1, 10*m + 3, 10*m + 7 or 10*m + 9, k and h are non-null positive integers and q – p = 10*k.

Conjecture 37: For any Poulet number N not divisible by 3 there exist at least a pair of numbers [p, q], where p is prime and q is prime or square of prime, such that N = p^2 + q – 1.

Conjecture 38: There exist an infinity of Poulet numbers of the form n^2 + 120*n, where n is prime or a composite positive integer.

Conjecture 39: There exist an infinity of duplets of primes [p, q] such that p – q = 120; there also exist an infinity of triplets of primes [p1, p2, q] such that p1*p2 – q = 120; there also exist an infinity of quadruplets of primes [p1, p2, p3, q] such that p1*p2*p3 – q = 120; generally, for any non-null positive integer i there exist i primes p1, p2, ..., pi and a prime q such that p1*p2*...*pi – q = 120.

Conjecture 40: For any non-null positive integer i there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that p1*p2*...*pi – q = 2.

Conjecture 41: For any n even positive integer and for any i non-null positive integer there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that p1*p2*...*pi – q = n.

Conjecture 42: For any n even positive integer and for any i and j non-null positive integers there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi – q1*q2*...*qj = n.

Conjecture 43: For any n even positive integer and for any i, j, k, l non-null positive integers, for any k given primes a1, a2, ..., ak and for any l given primes b1, b2, ..., bl, there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi*a1*a2*...*ak – q1*q2*...*qj*b1*b2*...bl = n.

Conjecture 44: For any square of a prime p of the form p = 6*k – 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime q*r where r – q = 8*k and a number congruent to 2, 3 or 5 modulo 6;
(3) p^2 can be deconcatenated into a semiprime 3*q, where q is of the form 10*k + 7, and a number congruent to 1 modulo 6;
(4) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(5) p^2 can be deconcatenated into a number of the form 121 + 48*k and a number congruent to 0 modulo 6;
(6) p^2 is a palindromic number.

Conjecture 45: For any square of a prime p of the form p = 6*k + 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime 3^n*q and a number congruent to 1 modulo 6;
(3) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 1;
(4) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 9;
(5) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(6) p^2 can be deconcatenated into a number of the form 121 + 24*k and a number congruent to 0 modulo 6.

Conjecture 46: For any prime p there exist an infinity of positive integers n such that the number n*p – n + 1 is prime.

Conjecture 47: For any prime p there exist an infinity of positive integers n such that the number n*p + n - 1 is prime.

Conjecture 48: For any prime p there exist an infinity of positive integers n such that the number n^2*p – n + 1 is prime.

Conjecture 49: For any prime p there exist an infinity of positive integers n such that the number n^2*p + n - 1 is prime.

Conjecture 50: For any prime p there exist an infinity of positive integers n such that the number n*p – p + n is prime.

Conjecture 51: For any prime p there exist an infinity of positive integers n such that the number n*p - p - n is prime.

Conjecture 52: For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p + n is prime.

Conjecture 53: For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p - n is prime.

Conjecture 54: For any two distinct primes greater than three p and q there exist an infinity of positive integers n such that the number (p^2 – 1)*n + q^2 is prime, also an infinity of positive integers m such that the number (q^2 – 1)*n + p^2 is prime.

Conjecture 55: For any p, q distinct primes, p > 30, there exist n positive integer such that p – 30*n and q + 30*n are both primes.

Conjecture 56: For any p, q, r distinct primes there exist n positive integer such that the numbers 30*n – p, 30*n – q and 30*n – r are all three primes.

Conjecture 57: There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q + (q - p) is prime.

Conjecture 58: There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

Conjecture 59: For any p prime there exist an infinity of primes q, q > p, where p and q are both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

Conjecture 60: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m + 1) – n and y = q*(n + 1) – m are both primes.

Conjecture 61: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m - 1) + n and y = q*(n - 1) + m are both primes.

Conjecture 62: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p + (m + 1)*n and y = q + m*n are both primes.

Conjecture 63: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n + 2*m are both primes.

Conjecture 64: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n - 2*m are both primes.

Conjecture 65: For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m + 2*n and y = q*n + 2*m are both primes.

Conjecture 66: There exist an infinity of positive integers n such that the numbers 30*n + 7, 60*n + 13 and 150*n + 31 are all three primes.

Conjecture 67: There exist an infinity of positive integers n such that the numbers 30*n - 23, 60*n - 47 and 90*n - 71 are all three primes.

Conjecture 68: There exist an infinity of positive integers n such that the numbers 30*n - 29, 60*n - 59 and 90*n - 89 are all three primes.

Conjecture 69: There exist an infinity of positive integers n such that the numbers 30*n - 23, 60*n - 47 and 90*n - 71 are all three primes.

Conjecture 70: There exist an infinity of positive integers n such that the numbers 30*n - 7, 90*n - 23 and 300*n - 79 are all three primes.

Conjecture 71: There exist an infinity of positive integers n such that the numbers 30*n - 17, 90*n - 53 and 150*n - 89 are all three primes.

Conjecture 72: There exist an infinity of positive integers n such that the numbers 60*n + 13, 180*n + 37 and 300*n + 61 are all three primes.

Conjecture 73: There exist an infinity of positive integers n such that the numbers 330*n + 7, 660*n + 13, 990*n + 19 and 1980*n + 37 are all four primes.

Conjecture 74: There exist an infinity of positive integers n such that the numbers 90*n + 1, 180*n + 1, 270*n + 1 and 540*n + 1 are all four primes.

Conjecture 75: There exist an infinity of pairs of primes [p, q] such that the numbers p + 30*n, q + 30*n and p*q + 30*n are all three primes.

Conjecture 76: There exist an infinity of primes p such that the numbers x = 30*n + p and y = 30*m*n + m*p – m + 1, where m, n are non-null positive integers, are both primes.

Conjecture 77: There exist an infinity of primes of the form 360*p*q + 1, where p, q are primes, both greater than or equal to 7.

Conjecture 79: There exist an infinity of primes of the form 360*p*q + r, where p, q, r are primes, all of them greater than or equal to 7.

Conjecture 80: There exist an infinity of primes of the form 2^m + n^2, where m is non-null positive integer and n odd integer.

Conjecture 81: There exist an infinity of primes of the form (2^n)^k + 2^n + 1, where n is non-null positive integer and k positive integer.

Conjecture 82: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Conjecture 83: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 3 respectively 10*z + 7.

Conjecture 84: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 7, 10*y + 7 respectively 10*z + 7.

Conjecture 85: Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 9 respectively 10*z + 9.

Conjecture 86: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Conjecture 87: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 3 respectively 10*z + 9.

Conjecture 88: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 3 respectively 10*z + 7.

Conjecture 89: Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms: 10*x + 7, 10*y + 7 respectively 10*z + 9.

Conjecture 90: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 3 respectively 10*z + 3.

Conjecture 91: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 7 respectively 10*z + 7.

Conjecture 92: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 7 respectively 10*z + 9.

Conjecture 93: Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms: 10*x + 9, 10*y + 9 respectively 10*z + 9.

Conjecture 94: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 1 respectively 10*z + 7.

Conjecture 95: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 3 respectively 10*z + 3.

Conjecture 96: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 3, 10*y + 7 respectively 10*z + 9.

Conjecture 97: Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms: 10*x + 1, 10*y + 9 respectively 10*z + 9.

Conjecture 98: Any prime greater than or equal to 5 can be written at least in one way as (9*p^2 – q^2)/(2^n), where p and q are primes and n non-null positive integer.

Conjecture 99: Any prime p of the form 6*k + 1 greater than or equal to 13 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Conjecture 100: Any prime p of the form 6*k – 1 greater than or equal to 11 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Conjecture 101: Any odd prime p can be written at least in one way as p = (q*2^a*3^b*5^c ± 1)*2^n ± 1, where q is an odd prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Conjecture 102: Any pair of twin primes [p_1, p_2] can be written as [p_1 = (q*2^a*3^b*5^c ± 1)*2^n - 1, p_2 = (q*2^a*3^b*5^c ± 1)*2^n + 1], where q is prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Conjecture 103: For any pair of primes, greater than 3, [p_1, q_1], where q_1 – p_1 = d, there exist at least a pair of positive integers [m, n], where n – m = d, such that the numbers p_2 = p_1*q_1 – n + 1 and q_2 = p_1*q_1 – m + 1 are both primes.

Conjecture 104: For any even number d there exist an infinity of pairs of primes [p_1, q_1], where q_1 – p_1 = d, such that the numbers p_2 = p_1*q_1 – p_1 + 1 and q_2 = p_1*q_1 – q_1 + 1 are both primes.

Conjecture 105: For any n non-null positive integer there exist q, r primes such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 106: For any q odd prime there exist n non-null positive integer and r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 107: For any q, r odd primes there exist n non-null positive integer such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 108: For any n non-null positive integer and any q prime there exist r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 109: For any n non-null positive integer there exist q prime such that 120*n*q^2 + 1 = p^2, where p is prime or a power of prime.

Conjecture 110: There exist an infinity of primes p of the form p = 25^n + 906304.

Conjecture 111: There exist an infinity of infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 112: For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 113: For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is a multiple of p(2) - 1, the period of the rational number a(2) is a multiple of p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 114: For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) divides p(2) - 1, the period of the rational number a(2) divides p(3) – 1, the period of the rational number a(n) divides a(n) – 1.

Conjecture 115: For any Poulet number P there exist a rational number r equal to a sum of unit fractions 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1), p(2), p(3)... are distinct odd primes, such that the period of r is equal to P – 1.

Conjecture 116: Any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones: 10k + 1, 10k + 3, 10k + 7 or 10k + 9.

Conjecture 117: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 1.

Conjecture 118: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 3.

Conjecture 119: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 7.

Conjecture 120: There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 9.

Conjecture 121: Any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form 10k + 3 or all three of the form 10k + 7.

Conjecture 122. Any square of a prime p^2, where p is greater than or equal to 7, can be written as p^2 = 2*m + n, where m and n are distinct primes, both of the form 10k + 3 or both of the form 10k + 7.

Conjecture 123. There exist an infinity of positive integers k such that 6*k – 1 and 18*k – 5 are both primes.

Conjecture 124. There exist an infinity of positive integers k such that 6*k + 1 and 12*k + 1 are both primes.

Conjecture 125. There exist an infinity of positive integers k such that 6*k + 1 and 18*k + 1 are both primes.

Conjecture 126. There exist an infinity of positive integers k such that 6*k - 5 and 24*k - 5 are both primes.

Conjecture 127. There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 1 are all three primes.

Conjecture 128. There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 13 are all three primes.

Conjecture 129. There exist an infinity of positive integers k such that k, 2*k - 1 and 5*k - 4 are all three primes.

Conjecture 130. There exist an infinity of positive integers k such that k, 2*k - 1 and 3*k - 2 are all three primes.

Conjecture 131. There exist an infinity of positive integers k such that k, 3*k - 2 and 4*k - 3 are all three primes.

Conjecture 132. There exist an infinity of positive integers k such that 40*k + 1, 60*k + 1 and 100*k + 1 are all three primes.

Conjecture 133. There exist an infinity of positive integers k such that k, 2*k – 1, 7*k - 6 and 14*k - 13 are all four primes.

Conjecture 134. There exist an infinity of positive integers k such that k, 2*k – 1, 6*k - 5 and 12*k - 11 are all four primes.

Conjecture 135. There exist an infinity of pairs of distinct non-null positive integers m, n such that 60*m*n – 29 and 60*m*n – (60*m + 29) are both primes.

Conjecture 136. There exist an infinity of pairs of distinct non-null positive integers [m, n] such that 40*m – 10*n - 29 and 40*m – 10*n – 129 are both primes.

Conjecture 137. For any pair of twin primes [q, r] there exist an infinity of primes p of the form p = 7200*q*r*n + 1, where n is positive integer.

Conjecture 138. There exist an infinity of primes p of the form p = n*s(p) – n + 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 139. There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 140. There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 141. There exist an infinity of primes p of the form p = m*n + m - n, where m and n are distinct odd primes.

Conjecture 142. There exist an infinity of primes p of the form p = m^2 – m*n + n, where m and n are distinct odd primes.

Conjecture 143. There exist an infinity of primes p of the form p = (q + 5^k)/ 10, where q is prime and k positive integer.

Conjecture 144. There exist an infinity of primes p of the form p = (q + 5^k)/ 30, where q is prime and k positive integer.

Conjecture 145. If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of numbers which are primes or squares of primes.

Conjecture 146. If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of semiprimes q = m*n, where m < n, with the following property: the number n – m + 1 is a prime or a square of a prime.

Conjecture 147. Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of primes of the form a(i)/p as well as an infinity of primes of the form b(i)/q for any pair [p, q].

Conjecture 148. Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of pairs [p, q] such that the sequence of primes a(i)/p is the same with the sequence of primes b(i)/q.

Conjecture 149. There exist an infinity of primes, for k positive integer, of the form n*2^k + 1, for n equal to 5, 6, 29, 49 or 99 (note that this conjecture is a consequence of Conjecture 1 and the examples observed above).

Conjecture 150. There exist an infinity of positive integers n such that the sequence n*2^k + 1, where k is positive integer, contains an infinity of primes.

Conjecture 151. Any Mersenne number 2^n – 1 with odd exponent n, where n is greater than or equal to 3, also n is not a power of 3, is either prime either divisible by a 2-Poulet number.

Conjecture 152. Any Mersenne-Coman number of the form P = ((2^m)^n – 1)/3^k, where m is non-null positive integer, n is odd, greater than or equal to 5, also n is not a power of 3, and k is equal to 0 or is equal to the greatest positive integer such that P is integer, is either a prime either divisible by at least a 2-Poulet number.

Conjecture 153. For any prime p greater than or equal to 5 the number (4^p – 1)/3 is either prime either a product of primes $p_1*p_2*...p_n$ such that all the numbers p_i*p_j are 2-Poulet numbers for $1 \le i < j \le n$.

Conjecture 154. Any Fermat number F = 2^(2^n) + 1 is either prime either divisible by a 2-Poulet number.

Conjecture 155. Any Fermat-Coman number of the form N = ((2^m)^p + 1)/3^k, where m is non-null positive integer, p is prime, greater than or equal to 7, and k is equal to 0 or is equal to the greatest positive integer such that N is integer, is either a prime either divisible by at least a 2-Poulet number.

Conjecture 156. For any prime p greater than or equal to 7 the number (4^p + 1)/5 is either prime either a product of primes $p_1*p_2*...p_n$ such that all the numbers p_i*p_j are 2-Poulet numbers for $1 \le i < j \le n$.

Part two. Forty articles on primes

1. A conjecture about a way in which the squares of primes can be written and five other related conjectures

Abstract. I was playing with randomly formed formulas based on two distinct primes and the difference of them, when I noticed that the formula p + q + 2*(q – p) – 1, where p, q primes, conducts often to a result which is prime, semiprime, square of prime or product of very few primes. Starting from here, I made a conjecture about a way in which any square of a prime seems that can be written. Following from there, I made a conjecture about a possible infinite set of primes, a conjecture regarding the squares of primes and Poulet numbers and yet three other related conjectures.

Conjecture 1:

The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q and r are distinct primes, q ≥ 5 and r ≥ 5.

Comment:

I really have no idea yet how it could be proved the conjecture or what implications it could have if it were true, so I'll just check it for the first few squares of primes.

Verifying the conjecture:
(for the first few squares of primes)

: 5^2 = 3*11 – 7 – 1, so [p, q, r] = [5, 11, 7];
: 7^2 = 3*19 – 7 – 1, so [p, q, r] = [7, 19, 7];
: 11^2 = 3*43 – 7 – 1, so [p, q, r] = [11, 43, 7];
: 13^2 = 3*59 – 7 – 1, so [p, q, r] = [13, 59, 7];
: 17^2 = 3*101 – 13 – 1, so [p, q, r] = [17, 101, 13];
: 19^2 = 3*127 – 19 – 1, so [p, q, r] = [19, 127, 19];
: 23^2 = 3*179 – 7 – 1, so [p, q, r] = [23, 179, 7];
: 29^2 = 3*283 – 7 – 1, so [p, q, r] = [29, 283, 7];
: 31^2 = 3*331 – 31 – 1, so [p, q, r] = [31, 331, 31];
: 37^2 = 3*461 – 13 – 1, so [p, q, r] = [37, 461, 13];
: 41^2 = 3*563 – 7 – 1, so [p, q, r] = [41, 563, 7];
: 43^2 = 3*619 – 7 – 1, so [p, q, r] = [43, 619, 7];
: 47^2 = 3*739 – 7 – 1, so [p, q, r] = [47, 739, 7].

Note:

It can be seen that in few cases from the ones above we have p = r, so we make yet another conjecture:

Conjecture 2:

There exist an infinity of primes p that can be written as p = (q^2 + q + 1)/3, where q is also a prime.

Examples of such primes:

: 19 = (7^2 + 7 + 1)/3, so [p, q] = [19, 7];
: 127 = (19^2 + 19 + 1)/3, so [p, q] = [127, 19];
: 331 = (31^2 + 31 + 1)/3, so [p, q] = [331, 31].

Conjecture 3:

The square of any prime p, p ≥ 5, can be written at least in one way as p^2 = 3*q – r – 1, where q is a Poulet number and r a prime, r ≥ 5.

Verifying the conjecture:
(for the first few squares of primes)

: 5^2 = 3*341 – 997 – 1, so [p, q, r] = [5, 341, 997];
: 7^2 = 3*1387 – 4111 – 1, so [p, q, r] = [7, 1387, 4111];
: 11^2 = 3*4371 – 13921 – 1, so [p, q, r] = [11, 4371, 13921];
: 13^2 = 3*341 – 853 – 1, so [p, q, r] = [13, 341, 853].

Comment:

Considering the results from the three conjectures above, I make three other related conjectures.

Conjecture 4:

For any prime p, p ≥ 5, there exist an infinity of pairs of distinct primes [q, r] such that p = sqrt(3*q – r – 1).

Example:
(for p = 7)

: 7 = sqrt(3*19 – 7 – 1), so [q, r] = [19, 7];
: 7 = sqrt(3*23 – 19 – 1), so [q, r] = [23, 19];
: 7 = sqrt(3*29 – 37 – 1), so [q, r] = [29, 37];
: 7 = sqrt(3*31 – 43 – 1), so [q, r] = [31, 43];
: 7 = sqrt(3*37 – 61 – 1), so [q, r] = [37, 61];
: 7 = sqrt(3*41 – 73 – 1), so [q, r] = [41, 73] (...).

Conjecture 5:

For any prime p, p ≥ 5, there exist at least a pair of distinct primes [q, r] such that p = (q^2 + r + 1)/3.

Verifying the conjecture:
(for the first few primes)

: 5 = (3^2 + 5 + 1)/3, so [p, q, r] = [5, 3, 5];
: 7 = (3^2 + 11 + 1)/3, so [p, q, r] = [7, 3, 11];
: 11 = (3^2 + 5 + 1)/3, so [p, q, r] = [5, 3, 5];
: 13 = (5^2 + 13 + 1)/3, so [p, q, r] = [13, 5, 13];
: 17 = (3^2 + 41 + 1)/3, so [p, q, r] = [17, 3, 41];
: 19 = (5^2 + 31 + 1)/3, so [p, q, r] = [19, 5, 31].

Conjecture 6:

For any prime p of the form p = 6*k + 1 there exist an infinity of pairs of distinct primes [q, r] such that p = 3*q – p^2 – 1.

Example:
(for p = 37)

: 37 = 3*29 – 7^2 – 1, so [q, r] = [29, 7];
: 37 = 3*53 – 11^2 – 1, so [q, r] = [53, 11];
: 37 = 3*109 – 17^2 – 1, so [q, r] = [109, 17];
: 37 = 3*293 – 29^2 – 1, so [q, r] = [293, 29];
: 37 = 3*1693 – 71^2 – 1, so [q, r] = [1693, 71];
: 37 = 3*1789 – 73^2 – 1, so [q, r] = [1789, 73] (...).

2. A conjecture about an infinity of sets of integers, each one having an infinite number of primes

Abstract. In this paper, inspired by one of my previous papers posted on Vixra, I make, considering the sum of the digits of an odd integer, a conjecture about an infinity of sets of integers, each one having an infinite number of primes and I also make, considering the sum of the digits of a prime number, two other conjectures.

Conjecture 1:

For an infinity of odd positive integers m there is an infinite set of primes with the property that the sum of their digits is equal to m + 1.

Conjecture 2:

For an infinity of primes p there is an infinite set of primes with the property that the sum of their digits is equal to p + 1.

Comment:

Such a prime p I conjectured to be, in a previous paper posted on Vixra, the number 13.

Conjecture 3:

There is an infinite number of values the sum of the digits of the numbers p + 1, where p is odd prime, may have.

Note:

For a list with prime numbers with the property that the sum of their digits is equal to an even number see the sequence A119449 in OEIS.

Note:

We will refer hereinafter with D(m) to the set of primes with the property that the sum of their digits is equal to m + 1, where m is an odd integer.

The sequence D(1):

: 101 (...).

The sequence D(3):

: 13, 31, 103, 211, 1021, 1201 (...).

The sequence D(5):

: (...).

The sequence D(7):

: 17, 53, 71, 107, 233, 251, 431, 503, 521, 701, 1061, 1151, 1223 (...).

The sequence D(9):

: 19, 37, 73, 109, 127, 163, 181, 271, 307, 433, 523, 541, 613, 631, 811, 1009, 1063, 1117, 1153, 1171 (...).

The sequence D(11):

: (...).

The sequence D(13):

: 59, 149, 167, 239, 257, 293, 347, 419, 491, 563, 617, 653, 743, 761, 941, 1049, 1193, 1229, 1283, 1319 (...).

The sequence D(15):

: 79, 97, 277, 349, 367, 383, 439, 457, 547, 619, 673, 691, 709, 727, 853, 907, 1069, 1087, 1249 (...).

The sequence D(17):

: (...).

The sequence D(19):

: 389, 479, 569, 587, 659, 677, 839, 857, 929, 947, 983, 1289 (...).

The sequence D(21):

: 499, 769, 787, 859, 877, 967 (...).

Note:

It can easily be seen that for some values of odd integers m were obtained much more primes with the sum of the digits equal to m + 1 than for other values of m; for instance were obtained, from the first hundred of primes having the sum of digits equal to an even number, 20 such primes for which m = 9, 21 such primes for which m = 13, 19 such primes for which m = 15, but no such primes at all for which m = 5, m = 11 or m = 17.

3. A trivial but notable observation about a relation between the twin primes and the number 14

Abstract. There are known few interesting properties which distinquish twin primes from the general set of primes, like for instance that 46% of primes smaller than 19000 are Ramanujan primes while about 78% of the lesser of twin primes smaller than 19000 are Ramanujan primes. But seems that a much more trivial observation about the lesser of twin primes escaped attention: from the first 500 numbers which are lesser in a pair of twin primes, 66 of them have the following remarkable property: the sum of their digits is equal to 14.

Note:

For a list of lesser of twin primes and also for the property mentioned in Abstract regarding Ramanujan primes see the sequence A001359 in OEIS.

Observation:

Like I mentioned in abstract, this paper is a trivial observation of a fact: from the first 100 numbers which are lesser in a pair of twin primes, 20 of them have the property that the sum of their digits is equal to 14; from the first 500 numbers which are lesser in a pair of twin primes, 66 of them have this property; these numbers are:

: 59, 149, 239, 347, 419, 617, 1049, 1229, 1319, 1427, 1481, 1607, 2129, 2237, 2309, 2381, 3119, 3371, 3461, 3821, 4019, 4091, 4127, 4217, 4271, 4721, 5009, 5441, 6701, 7331, 8231, 9041, 10067, 10139, 10427, 11057, 12821, 13217, 13721, 13901, 14009, 14081, 16061, 18041, 18131, 18311, 19211, 20147, 20507, 21191, 21317, 22037, 22091, 22109, 22271, 22541, 23027, 24107, 25601, 29021, 30137, 31181, 31541, 31721, 32027, 32117.

Conjecture:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that the sum of its digits it’s equal to 14.

Note:

The conjecture above implies the following one:

Conjecture:

There is an infinity of primes with the property that the sum of their digits is equal to 14.

4. An observation about the digital root of the twin primes, few conjectures and an open problem on primes

Abstract. Few interesting properties which distinquish twin primes from the general set of primes there are already known. I wrote myself an article regarding an interesting property of a set of (pairs of) twin primes based on the sum of the digits of the lesser (implicitly greater) prime from a pair of twin primes. This paper notes a property regarding twin primes based on their digital root.

Observation:

The digital root of the lesser prime p from a pair of twin primes [p, q], under the condition that p > 3, is, for the first such 100 primes (for a list of lesser of twin primes see the sequence A001359 in OEIS):

: 5, 2, 8, 2, 5, 5, 8, 2, 8, 2, 5, 8, 2, 8, 2, 5, 8, 2, 5, 5, 8, 2, 8, 2, 5, 5, 2, 2, 8, 2, 8, 2, 8, 2, 5, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 5, 2, 5, 8, 2, 5, 2, 5, 8, 5, 5, 5, 8, 2, 2, 8, 5, 5, 8, 5, 8, 5, 8, 5, 2, 8, 2, 5, 2, 2, 8, 2, 8, 2, 5, 8, 2, 8, 5, 8, 2, 5, 5, 5, 2, 8, 2, 2, 8, 8, 5, 5.

Note:

Obviously the digital root of a lesser from a pair of twin primes can never be equal to 3, 6 or 9 (it would be then a number divisible by 3 not a prime) or 1, 4 or 7 (the greater from the pair of twin primes would be in this case divisible by 3).

Conjecture 1:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 2.

Conjecture 2:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 5.

Conjecture 3:

There is an infinity of pairs of twin primes for which the lesser term of the pair has the property that its digital root is equal to 8.

Note:

Is remarkable that the three subsets of the set of (lesser of) twin primes seem to have (of course, for a given term great enough) an approximately equal number of terms; for instance, from the first 100 from the lesser of twin primes, 33 of them have the digital root equal to 2, 35 have the digital root equal to 5 and 32 have the digital root equal to 7.

Conjecture 4:

Let a_i be the sequence of the lesser of twin primes whose digital root is equal to 2, b_i be the sequence of the lesser of twin primes whose digital root is equal to 5 and c_i be the sequence of the lesser of twin primes whose digital root is equal to 8. Than:

: there exist an infinity of terms n of a_i for which the number of the terms of b_i smaller than n is equal to the number of the terms of c_i smaller than n;

: there exist an infinity of terms n of b_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of c_i smaller than n;

: there exist an infinity of terms n of c_i for which the number of the terms of a_i smaller than n is equal to the number of the terms of b_i smaller than n.

Conjecture 5:

There is an infinity of integers n for which the set of the lesser (greater) of the twin primes smaller than n is divided in three subsets whith an equal number of terms, a_i with the property that the digital root of its terms is equal to 2 (4), b_i with the property that the digital root of its terms is equal to 5 (7) and c_i with the property that the digital root of its terms is equal to 8 (1).

Open problem:

Is there any other prime p beside p = 23 with the property that the following six subsets of odd primes have an equal number of terms smaller than p? The terms of the six subsets are the primes whose digital root is equal to 1, 2, 4, 5, 7 respectively 8 (it can be seen that, for p = 23, we have the following odd primes smaller than 23 that belong to the six subsets: 5, 7, 11, 13, 17, 19, whose digital root is 5, 7, 2, 4, 8, 1).

5. A conjecture on primes involving the pairs of sexy primes

Abstract. This paper states a conjecture on primes involving two types of pairs of primes: the pairs of sexy primes, which are the two primes that differ from each other by six and the pairs of primes of the form [p, q], where q = p + 6*r, where r is positive integer.

Conjecture:

If n and n + 6 are both primes (in other words if [n, n + 6] is a pair of sexy primes), where n ≥ 7, then the number m = n + 3 can be written at least in one way as m = p + q, where p and q are primes, q = p + 6*r and r is positive integer.

Verifying the conjecture:
(for the first fifteen pairs of sexy primes)

: for [n, n + 6] = [7, 13] we have [p, q, r] = [5, 5, 0];
: for [n, n + 6] = [11, 17] we have [p, q, r] = [7, 7, 0];
: for [n, n + 6] = [13, 19] we have [p, q, r] = [5, 11, 1];
: for [n, n + 6] = [17, 23] we have [p, q, r] = [7, 13, 1];
: for [n, n + 6] = [23, 29] we have [p, q, r] = [13, 13, 0];
: for [n, n + 6] = [31, 37] we have [p, q, r] = [5, 29, 1] or [17, 17, 0];
: for [n, n + 6] = [37, 43] we have [p, q, r] = [11, 29, 3] or [17, 23, 1];
: for [n, n + 6] = [41, 47] we have [p, q, r] = [7, 37, 1] or [13, 31, 3];
: for [n, n + 6] = [47, 53] we have [p, q, r] = [7, 37, 1] or [13, 37, 4] or [19, 31, 2];
: for [n, n + 6] = [53, 59] we have [p, q, r] = [13, 43, 5] or [19, 37, 3];
: for [n, n + 6] = [61, 67] we have [p, q, r] = [17, 47, 5] or [23, 41, 3];
: for [n, n + 6] = [67, 73] we have [p, q, r] = [11, 59, 8] or [17, 53, 6] or [23, 47, 4] or [29, 41, 2];
: for [n, n + 6] = [73, 79] we have [p, q, r] = [17, 59, 7] or [23, 53, 5] or [29, 47, 3];
: for [n, n + 6] = [83, 89] we have [p, q, r] = [7, 79, 12] or [13, 73, 10] or [19, 67, 8] or [43, 43, 0];
: for [n, n + 6] = [97, 103] we have [p, q, r] = [11, 89, 13]; or [17, 83, 11] or [29, 71, 7] or [47, 53, 1].

6. A conjecture on the pairs of primes p, q, where q is equal to the sum of p and a primorial number

Abstract. In a previous paper I stated a conjecture on primes involving the pairs of sexy primes, which are the two primes that differ from each other by six. In this paper I extend that conjecture on the pairs of primes [p, q], where q is of the form p + p(n)#, where p(n)# is a primorial number, which means the product of first n primes.

Conjecture:

If p and p + p(n)# are both primes, where p > p(n)#, n ≥ 2 and p(n)# is a primorial number (which means the product of first n primes), then the number m = p + p(n)#/2 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + p(n)#*r and r is positive integer.

Note:

For a list of primorial numbers, see the sequence A002110 in OEIS; the first ten primorial numbers are: 1, 2, 6, 30, 210, 2310, 30030, 510510, 9699690, 223092870.

Note:

Because p(0)# is, by convention or justified through the concept of "empty product", equal to 1, then p(1)# is equal to 2, p(2)# is equal to 6, p(3)# is equal to 30, p(4)# is equal to 210 and so on.

Comment:

The conjecture it will be then formulate:

: for p(2)# = 6:

If p and p + 6 are both primes, where p > 6, then the number m = p + 3 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 6*r and r is positive integer (this is the conjecture which I made in a previous paper, where I also verified it for the first fifteen pairs of sexy primes, with the difference that in the formulation from there x and y were "primes" not "primes or squares of primes");

: for p(3)# = 30:

If p and p + 30 are both primes, where p > 30, then the number m = p + 15 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 30*r and r is positive integer;

: for p(4)# = 210:

If p and p + 210 are both primes, where p > 210, then the number m = p + 105 can be written at least in one way as m = x + y, where x and y are primes or squares of primes, y = x + 210*r and r is positive integer;

: for p(5)# = 2310:

If p and p + 2310 are both primes, where p > 2310, then the number m = p + 1155 can be written at least in one way as m = x + y, where x and y are primes, y = x + 2310*r and r is positive integer.

Verifying the conjecture:
(for the first ten pairs of primes [p, p + 30], where p > 30)

: for [p, p + 30] = [31, 61] we have [x, y, r] = [23, 23, 0];
: for [p, p + 30] = [37, 67] we have [p, q, r] = [11, 41, 1];
: for [p, p + 30] = [41, 71] we have [p, q, r] = [13, 43, 1];
: for [p, p + 30] = [43, 73] we have [p, q, r] = [29, 29, 0];
: for [p, p + 30] = [53, 83] we have [p, q, r] = [19, 49, 1];
: for [p, p + 30] = [59, 89] we have [p, q, r] = [37, 37, 0] or [7, 67, 2];
: for [p, p + 30] = [67, 97] we have [p, q, r] = [41, 41, 0] or [11, 71, 2];
: for [p, p + 30] = [71, 101] we have [p, q, r] = [43, 43, 0] or [13, 73, 2];
: for [p, p + 30] = [73, 103] we have [p, q, r] = [29, 59, 1];
: for [p, p + 30] = [79, 109] we have [p, q, r] = [47, 47, 0].

Verifying the conjecture:
(for the first eight pairs of primes [p, p + 210], where p > 210)

: for [p, p + 210] = [13, 223] we have [x, y, r] = [59, 59, 0] or [29, 89, 2];
: for [p, p + 210] = [17, 227] we have [x, y, r] = [61, 61, 0];
: for [p, p + 210] = [19, 229] we have [x, y, r] = [17, 107, 3];
: for [p, p + 210] = [23, 233] we have [x, y, r] = [19, 109, 3] or [49, 79, 1];
: for [p, p + 210] = [29, 239] we have [x, y, r] = [67, 67, 0] or [7, 127, 4] or [37, 97, 2];
: for [p, p + 210] = [31, 241] we have [x, y, r] = [23, 113, 3] or [53, 83, 1];
: for [p, p + 210] = [41, 251] we have [x, y, r] = [73, 73, 0] or [43, 103, 2];
: for [p, p + 210] = [47, 257] we have [x, y, r] = [31, 121, 3].

Verifying the conjecture:
(for the first pair of primes [p, p + 2310], where p > 2310)

: for [p, p + 2310] = [23, 2333] we have [x, y, r] = [49, 1129, 36] or [109, 1069, 32] or [139, 1039, 30] or [169, 1009, 28] or [349, 829, 16] or [469, 709, 12] or [439, 739, 10].

7. Two conjectures involving the sum of a prime and a factorial number

Abstract. In this paper I state two conjectures about the sum of a prime and a factorial.

Note:

For a list of factorial numbers see the sequence A000142 in OEIS.

Conjecture 1:

For any odd prime p there exist at least one prime q such that p + n! = q, where n is a positive integer, n < p.

Verifying the conjecture:
(for the first five odd primes p)

: 3 + 2! = 5, so [p, q, n] = [3, 5, 2];
: 5 + 2! = 7 and 5 + 3! = 11 and 5 + 4! = 29, so [p, q, n] = [5, 7, 2] or [5, 11, 3] or [5, 29, 4];
: 7 + 3! = 13 and 7 + 4! = 31 and 7 + 5! = 127 and 7 + 6! = 727 so [p, q, n] = [7, 13, 3] or [7, 31, 4] or [7, 127, 5] or [7, 727, 6];
: 11 + 2! = 13 and 11 + 6! = 17 and 11 + 5! = 131 and 11 + 7! = 5051 and 11 + 10! = 3628811 so [p, q, n] = [11, 13, 2] or [11, 17, 6] or [11, 131, 10] or [11, 5051, 7] or [11, 3628811, 10];
: 13 + 3! = 19 and 13 + 4! = 37 and 13 + 6! = 733 so [p, q, n] = [13, 19, 3] or [13, 37, 4] or [13, 733, 6].

Note:

From the primes q, q ≥ 5, q ≤ 401, just three primes can't be written as p + n!, where p is a lesser odd prime and n is a positive integer, *i.e.* the primes 41, 101, 367 (but, interesting, 367 can be written as 7^3 + 5!); indeed:

: q can be written as p + 2! for q = 5, 7, 31, 43, 61, 73, 103, 109, 139, 151, 181, 193, 199, 229, 241, 271, 283, 313, 349 [...];
: q can be written as p + 3! for q = 11, 13, 17, 23, 29, 37, 47, 53, 59, 67, 79, 89, 107, 113, 157, 163, 173, 179, 197, 239, 251, 257, 263, 269, 277, 337, 359, 373, 379, 389 [...];
: q can be written as p + 4! for q = 71, 83, 97, 127, 131, 191, 223, 251, 281, 293, 307, 317, 331, 383, 397 [...];
: q can be written as p + 5! for q = 149, 167, 227, 233, 311, 347, 353, 401 [...].

Conjecture 2:

For any odd prime p, p ≥ 5, there exist an infinity of primes q of the form q = (p + n!)/n^k, where n and k are positive integers and n ≥ p.

Examples:

: for p = 5, we have:
  : q = 29 = (6! + 5)/5^2;
  : q = 1009 = (7! + 5)/5^1;:
  : q = 1613 = (8! + 5)/5^2;
  : q = 72577 = (9! + 5)/5^1;
  [...]
: for p = 7, we have:
  : q = 103 = (7! + 7)/7^2;
  : q = 823 = (8! + 7)/7^2;
  [...]
: for p = 11, we have:
  : q = 329891 = (11! + 11)/11^2;
  [...]
: for p = 13, we have:
  : q = 2834329 = (13! + 13)/13^3;
  : q = 515847877 = (13! + 13)/13^2;
  [...].

8. Seven conjectures on a certain way to write primes including two generalizations of the twin primes conjecture

Abstract. In this paper I make few conjectures about a way to write an odd prime p, id est p = q – r + 1, where q and r are also primes; two of these conjectures can be regarded as generalizations of the twin primes conjecture, which states that there exist an infinity of pairs of twin primes.

Conjecture 1
(Which can be regarded as a generalization of the twin primes conjecture)

Any odd prime p can be written in an infinity of distinct ways like p = q – r + 1, where q and r are also primes; in other words, there exist an infinity of pairs of primes (q, r) such that q – r = p – 1, for any odd prime p (it can be seen that for p = 3 the conjecture states the same thing with the twin primes conjecture).

Conjecture 2

Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i – 1 and, where h and i are positive integers.

Example: the prime p = 7 can be written as 11 – 5 + 1; 17 – 11 + 1; 23 – 17 + 1 etc.; in fact, for p = 7 the conjecture states that there exist an infinity of pairs of sexy primes (q, r), both of the form 6*k - 1 (sexy primes are the primes that differ by each other by six).

Conjecture 3

Any prime p of the form p = 6*k + 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Example: the prime p = 7 can be written as 13 – 7 + 1; 19 – 13 + 1; 37 – 31 + 1 etc.; in fact, for p = 7 the conjecture states that there exist an infinity of pairs of sexy primes (q, r), both of the form 6*k + 1.

Conjecture 4

Any prime p of the form p = 6*k – 1, where k is positive integer, can be written in an infinity of distinct ways like p = q – r + 1, where q is a prime of the form q = 6*h – 1 and r is a prime of the form q = 6*i + 1 and, where h and i are positive integers.

Conjecture 5
(Which can be regarded as a generalization of the twin primes conjecture)

There exist an infinity of pairs of primes (p, q), where p is of the form 6*k – 1 and q is of the form 6*h + 1, such that q – p + 1 = 3^n, for any n non-null positive integer (it can be seen that for n = 1 the conjecture states the same thing with the twin primes conjecture).

Example: for n = 2 we have the pairs of primes (p, q): (11, 19); (23, 31) etc.; for n = 3 we have the pairs of primes (5, 31); (11, 37) etc.

Conjecture 6

Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h + 1 and r is a prime of the form q = 6*i + 1.

Example: the number 49 = 7^2 can be written as 61 – 13 + 1; 67 – 19 + 1; 79 – 31 + 1 etc.

Conjecture 7

Any square of prime p^2, p ≥ 5, can be written in an infinity of distinct ways like p^2 = q – r + 1, where q is a prime of the form q = 6*h 1 and r is a prime of the form q = 6*i –1.

Example: the number 49 = 7^2 can be written as 53 – 5 + 1; 59 – 11 + 1; 71 – 23 + 1 etc.

9. An interesting formula for generating primes and five conjectures about a certain type of pairs of primes

Abstract. In this paper I just enunciate a formula which often leads to primes and products of very few primes and I state five conjectures about the pairs of primes of the form [(q^2 – p^2 – 2*r)/2,(q^2 – p^2 + 2*r)/2], where p, q, r are odd primes.

Conjecture 1:

For any r prime, r ≥ 5, there exist an infinity of pairs of primes (p, q) such that the numbers (q^2 – p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Conjecture 2:

For any pair of primes (p, r), p ≥ 5, r ≥ 5, there exist an infinity of primes q such that the numbers (q^2 – p^2 – 2*r)/2 and (q^2 – p^2 + 2*r)/2 are both primes.

Note:

The numbers m = (q^2 – p^2 – 2*r)/2 and n = (q^2 – p^2 + 2*r)/2, where p, q, r are odd primes, seems to be often primes and generally products of very few primes.

Examples:

: For (p, q, r) = (13, 11, 5) we have (m, n) = (19, 29);
: For (p, q, r) = (17, 11, 5) we have (m, n) = (79, 89);
: For (p, q, r) = (37, 11, 5) we have (m, n) = (1039, 1049);

: For (p, q, r) = (13, 11, 7) we have (m, n) = (17, 31);
: For (p, q, r) = (19, 11, 7) we have (m, n) = (113, 127);
: For (p, q, r) = (23, 11, 7) we have (m, n) = (197, 211);

: For (p, q, r) = (17, 13, 7) we have (m, n) = (53, 67);
: For (p, q, r) = (19, 13, 7) we have (m, n) = (89, 103);
: For (p, q, r) = (37, 13, 7) we have (m, n) = (593, 607);

: For (p, q, r) = (19, 17, 7) we have (m, n) = (29, 43);
: For (p, q, r) = (23, 17, 7) we have (m, n) = (113, 127);
: For (p, q, r) = (29, 17, 7) we have (m, n) = (269, 283);
: For (p, q, r) = (11, 7, 7) we have (m, n) = (29, 43);
: For (p, q, r) = (13, 7, 7) we have (m, n) = (53, 67);
: For (p, q, r) = (17, 7, 7) we have (m, n) = (113, 127);

: For (p, q, r) = (19, 11, 11) we have (m, n) = (109, 131);
: For (p, q, r) = (31, 11, 11) we have (m, n) = (409, 431);
: For (p, q, r) = (61, 11, 11) we have (m, n) = (1789, 1811).

Conjecture 3:

For any p prime, p ≥ 5, there exist an infinity of primes q such that the numbers (q^2 – p^2 – 2*p)/2 and (q^2 – p^2 + 2*p)/2 are both primes.

Conjecture 4:

If x, y and r are odd primes such that y = x + 2*r, where r ≥ 5, then there exist p and q also primes such that x = (q^2 – p^2 – 2*r)/2 and y = (q^2 – p^2 + 2*r)/2.

Examples:

: For (x, y, r) = (17, 31, 7), we have (p, q) = (11, 13);
: For (x, y, r) = (29, 43, 7), we have (p, q) = (17, 19);
: For (x, y, r) = (53, 67, 7), we have (p, q) = (13, 17).

Conjecture 5:

For any p prime, p ≥ 7, there exist a pair of smaller primes (q, r) such that the numbers x = (p^2 – q^2 – 2*r)/2 and y = (p^2 – q^2 + 2*r)/2 are both primes.

Examples:

: For p = 7, (q, r) = (5, 5) and (x, y) = (7, 17);
: For p = 11, (q, r) = (5, 5) and (x, y) = (43, 53) and also (q, r) = (7, 7) and (x, y) = (43, 53) and also (q, r) = (7, 5) and (x, y) = (31, 41);
: For p = 13, (q, r) = (7, 7) and (x, y) = (53, 67) and also (q, r) = (11, 5) and (x, y) = (19, 29) and also (q, r) = (11, 7) and (x, y) = (17, 31).

10. Few possible infinite sets of triplets of primes related in a certain way and an open problem

Abstract. In this paper I make three conjectures about a type of triplets of primes related in a certain way, i.e. the triplets of primes [p, q, r], where 2*p^2 – 1 = q*r and I raise an open problem about the primes of the form q = (2*p^2 – 1)/r, where p, r are also primes.

Conjecture 1:

There exist an infinity of primes p such that 2*p^2 – 1 = q*r, where q and r are also primes.

Examples: such primes are: 5, 19, 23, 29, 31, 47, 53, 61, 67, 71, 79, 83, 97 (...).

Conjecture 2:

If p is prime and 2*p^2 – 1 = q*r, where q and r are also primes, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)*(r + n), such that p + m, q + n and r + n are also primes.

Examples:

: for p = 5, [q, r] = [7, 7]; for [m, n] = [24, 34], [p + n, q + n, r + n] = [29, 41, 41];

: for p = 19, [q, r] = [7, 103]; for [m, n] = [34, 34], [p + n, q + n, r + n] = [53, 41, 137];

: for p = 23, [q, r] = [7, 151]; for [m, n] = [44, 40], [p + n, q + n, r + n] = [67, 47, 191];

: for p = 31, [q, r] = [17, 113]; for [m, n] = [22, 24], [p + n, q + n, r + n] = [53, 41, 137];

: for p = 71, [q, r] = [17, 593]; for [m, n] = [26, 13], [p + n, q + n, r + n] = [97, 31, 607];

: for p = 83, [q, r] = [23, 599]; for [m, n] = [254, 210], [p + n, q + n, r + n] = [307, 233, 809]; also for [m, n] = [198, 258], [p + n, q + n, r + n] = [347, 281, 857];

: for p = 139, [q, r] = [17, 2273]; for [m, n] = [250, 110], [p + n, q + n, r + n] = [389, 127, 2383].

Conjecture 3:

If p is prime and 2*p^2 – 1 = q^2, where q is also prime, there exist an infinity of pairs of even positive integers [m, n] such that 2*(p + m)^2 – 1 = (q + n)^2, such that p + m and q + n are also primes.

Example: for p = 5, q = 7; for [m, n] = [24, 34], [p + n, q + n] = [29, 41].

Open problem:

Which primes q can be written as q = (2*p^2 – 1)/r, where p, r are also primes?

11. Two types of pairs of primes that could be associated to Poulet numbers

Abstract. In this paper I combine two of my objects of study, the Poulet numbers and the different types of pairs of primes and I state two conjectures about few ways in which types of Poulet numbers could be associated with types of pairs of primes.

Conjecture 1:

Any Poulet number of the form 10*n + 1 or 10*n + 9 can be written at least in one way as p*q + 10*k*h, where p and q are primes or powers of primes of the same form from the following four ones: 10*m + 1, 10*m + 3, 10*m + 7 or 10*m + 9, k and h are non-null positive integers and q – p = 10*k.

Verifying the conjecture:
(for the first six such Poulet numbers)

: 341 = 9*(9 + 20) + 4*20 = 9*(9 + 10) + 17*10, so [p, q] = [3^2, 29] or [3^2, 19];
: 561 = 19*(29 + 10) + 1*10 = 9*(9 + 50) + 3*10, so [p, q] = [19, 29] or [3^2, 59];
: 1729 = 23*(23 + 50) + 1*50 = 17*(17 + 80) + 1*80 = 23*(23 + 30) + 17*30 = 27*(27 + 10) + 73*10 = 23*(23 + 20) + 37*20 = 13*(13 + 60) + 13*60 = 7*(7 + 120) + 7*120 = 17*(17 + 30) + 31*30 = 13*(13 + 40) + 26*40, so [p, q] = [23, 73] or [17, 97] or [23, 53] or [3^3, 37] or [23, 43] or [13, 73] or [7, 127] or [17, 47] etc.;
: 2701 = 29*(29 + 60) + 2*60, so [p, q] = [29, 89] etc.;
: 2821 = 29*(29 + 60) + 4*60, so [p, q] = [29, 89] etc.;
: 4369 = 27*(27 + 130) + 1*130, so [p, q] = [3^3, 157] etc.

Note 1:

Some such Poulet numbers can be written as p*q + (q – p), where p, q primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this: 1729 = 23*53 + (53 – 23) = 17*97 + (97 – 17).

Note 2:

Probably this conjecture can stipulate for h to be equal to 1 or prime or power of prime (in the examples above, we found that h is equal to: 2^2 or 17; 1 or 3; 1 or 17 or 73 or 37 or 13 or 7 or 31; 2; 2^2; 1.

Conjecture 2:

For any Poulet number N not divisible by 3 there exist at least a pair of numbers [p, q], where p is prime and q is prime or square of prime, such that N = p^2 + q – 1.

Verifying the conjecture:
(for the first six such Poulet numbers)

: 341 = 7^2 + 293 – 1 = 13^2 + 173 – 1 = 17^2 + 53 – 1, so [p, q] = [7, 293] or [13, 173] or [17, 53];

: 1105 = 13^2 + 937 – 1 = 23^2 + 577 – 1, so [p, q] = [13, 937] or [23, 577];

: 1387 = 23^2 + 859 – 1 = 29^2 + 547 – 1 = 37^2 + 19 – 1, so [p, q] = [23, 859] or [29, 547] or [37, 19];

: 1729 = 7^2 + 41^2 – 1 = 11^2 + 1609 – 1 = 19^2 + 37^2 – 1 = 23^2 + 1201 – 1 = 31^2 + 769 – 1, so [p, q] = [7, 41^2] or [41, 7^2] or [11, 1609] or [19, 37^2] or [37, 19^2] or [23, 1201] or [31, 769].

Note:

Some such Poulet numbers can be written as p^2 + q^2 – 1, where p, q are primes; for instance, the Hardy-Ramanujan number 1729 can be written in two different ways like this: 1729 = 7^2 + 41^2 – 1 = 19^2 + 37^2 – 1.

12. A set of Poulet numbers and generalizations of the twin primes and de Polignac's conjectures inspired by this

Abstract. In this paper I show a set of Poulet numbers, each one of them having the same interesting relation between its prime factors, and I make four conjectures, one about the infinity of this set, one about the infinity of a certain type of duplets respectively triplets respectively quadruplets and so on of primes and finally two generalizations, of the twin primes conjecture respectively of de Polignac's conjecture.

Conjecture 1:

There exist an infinity of Poulet numbers of the form n^2 + 120*n, where n is prime or a composite positive integer.

Note:

In the first case, obviously n is a prime factor of such a Poulet number and the product of the other prime factors is equal to n + 120; for instance, the number 1729 is a part of this set of Poulet numbers because 1729 = 7*13*19 can be written as 13^2 + 13*120 and implicitly 7*19 = 13 + 120. First few such Poulet numbers are:

: 1729 = 7*13*19 = 13^2 + 13*120;
: 4681 = 31*151 = 31^2 + 31*120;
: 6601 = 7*23*41 = 41^2 + 41*120.

Note:

In the second case, obviously n is a product of few prime factors of such a Poulet number and the product of the other prime factors is equal to n + 120. Such a Poulet number is 75361 = 11*13*17*31 = 221^2 + 221*120 and implicitly 11*31 = 13*17 + 120.

Conjecture 2:

There exist an infinity of duplets of primes [p, q] such that p – q = 120; there also exist an infinity of triplets of primes [p1, p2, q] such that p1*p2 – q = 120; there also exist an infinity of quadruplets of primes [p1, p2, p3, q] such that p1*p2*p3 – q = 120; generally, for any non-null positive integer i there exist i primes p1, p2, ..., pi and a prime q such that p1*p2*...*pi – q = 120.

Examples:

: 151 – 31 = 120;
: 7*19 – 13 = 120;
: 7*17*37 – 4283 = 120.

Conjecture 3:
(generalization of the twin primes conjecture)

For any non-null positive integer i there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that p1*p2*...*pi – q = 2.

Conjecture 4:
(generalization of de Polignac’s conjecture)

For any n even positive integer and for any i non-null positive integer there exist an infinity of sets of i + 1 primes p1, p2, ..., pi, q such that p1*p2*...*pi – q = n.

13. A very exhaustive generalization of de Polignac's conjecture

Abstract. In a previous paper I made a generalization of de Polignac's conjecture. In this paper I extend that generalization as much as is possible.

Conjecture:

For any n even positive integer and for any i and j non-null positive integers there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi – q1*q2*...*qj = n.

Case [i, j, n] = [1, 1, 2]:

In this case we have p – q = 2, which gave us the twin primes conjecture.

Case [i, j, n] = [1, 1, n]:

In this case we have p – q = n, which gave us de Polignac's conjecture.

Case [i, j, n] = [2, 1, 2]:

In this case we have p1*p2 – q = 2.

Such triplets of primes [p1, p2, q], are: [7, 13, 89], [7, 19, 131], [7, 37, 257]...Note that the conjecture can be further extended in this case to: for any p1 odd prime there exist an infinity of pairs of primes [p2, q] such that p1*p2 – q = 2.

Case [i, j, n] = [1, 2, 2]:

In this case we have p – q1*q2 = 2.

Such triplets of primes [p, q1, q2], are: [79, 11, 7], [163, 23, 7], [331, 47, 7]...Note that the conjecture can be further extended in this case to: for any q1 odd prime there exist an infinity of pairs of primes [p, q2] such that p – q1*q2 = 2.

Conjecture:
(the most exhaustive generalization of de Polignac's conjecture)

For any n even positive integer and for any i, j, k, l non-null positive integers, for any k given primes a1, a2, ..., ak and for any l given primes b1, b2, ..., bl, there exist an infinity of distinct sets of i primes p1, p2, ..., pi and also an infinity of distinct sets of j primes q1, q2, ..., qj such that p1*p2*...*pi*a1*a2*...*ak – q1*q2*...*qj*b1*b2*...bl = n.

14. A formula which conducts to primes or to a type of composites that could form a class themselves

Abstract. In this paper I present a very simple formula which conducts often to primes or composites with very few prime factors; for instance, for the first 27 consecutive values introduced as "input" in this formula were obtained 10 primes, 4 squares of primes and 12 semiprimes; just 2 from the numbers obtained have three prime factors; but the most interesting thing is that the composites obtained have a special property that make them form a class of numbers themselves.

Observation:

The numbers C = 3^3*(3^3 + n*10) + n*10, where n is a positive integer of the form 4 + 9*k, or in other words C = 2520*k + 1849, are very often primes or numbers with very few prime factors, composites that have certain very interesting properties. Let's see the case of the first 27 consecutive such numbers C; we will consider all 27 numbers but we will list them separatelly in three different lists: the case C is prime or square of prime, the case C is Coman semiprime and the case of the other numbers C (note that a Coman semiprime is a semprime p*q with the property that p – q + 1 is a prime or a square of prime; this is a class of numbers that I met it often in my research, for instance in the study of 2-Poulet numbers, many of these semiprimes having this property, but as well in the study of the prime factors of Carmichael numbers):

The case C is prime or square of prime:

: for k = 0 we have C = 43^2 where 43 prime;
: for k = 1 we have C = 4369 prime;
: for k = 2 we have C = 83^2 where 83 prime;
: for k = 3 we have C = 97^2 where 97 prime;
: for k = 5 we have C = 14449 prime;
: for k = 7 we have C = 19489 prime;
: for k = 11 we have C = 29569 prime;
: for k = 12 we have C = 32089 prime;
: for k = 16 we have C = 42169 prime;
: for k = 19 we have C = 223^2 where 223 prime;
: for k = 20 we have C = 52249 prime;
: for k = 23 we have C = 59809 prime;

The case C is Coman semiprime:

: for k = 4 we have C = 79*151 and 151 – 79 + 1 = 73 prime;
: for k = 6 we have C = 71*239 and 239 – 71 + 1 = 13^2, where 13 prime;
: for k = 8 we have C = 13*1693 and 1693 – 13 + 1 = 41^2, where 41 prime;
: for k = 13 we have C = 53*653 and 653 – 53 + 1 = 601 prime;
: for k = 14 we have C = 107*347 and 347 – 107 + 1 = 241 prime;
: for k = 15 we have C = 31*1279 and 1279 – 31 + 1 = 1249 prime;
: for k = 24 we have C = 157*397 and 397 – 157 + 1 = 241 prime.

: for k = 25 we have C = 64849 prime;
: for k = 26 we have C = 67369 prime.

The other numbers C:

: for k = 9 we have C = 19*1291 and 1291 – 19 + 1 = 19*67 and 67 – 19 + 1 = 7^2, where 7 prime;
: for k = 10 we have C = 11*2459 and 2459 – 11 + 1 = 31*79 and 79 – 31 + 1 = 7^2, where 7 prime;
: for k = 17 we have C = 23*29*67 and 23*29 – 67 + 1 = 601 prime, 29*67 – 23 + 1 = 17*113 where 113 – 17 + 1 = 97 prime and 23*67 – 28 = 17*89 where 89 – 17 + 1 = 73 prime;
: for k = 18 we have C = 17*2777 and 2777 – 17 + 1 = 11*251 and 251 – 11 + 1 = 241 prime;
: for k = 21 we have C = 11*13*383 and 11*13 – 383 + 1 = -239 prime in absolute value, 11*383 – 13 + 1 = 4201 prime, 13*383 – 11 + 1 = 4969 prime;
: for k = 22 we have C = 59*971 and 971 – 59 + 1 = 11*83 and 83 – 11 + 1 = 73 prime;
: for k = 27 we have C = 47*1487 and 1487 – 47 + 1 = 11*131 and 131 – 11 + 1 = 11^2, where 11 prime.

Note:

It can be seen that also "the other numbers C" have special properties; for instance, the semiprimes can be considered a kind of "extended Coman semiprimes" because of the iterative process that ends also in a prime or in a square of prime: let N = p1*q1; than p1- q1 + 1 = p2*q2 then p2 – q2 + 1 = p3*q3 and so on until is obtained a prime. On the other side, the numbers with three prime factors obtained p*q*r have the property that p*q – r + 1, p*r – q + 1 and q*r – p + 1 are primes or (extended) Coman semiprimes.

15. Four sequences of numbers obtained through concatenation, rich in primes and semiprimes

Abstract. In this paper I will define four sequences of numbers obtained through concatenation, definitions which also use the notion of “sum of the digits of a number”, sequences that have the property to produce many primes, semiprimes and products of very few prime factors.

Observation 1:

Let x be a number with the sum of the digits equal to p, where p is prime, and y the number obtained through concatenation of x and p; then y is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 2911 (semiprime), 9211 (semiprime), 4913 (cube of prime), 9413 (prime), 8917 (semiprime), 9817 (prime), 99119 (prime), 91919 (semiprime), 19919 (prime), 99523 (prime), 95923 (prime), 59923 (semiprime), 999431 (prime), 949931 (prime), 499931 (semiprime) etc.

Observation 2:

Let x be a number equal to the sum of the digits of p, where p is prime, and y the number obtained through concatenation of x and p; then y is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 211 (prime), 413 (semiprime), 817 (semiprime), 1019 (prime), 523 (prime), 1129 (prime), 431 (prime), 1037 (semiprime), 541 (prime), 743 (prime), 1147 (semiprime), 853 (prime), 1459 (prime), 761 (prime), 1367 (prime), 871 (semiprime), 1073 (semiprime) etc.

Observation 3:

Let x be a number whose sum of the digits is equal to the sum of the digits of p, where p is prime, and y the number obtained through concatenation of x and p; then y is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: 1111 (semiprime), 10111 (prime), 2011 (prime), 20011 (prime), 200011 (semiprime), 2213 (prime), 22013 (prime), 3113 (semiprime), 4013 (prime), 40013 (prime), 400013 (semiprime), 4000013 (semiprime) etc.

Observation 4:

Let x be a number with the sum of the digits equal to p, where p is prime, let y = 6*x + 5 and z the number obtained through concatenation of y and p; then z is often a prime, a semiprime or a product of very few prime factors.

Numbers that belong to this sequence are:

: For [p, x, y] = [11, 29, 179] we have z = 17911 prime;
: For [p, x, y] = [11, 92, 557] we have z = 55711 prime;
: For [p, x, y] = [11, 902, 5417] we have z = 541711 prime;
: For [p, x, y] = [29, 9299, 55799] we have z semiprime;
: For [p, x, y] = [29, 9929, 59579] we have z semiprime;
: For [p, x, y] = [29, 9992, 59957] we have z = 5995729 prime;
: For [p, x, y] = [29, 2999, 17999] we have z = 1799929 prime;
: For [p, x, y] = [29, 9299, 55799] we have z semiprime;
: For [p, x, y] = [29, 9929, 59579] we have z semiprime;
: For [p, x, y] = [29, 9992, 59957] we have z = 5995729 prime.

Note:

In order to see wherefrom the idea of this sequence originate see my previous paper “A conjecture about a large subset of Carmichael numbers related to concatenation”.

16. A conjecture on the squares of primes of the form 6k – 1

Abstract. In this paper I make a conjecture on the squares of primes of the form 6k – 1, conjecture that states that by a certain deconcatenation of those numbers (each one in other two numbers) it will be obtained similar results.

Conjecture:

For any square of a prime p of the form p = 6*k – 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime q*r where r – q = 8*k and a number congruent to 2, 3 or 5 modulo 6;
(3) p^2 can be deconcatenated into a semiprime 3*q, where q is of the form 10*k + 7, and a number congruent to 1 modulo 6;
(4) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(5) p^2 can be deconcatenated into a number of the form 121 + 48*k and a number congruent to 0 modulo 6;
(6) p^2 is a palindromic number.

Examples for case (1):

: for 5^2 = 25 we got 5 prime and 2 ≡ 2 (mod 6);
: for 17^2 = 289 we got 89 prime and 2 ≡ 2 (mod 6);
: for 23^2 = 529 we got 29 prime and 5 ≡ 5 (mod 6);
: for 29^2 = 841 we got 41 prime and 8 ≡ 2 (mod 6);
: for 53^2 = 2809 we got 809 prime and 2 ≡ 2 (mod 6);
: for 71^2 = 5041 we got 41 prime and 50 ≡ 2 (mod 6);
: for 83^2 = 6889 we got 89 prime and 68 ≡ 2 (mod 6);
: for 107^2 = 11449 we got 449 prime and 11 ≡ 5 (mod 6);
: for 167^2 = 27889 we got 89 prime and 278 ≡ 2 (mod 6);
: for 173^2 = 29929 we got 29 prime and 29 ≡ 5 (mod 6) also 929 prime and 2 ≡ 2 (mod 6);
: for 179^2 = 32041 we got 41 prime and 320 ≡ 2 (mod 6);
: for 191^2 = 36481 we got 6481 prime and 3 ≡ 3 (mod 6);
: for 197^2 = 38809 we got 809 prime and 38 ≡ 2 (mod 6);
: for 227^2 = 51529 we got 29 prime and 515 ≡ 5 (mod 6);
: for 233^2 = 54289 we got 89 prime and 542 ≡ 2 (mod 6);
: for 239^2 = 57121 we got 7121 prime and 5 ≡ 5 (mod 6);
: for 269^2 = 72361 we got 61 prime and 723 ≡ 3 (mod 6).

Examples for case (2):

: for 47^2 = 2209 we got 209 = 11*19 where 19 – 11 = 8*1 and 2 ≡ 2 (mod 6);
: for 59^2 = 3481 we got 481 = 13*37 where 37 – 13 = 8*3 and 3 ≡ 3 (mod 6);

: for 131^2 = 17161 we got 161 = 7*23 where 23 – 7 = 8*2 and 17 ≡ 5 (mod 6);
: for 149^2 = 22201 we got 2201 = 31*71 where 71 – 31 = 8*5 and 2 ≡ 2 (mod 6).

Examples for case (3):

: for 41^2 = 1681 we got 681 = 3*227 and 1 ≡ 1 (mod 6);
: for 89^2 = 7921 we got 921 = 3*307 and 7 ≡ 1 (mod 6).

Examples for case (4):

: for 83^2 = 6889 we got 889 = 49 + 120*7 and 6 ≡ 0 (mod 6);
: for 113^2 = 12769 we got 769 = 49 + 120*6 and 12 ≡ 0 (mod 6);
: for 137^2 = 18769 we got 769 = 49 + 120*6 and 18 ≡ 0 (mod 6);
: for 257^2 = 66049 we got 6049 = 49 + 120*50 and 6 ≡ 0 (mod 6);
: for 263^2 = 69169 we got 9169 = 49 + 120*76 and 6 ≡ 0 (mod 6).

Examples for case (5):

: for 251^2 = 63001 we got 3001 = 121 + 48*60 and 6 ≡ 0 (mod 6).

Examples for case (6):

: 11^2 = 121;
: 101^2 = 10201.

Note:

This conjecture is verified up to p = 269.

17. A conjecture on the squares of primes of the form 6k + 1

Abstract. In this paper I make a conjecture on the squares of primes of the form 6k + 1, conjecture that states that by a certain deconcatenation of those numbers (each one in other two numbers) it will be obtained similar results.

Conjecture:

For any square of a prime p of the form p = 6*k + 1 is true at least one of the following six statements:

(1) p^2 can be deconcatenated into a prime and a number congruent to 2, 3 or 5 modulo 6;
(2) p^2 can be deconcatenated into a semiprime 3^n*q and a number congruent to 1 modulo 6;
(3) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 1;
(4) p^2 can be deconcatenated into a number n such that n + 1 is prime or power of prime and the digit 9;
(5) p^2 can be deconcatenated into a number of the form 49 + 120*k and a number congruent to 0 modulo 6;
(6) p^2 can be deconcatenated into a number of the form 121 + 24*k and a number congruent to 0 modulo 6.

Examples for case (1):

: for 67^2 = 4489 we got 89 prime and 44 ≡ 2 (mod 6);
: for 73^2 = 5329 we got 29 prime and 53 ≡ 5 (mod 6);
: for 79^2 = 6241 we got 41 prime and 62 ≡ 2 (mod 6);
: for 109^2 = 11881 we got 881 prime and 11 ≡ 2 (mod 6);
: for 163^2 = 26569 we got 569 prime and 26 ≡ 2 (mod 6);
: for 181^2 = 32761 we got 761 prime and 32 ≡ 5 (mod 6);
: for 199^2 = 39601 we got 601 prime and 39 ≡ 3 (mod 6).

Examples for case (2):

: for 13^2 = 169 we got 69 = 3*23 and 1 ≡ 1 (mod 6);
: for 37^2 = 1369 we got 369 = 3^2*41 and 1 ≡ 1 (mod 6);
: for 43^2 = 1849 we got 849 = 3*283 and 1 ≡ 1 (mod 6);
: for 61^2 = 3721 we got 21 = 3*7 and 37 ≡ 1 (mod 6);
: for 127^2 = 16129 we got 6129 = 3^3*227 and 1 ≡ 1 (mod 6);
: for 193^2 = 37249 we got 249 = 3*83 and 37 ≡ 1 (mod 6).

Examples for case (3):

: for 19^2 = 361 we got 36 + 1 = 37 prime;
: for 31^2 = 961 we got 96 + 1 = 97 prime;
: for 79^2 = 6241 we got 624 + 1 = 625 power of prime.

: for 139^2 = 19321 we got 1932 + 1 = 1933 prime;
: for 151^2 = 22801 we got 2280 + 1 = 2281 prime.

Examples for case (4):

: for 7^2 = 49 we got 4 + 1 = 5 prime;
: for 97^2 = 9409 we got 940 + 1 = 941 prime;
: for 103^2 = 10609 we got 1060 + 1 = 1061 prime.

Examples for case (5):

: for 157^2 = 24649 we got 649 = 49 + 120*5 and 24 ≡ 0 (mod 6).

Examples for case (6):

: for 79^2 = 6241 we got 241 = 121 + 24*5 and 6 ≡ 0 (mod 6).

Note:

This conjecture is verified up to p = 199.

Note:

I mention that this conjecture and the one from my previous paper "A conjecture on the squares of primes of the form 6k – 1" were made with the title of *jocandi causa*. It is not relevant if they are not true if they raise interesting questions about squares of primes.

18. Nine conjectures on the infinity of certain sequences of primes

Abstract. In this paper I enunciate nine conjectures on primes, all of them on the infinity of certain sequences of primes.

Conjecture 1:

For any prime p there exist an infinity of positive integers n such that the number n*p – n + 1 is prime.

Examples:

: For p = 19 we have the following primes: 2*19 – 1 = 37; 4*19 – 3 = 73; 6*19 – 5 = 109; 7*19 – 6 = 127; 9*19 – 8 = 163; 10*19 - 9 = 181 etc.

Conjecture 2:

For any prime p there exist an infinity of positive integers n such that the number n*p + n - 1 is prime.

Examples:

: For p = 11 we have the following primes: 2*11 + 1 = 23; 4*11 + 3 = 47; 5*11 + 4 = 59; 6*11 + 5 = 71; 7*11 + 6 = 83; 9*11 + 8 = 107 etc.

Conjecture 3:

For any prime p there exist an infinity of positive integers n such that the number n^2*p – n + 1 is prime.

Examples:

: For p = 7 we have the following primes: 3^2*7 – 2 = 61; 4^2*7 – 3 = 109; 7^2*7 – 6 = 337; 10^2*7 – 9 = 691; 12^2*7 – 11 = 997 etc.

Conjecture 4:

For any prime p there exist an infinity of positive integers n such that the number n^2*p + n - 1 is prime.

Examples:

: For p = 11 we have the following primes: 3^2*11 + 2 = 101; 4^2*11 + 3 = 179; 6^2*11 + 5 = 401; 10^2*11 + 9 = 1109; 13^2*11 + 12 = 1871 etc.

Conjecture 5:

For any prime p there exist an infinity of positive integers n such that the number n*p – p + n is prime.

Examples:

: For p = 5 we have the following primes: 1*5 + 2 = 7; 2*5 + 3 = 13; 3*5 + 4 = 19; 5*5 + 6 = 31; 6*5 + 7 = 37; 7*5 + 8 = 43 etc.

Conjecture 6:

For any prime p there exist an infinity of positive integers n such that the number n*p - p - n is prime.

Examples:

: For p = 5 we have the following primes: 1*5 - 2 = 3; 2*5 - 3 = 7; 5*5 - 6 = 19; 6*5 - 7 = 23; 8*5 - 9 = 31; 11*5 - 12 = 43 etc.

Conjecture 7:

For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p + n is prime.

Examples:

: For p = 7 we have the following primes: 2^2*7 + 3 = 31; 3^2*7 + 4 = 67; 5^2*7 + 4 = 179; 6^2*7 + 5 = 257; 7^2*7 + 6 = 349 etc.

Conjecture 8:

For any prime p there exist an infinity of positive integers n such that the number (n – 1)^2*p - n is prime.

Examples:

: For p = 7 we have the following primes: 3^2*7 - 4 = 59; 4^2*7 - 5 = 107; 8^2*7 - 9 = 439; 9^2*7 - 10 = 557; 15^2*7 - 16 = 1559 etc.

Conjecture 9:

For any two distinct primes greater than three p and q there exist an infinity of positive integers n such that the number (p^2 – 1)*n + q^2 is prime, also an infinity of positive integers m such that the number (q^2 – 1)*n + p^2 is prime.

Examples:

: For (p, q) = (7, 11) we have the following primes of the form 48*n + 121: 313, 409, 457, 601, 937, 1033 etc. and the following primes of the form 120*n + 49: 409, 769, 1009 etc.

Note:
The idea of these sequences didn’t come to me from “nowhere”. Many from the types of primes presented in this paper are met in the study of Fermat peudoprimes.

19. Five conjectures on primes based on the observation of Poulet and Carmichael numbers

Abstract. In this paper I enunciate five conjectures on primes, based on the study of Fermat pseudoprimes and on the author's believe in the importance of multiples of 30 in the study of primes.

Conjecture 1:

For any p, q distinct primes, p > 30, there exist n positive integer such that p – 30*n and q + 30*n are both primes.

Note:

This conjecture is based on the observation of 2-Poulet numbers (see my paper "A conjecture about 2-Poulet numbers and a question about primes").

Conjecture 2:

For any p, q, r distinct primes there exist n positive integer such that the numbers 30*n – p, 30*n – q and 30*n – r are all three primes.

Note:

This enunciation is obviously equivalent to the enunciation that there exist m such that p + 30*m, q + 30*m and r + 30*m are all three primes (take x = 30*n – p, y = 30*n – q and z = 30*n – r. Then there exist k such that 30*k – 30*n + p, 30*k – 30*n + q and 30*k – 30*n + r are all three primes).

Note:

This conjecture implies of course that for any pair of twin primes (p, q) there exist a pair of primes (30*n – p, 30*n – q) so that there are infinitely many pairs of twin primes.

Note:

This conjecture is based on the observation of 3-Carmichael numbers (see my paper "A conjecture about primes based on heuristic arguments involving Carmichael numbers).

Conjecture 3:

There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q + (q - p) is prime.

Note:

This conjecture is based on the observation of Carmichael numbers.

Examples:

: 31*151 + (151 – 31) = 4801 prime;
: 37*127 + (127 – 37) = 4789 prime;
: 41*101 + (101 – 41) = 4201 prime;
: 13*103 + (103 – 13) = 1429 prime;
: 17*47 + (47 – 17) = 829 prime;
: 19*109 + (109 – 19) = 2161 prime;
: 23*53 + (53 – 23) = 1249 prime.

Conjecture 4:

There exist an infinity of pairs of distinct primes (p, q), where p < q, both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

Note:

This conjecture is based on the observation of Carmichael numbers.

Examples:

: 31*61 – (61 – 31) = 1861 prime;
: 7*37 – (37 – 7) = 229 prime;
: 11*41 – (41 – 11) = 421 prime;
: 13*73 – (73 – 13) = 919 prime;
: 17*47 – (47 – 17) = 769 prime;
: 19*139 - (139 – 19) = 2521 prime;
: 23*293 - (293 – 23) = 6469 prime.

Conjecture 5:

For any p prime there exist an infinity of primes q, q > p, where p and q are both of the same form from the following eight ones: 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 such that the number p*q - (q - p) is prime.

20. Six conjectures on primes based on the study of 3-Carmichael numbers and a question about primes

Abstract. In this paper are stated six conjectures on primes, more precisely on the infinity of some types of pairs of primes, all of them met in the study of 3-Carmichael numbers.

Conjecture 1:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m + 1) – n and y = q*(n + 1) – m are both primes.

Examples:
: for [p, q] = [3, 3] we have [x, y] = [5, 13] for [m, n] = [2, 4];
: for [p, q] = [7, 11] we have [x, y] = [29, 73] for [m, n] = [4, 6].

Conjecture 2:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*(m - 1) + n and y = q*(n - 1) + m are both primes.

Examples:
: for [p, q] = [7, 7] we have [x, y] = [11, 23] for [m, n] = [2, 4];
: for [p, q] = [5, 13] we have [x, y] = [11, 67] for [m, n] = [2, 6].

Conjecture 3:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p + (m + 1)*n and y = q + m*n are both primes.

Examples:
: for [p, q] = [5, 5] we have [x, y] = [17, 13] for [m, n] = [2, 4];
: for [p, q] = [5, 7] we have [x, y] = [29, 23] for [m, n] = [2, 8].

Conjecture 4:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n + 2*m are both primes.

Examples:
: for [p, q] = [11, 11] we have [x, y] = [23, 61] for [m, n] = [3, 5];
: for [p, q] = [11, 13] we have [x, y] = [23, 71] for [m, n] = [3, 5].

Conjecture 5:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m – 2*n and y = q*n - 2*m are both primes.

Examples:
: for [p, q] = [3, 3] we have [x, y] = [7, 17] for [m, n] = [11, 13];
: for [p, q] = [3, 5] we have [x, y] = [13, 61] for [m, n] = [17, 19].

Conjecture 6:

For any pair of odd primes [p, q] there exist an infinity of pairs of distinct positive integers [m, n] such that the numbers x = p*m + 2*n and y = q*n + 2*m are both primes.

Examples:
: for [p, q] = [5, 5] we have [x, y] = [29, 41] for [m, n] = [3, 7];
: for [p, q] = [5, 11] we have [x, y] = [19, 79] for [m, n] = [1, 7].

Question:

Are there an infinity of primes with the property that can be written as p*m + n – q as well as q*n + m – p, where p, q are distinct primes and m, n are distinct positive integers? But under the condition that m, n, p, q are all four primes? Such number is, for instance, 397 = 13*31 + 7 – 13 = 61*7 + 31 – 61.

Note:

Like I already said in Abstract, I met these pairs of primes in the study of 3-Carmichael numbers: see my previous paper “Connections between the three prime factors of a 3-Carmichael number”.

21. Ten conjectures on primes based on the study of Carmichael numbers, involving the multiples of 30

Abstract. In this paper are stated ten conjectures on primes, more precisely on the infinity of some types of triplets and quadruplets of primes, all of them using the multiples of the number 30 and also all of them met on the study of Carmichael numbers.

Conjecture 1:

There exist an infinity of positive integers n such that the numbers 30*n + 7, 60*n + 13 and 150*n + 31 are all three primes.

The sequence of these numbers n is: 0, 1, 2, 3 (...), corresponding to the triplets of primes [7, 13, 31], [37, 73, 181], [67, 73, 181], [97, 193, 481]...

Conjecture 2:

There exist an infinity of positive integers n such that the numbers 30*n - 23, 60*n - 47 and 90*n - 71 are all three primes.

The sequence of these numbers n is: 1, 2, 3 (...), corresponding to the triplets of primes [7, 13, 19], [37, 73, 109], [67, 133, 199]...

Conjecture 3:

There exist an infinity of positive integers n such that the numbers 30*n - 29, 60*n - 59 and 90*n - 89 are all three primes.

The sequence of these numbers n is: 8, 10 (...), corresponding to the triplets of primes [211, 421, 691], [271, 541, 811]...

Conjecture 4:

There exist an infinity of positive integers n such that the numbers 30*n - 7, 90*n - 23 and 300*n - 79 are all three primes.

The sequence of these numbers n is: 2, 9 (...), corresponding to the triplets of primes [53, 157, 521], [263, 787, 2621]...

Conjecture 5:

There exist an infinity of positive integers n such that the numbers 30*n - 17, 90*n - 53 and 150*n - 89 are all three primes.

The sequence of these numbers n is: 1, 2 (...), corresponding to the triplets of primes [13, 37, 61], [43, 127, 211]...

Conjecture 6:

There exist an infinity of positive integers n such that the numbers 60*n + 13, 180*n + 37 and 300*n + 61 are all three primes.

The sequence of these numbers n is: 2, 6 (...), corresponding to the triplets of primes [133, 397, 661], [373, 1117, 1861]...

Conjecture 7:

There exist an infinity of positive integers n such that the numbers 330*n + 7, 660*n + 13, 990*n + 19 and 1980*n + 37 are all four primes.

The sequence of these numbers n is: 1 (...), corresponding to the quadruplets of primes [133, 397, 661]...

Conjecture 8:

There exist an infinity of positive integers n such that the numbers 90*n + 1, 180*n + 1, 270*n + 1 and 540*n + 1 are all four primes.

The sequence of these numbers n is: 3 (...), corresponding to the quadruplets of primes [271, 541, 811, 1621]...

Conjecture 9:

There exist an infinity of pairs of primes [p, q] such that the numbers p + 30*n, q + 30*n and p*q + 30*n are all three primes.

Examples: [p, q] = [7, 7], [7, 11], [11, 7] etc. corresponding to the triplets [37, 67, 137], [37, 71, 167], [41, 67, 167] etc.

Conjecture 10:

There exist an infinity of primes p such that the numbers x = 30*n + p and y = 30*m*n + m*p – m + 1, where m, n are non-null positive integers, are both primes.

Examples:

: for p = 7 we have [x, y] = [30*n + 7, 30*m*n + 6*m + 1]; for [m, n] = [2, 1] we have [x, y] = [37, 73];

: for p = 11 we have [x, y] = [30*n + 11, 30*m*n + 10*m + 1]; for [m, n] = [2, 1] we have [x, y] = [41, 101];

Note:

Like I already said in Abstract, I met these triplets and quadruplets of primes in the study of Carmichael numbers: see my previous paper “A list of 13 sequences of Carmichael numbers based on the multiples of the number 30”.

22. Two sequences of primes whose formulas contain the number 360

Abstract. In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the number 360.

Conjecture 1:

There exist an infinity of primes of the form 360*p*q + 1, where p, q are primes, both greater than or equal to 7.

The first few such primes:
: 360*7*17 + 1 = 42841;
: 360*7*19 + 1 = 47881;
: 360*11*13 + 1 = 51481;
: 360*13*17 + 1 = 79561;
: 360*11*23 + 1 = 91081;
: 360*13*23 + 1 = 107641.

Conjecture 2:

There exist an infinity of primes of the form 360*p*q + r, where p, q, r are primes, all of them greater than or equal to 7.

The first few such primes for p = q = 7:
: 360*7*7 + 17 = 17657;
: 360*7*7 + 19 = 17659;
: 360*7*7 + 29 = 17669;
: 360*7*7 + 41 = 17681;
: 360*7*7 + 41 = 17683.

The first few such primes for p = 7, q = 11:
: 360*7*11 + 13 = 27733;
: 360*7*11 + 17 = 27737;
: 360*7*11 + 19 = 27739;
: 360*7*11 + 23 = 27743;
: 360*7*11 + 29 = 27749;
: 360*7*11 + 31 = 27751.

Note the six consecutive primes obtained above!

23. Two sequences of primes whose formulas contain the powers of the number 2

Abstract. In this paper I present two possible infinite sequences of primes, having in common the fact that their formulas contain the powers of the number 2.

Conjecture 1:

There exist an infinity of primes of the form 2^m + n^2, where m is non-null positive integer and n odd integer.

The first few such primes for [m, n] = [2, n]:
: 3^2 + 4 = 13 for n = 3;
: 5^2 + 4 = 29 for n = 5;
: 7^2 + 4 = 53 for n = 7;
: 13^2 + 4 = 173 for n = 13;
: 17^2 + 4 = 293 for n = 17.

The first few such primes for [m, n] = [4, n]:
: 5^2 + 16 = 41 for n = 5;
: 11^2 + 16 = 137 for n = 11;
: 29^2 + 16 = 857 for n = 29;
: 31^2 + 16 = 977 for n = 31;
: 41^2 + 16 = 1697 for n = 41.

The first few such primes for [m, n] = [8, n]:
: 5^2 + 256 = 281 for n = 5;
: 19^2 + 256 = 617 for n = 19;
: 29^2 + 256 = 1097 for n = 29;
: 31^2 + 256 = 1217 for n = 31;
: 71^2 + 256 = 5297 for n = 71.

The first few such primes for [m, n] = [m, 1]:
: 2^1 + 1 = 3 for m = 1;
: 2^2 + 1 = 5 for m = 2;
: 2^4 + 1 = 17 for m = 4;
: 2^8 + 1 = 257 for m = 8;
: 2^16 + 1 = 65537 for m = 16.

The first few such primes for [m, n] = [m, 3]:
: 2^1 + 9 = 11 for m = 1;
: 2^2 + 9 = 13 for m = 2;
: 2^3 + 9 = 17 for m = 3;
: 2^5 + 9 = 41 for m = 5;
: 2^6 + 9 = 73 for m = 6.

Conjecture 2:

There exist an infinity of primes of the form (2^n)^k + 2^n + 1, where n is non-null positive integer and k positive integer.

The first few such primes for [n, k] = [n, 1]:
: 5 for n = 1;
: 17 for n = 3;
: 257 for n = 7.

The first few such primes for [n, k] = [n, 2]:
: 7 for n = 1;
: 73 for n = 3;
: 262657 for n = 9.

The first few such primes for [n, k] = [n, 3]:
: 11 for n = 1;
: 521 for n = 3;
: 32801 for n = 5.

The first few such primes for [n, k] = [1, k]:
: 5 for k = 1;
: 7 for k = 2;
: 11 for k = 3.

The first few such primes for [n, k] = [3, k]:
: 17 for k = 1;
: 73 for k = 2;
: 521 for k = 3.

The first few such primes for [n, k] = [5, k]:
: 32801 for k = 3;
: 1048609 for k = 4;
: 1073741857 for k = 6.

24. Conjectures about a way to express a prime as a sum of three other primes of a certain type

Abstract. These conjectures state that any prime p greater than 60 can be written as a sum of three primes of a certain type from the following four ones: 10k + 1, 10k + 3, 10k + 7 and 10k + 9.

Conjecture 1a:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Examples:
: 61 = 11 + 31 + 19;
: 71 = 11 + 31 + 29 = 11 + 41 + 19.

Conjecture 1b:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 3 respectively 10*z + 7.

Examples:
: 61 = 41 + 13 + 7 = 31 + 23 + 7 = 31 + 13 + 17;
: 71 = 41 + 23 + 7 = 41 + 13 + 17 = 31 + 23 + 7.

Conjecture 1c:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 7, 10*y + 7 respectively 10*z + 7.

Examples:
: 61 = 7 + 17 + 37 = 7 + 7 + 47;
: 71 = 17 + 17 + 37 = 7 + 17 + 47.

Conjecture 1d:

Any prime p of the form 10*k + 1, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 9 respectively 10*z + 9.

Examples:
: 61 = 13 + 19 + 29 = 23 + 19 + 19;
: 71 = 23 + 19 + 29 = 13 + 29 + 29.

Conjecture 2a:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 1 respectively 10*z + 1.

Conjecture 2b:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 3 respectively 10*z + 9.

Conjecture 2c:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 3 respectively 10*z + 7.

Conjecture 2d:

Any prime p of the form 10*k + 3, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 7, 10*y + 7 respectively 10*z + 9.

Conjecture 3a:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 3 respectively 10*z + 3.

Conjecture 3b:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 7 respectively 10*z + 7.

Conjecture 3c:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 7 respectively 10*z + 9.

Conjecture 3d:

Any prime p of the form 10*k + 7, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 9, 10*y + 9 respectively 10*z + 9.

Conjecture 4a:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 1 respectively 10*z + 7.

Conjecture 4b:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 3 respectively 10*z + 3.

Conjecture 4c:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 3, 10*y + 7 respectively 10*z + 9.

Conjecture 4d:

Any prime p of the form 10*k + 9, p > 60, can be written as a sum of three primes of the following forms:
: 10*x + 1, 10*y + 9 respectively 10*z + 9.

Addenda

In one of my previous papers, "Two conjectures that relates any Poulet number by a type of triplets respectively of duplets of primes" I made the following two conjectures:

Conjecture:

Any square of a prime of the form p^2 = 10*k + 1 can be written as p^2 = x + y + z, where x, y, z are primes, not necessarily all three distinct, of the form 10*k + 7.
Examples:
: 11^2 = 121 = 37 + 37 + 47;
: 19^2 = 361 = 7 + 37 + 317.

Conjecture:

Any square of a prime of the form p^2 = 10*k + 9 can be written as p^2 = x + y + z, where x, y, z are primes, not necessarily all three distinct, of the form 10*k + 3.
Examples:
: 7^2 = 49 = 13 + 13 + 23;
: 19^2 = 169 = 13 + 43 + 113.

25. A bold conjecture about a way in which any prime can be written

Abstract. In this paper I make a conjecture which states that any prime greater than or equal to 5 can be written in a certain way, in other words that any such prime can be expressed using just two other primes and a power of the number 2.

Conjecture:

Any prime greater than or equal to 5 can be written at least in one way as (9*p^2 – q^2)/(2^n), where p and q are primes and n non-null positive integer.

Verifying the conjecture:
(For the first nine such primes)

: 5 = (9*7^2 – 11^2)/64, so [p, q, n] = [7, 11, 6] but also 5 = (9*7^2 – 19^2)/16 so [p, q, n] = [7, 19, 4];
: 7 = (9*5^2 – 13^2)/64, so [p, q, n] = [5, 13, 3];
: 11 = (9*5^2 – 7^2)/16, so [p, q, n] = [5, 7, 4];
: 13 = (9*5^2 – 11^2)/8, so [p, q, n] = [5, 11, 3] but also 13 = (9*7^2 – 5^2)/32 so [p, q, n] = [7, 5, 5];
: 17 = (9*7^2 – 13^2)/16, so [p, q, n] = [7, 13, 4];
: 19 = (9*7^2 – 17^2)/8, so [p, q, n] = [7, 17, 3] but also 19 = (9*13^2 – 37^2)/8 so [p, q, n] = [13, 37, 3] but also 19 = (9*17^2 – 13^2)/128 so [p, q, n] = [17, 11, 7];
: 23 = (9*13^2 – 7^2)/64, so [p, q, n] = [13, 7, 6];
: 31 = (9*11^2 – 29^2)/8, so [p, q, n] = [11, 29, 3] but also 31 = (9*13^2 – 23^2)/32 so [p, q, n] = [13, 23, 5];
: 37 = (9*23^2 – 5^2)/128, so [p, q, n] = [23, 5, 7].

Note:

For the prime 29 I didn’t find primes solution [p, q] up to the denominator 2^12, but surely I conjecture that there exist such solutions.

Note:

For some of the primes we found that they verify also the formula (9*p^2 – q^4)/(2^n).

26. Two conjectures, on the primes of the form 6k + 1 respectively of the form 6k – 1

Abstract. In this paper I make two conjectures, one about how could be expressed a prime of the form 6k + 1 and one about how could be expressed a prime of the form 6k – 1.

Conjecture 1:

Any prime p of the form 6*k + 1 greater than or equal to 13 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Note:
Because we have 5^2 – 5 = 20, 11^2 – 11 = 110, 17^2 – 17 = 272, 23^2 – 23 = 506 and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers 3*p – 20, 3*p – 110, 3*p – 272, 3*p – 506 and so on.

Verifying the conjecture:
(up to p = 229)

: 13*3 – 20 = 19, prime, so [p, q, r] = [13, 5, 19];
: 19*3 – 20 = 37, prime, so [p, q, r] = [19, 5, 37];
: 31*3 – 20 = 73, prime, so [p, q, r] = [31, 5, 73];
: 37*3 – 110 = 1, so [p, q, r] = [37, 11, 1];
: 43*3 – 20 = 109, prime, so [p, q, r] = [43, 5, 109] and also 43*3 – 110 = 29, prime, so [p, q, r] = [43, 11, 29];
: 61*3 – 20 = 163, prime, so [p, q, r] = [61, 5, 163] and also 61*3 – 110 = 73, prime, so [p, q, r] = [61, 11, 73];

We also found the following triplets [p, q, r]: [67, 5, 181], [73, 5, 199], [73, 11, 109], [79, 11, 127], [97, 5, 271], [97, 11, 181], [97, 17, 19], [103, 11, 199], [109, 5, 307], [127, 11, 271], [127, 17, 109], [139, 5, 397], [139, 11, 307], [151, 5, 433], [151, 17, 71], [157, 17, 199], [163, 1, 379], [181, 5, 523], [181, 11, 433], [181, 17, 271], [181, 23, 37], [193, 17, 307], [193, 23, 73], [199,5, 577], [199, 11, 487], [211, 5, 613], [211, 11, 523], [211, 17, 19^2], [211, 23, 127], [223, 17, 397], [223, 506, 163], [229, 11, 577], [229, 23, 181], so the conjecture is verified up to p = 229.

Conjecture 2:

Any prime p of the form 6*k – 1 greater than or equal to 11 can be written as (q^2 – q + r)/3, where q is prime of the form 6*k – 1 and r is prime or power of prime or number 1.

Note:
Because we have 5^2 – 5 = 20, 11^2 – 11 = 110, 17^2 – 17 = 272, 23^2 – 23 = 506 and so on, the conjecture is equivalent to the existence of a prime or power of prime among the numbers 3*p – 20, 3*p – 110, 3*p – 272, 3*p – 506 and so on.

Verifying the conjecture:
(up to p = 179)

: 11*3 – 20 = 13, prime, so [p, q, r] = [11, 5, 13];
: 17*3 – 20 = 31, prime, so [p, q, r] = [17, 5, 31];
We also found the following triplets [p, q, r]: [29, 5, 67], [41, 5, 103}, [47, 11, 31], [53, 5, 129], [59, 5, 157], [59, 11, 67], [71, 5, 193], [71, 11, 103], [83, 5, 229], [83, 11, 139], [891, 11, 157], [101, 5, 283], [101, 11, 193], [101, 17, 31], [107, 11, 211], [113, 11, 229], [113, 17, 67], [131, 5, 373], [131, 11, 283], [137, 17, 139], [149, 11, 337], [167, 17, 229], [173, 5, 449], [173, 11, 409], [173, 23, 13], [179, 23, 31] so the conjecture is verified up to p = 179.

Comment:

In the case that the conjectures are invalidated, still remain two open problems:
(1) Which are the smallest primes that don't satisfy each from the two conjectures?;
(2) Which is the maximum length of a chain formed in the following way: p_2 = 3*p_1 – (q^2 – q), p_3 = 3*p_2 – (q^2 – q), ..., p_n = 3*p_{n-1} – (q^2 – q)? For instance, such a chain of length 3 is [43, 109, 307] for q = 5.

27. A possible way to write any prime, using just another prime and the powers of the numbers 2, 3 and 5

Abstract. In this paper I make a conjecture which states that any odd prime can be written in a certain way, in other words that any such prime can be expressed using just another prime and the powers of the numbers 2, 3 and 5. I also make a related conjecture about twin primes.

Conjecture:

Any odd prime p can be written at least in one way as p = (q*2^a*3^b*5^c ± 1)*2^n ± 1, where q is an odd prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Verifying the conjecture:
(For the first five odd primes)

: 3 = (1*2^1*3^0*5^0 – 1)*2^1 + 1, but also 3 = (1*2^0*3^1*5^0 – 1)*2^1 - 1;

: 5 = (1*2^1*3^0*5^0 + 1)*2^1 - 1, but also 5 = (1*2^0*3^1*5^0 – 1)*2^1 + 1, also 5 = (1*2^2*3^0*5^0 - 1)*2^1 - 1;

: 7 = (1*2^0*3^1*5^0 + 1)*2^1 - 1, but also 7 = (1*2^1*3^0*5^0 + 1)*2^1 + 1, also 7 = (3*2^0*3^0*5^0 + 1)*2^1 – 1, also 7 = (5*2^0*3^0*5^0 - 1)*2^1 – 1, also 7 = (1*2^2*3^0*5^0 - 1)*2^1 + 1;

: 11 = (1*2^1*3^1*5^0 - 1)*2^1 + 1, but also 11 = (1*2^0*3^0*5^1 + 1)*2^1 - 1, also 11 = (3*2^1*3^0*5^0 - 1)*2^1 + 1, also 11 = (5*2^0*3^0*5^0 + 1)*2^1 – 1, also 11 = (7*2^0*3^0*5^0 - 1)*2^1 – 1, also 11 = (1*2^2*3^0*5^0 + 1)*2^1 + 1;

: 13 = (1*2^1*3^1*5^0 + 1)*2^1 + 1, but also 13 = (1*2^0*3^0*5^1 + 1)*2^1 + 1, also 13 = (3*2^1*3^0*5^0 + 1)*2^1 + 1, also 13 = (5*2^0*3^0*5^0 + 1)*2^1 + 1, also 13 = (7*2^0*3^0*5^0 - 1)*2^1 + 1.

Conjecture:

Any pair of twin primes [p_1, p_2] can be written as [p_1 = (q*2^a*3^b*5^c ± 1)*2^n - 1, p_2 = (q*2^a*3^b*5^c ± 1)*2^n + 1], where q is prime or is equal to 1, where a, b and c are non-negative integers and n is non-null positive integer.

Verifying the conjecture:
(For the first three pairs of twin primes)

: 3 = (1*2^0*3^1*5^0 – 1)*2^1 - 1 and
5 = (1*2^0*3^1*5^0 – 1)*2^1 + 1;

: 5 = (1*2^1*3^0*5^0 + 1)*2^1 - 1 and
7 = (1*2^1*3^0*5^0 + 1)*2^1 + 1, also

5 = (1*2^2*3^0*5^0 - 1)*2^1 – 1 and
7 = (1*2^2*3^0*5^0 - 1)*2^1 + 1;

: 11 = (1*2^0*3^0*5^1 + 1)*2^1 – 1 and
13 = (1*2^1*3^1*5^0 + 1)*2^1 + 1, also

11 = (5*2^0*3^0*5^0 + 1)*2^1 – 1 and
13 = (5*2^0*3^0*5^0 + 1)*2^1 + 1.

28. Two conjectures about the pairs of primes separated by a certain distance

Abstract. In this paper I make two conjectures abut the pairs of primes [p1, q1], where the difference between p1 and q1 is a certain even number d. I state that any such pair has at least one other corresponding, in a specified manner, pair of primes [p2, q2], such that the difference between p2 and q2 is also equal to d.

Conjecture 1:

For any pair of primes, greater than 3, $[p_1, q_1]$, where $q_1 - p_1 = d$, there exist at least a pair of positive integers [m, n], where $n - m = d$, such that the numbers $p_2 = p_1*q_1 - n + 1$ and $q_2 = p_1*q_1 - m + 1$ are both primes.

Examples:

: For $[p_1, q_1] = [5, 7]$ there exist the pair [m, n] = [5, 7] such that $p_2 = 5*7 - 7 + 1 = 29$ and $q_2 = 5*7 - 5 + 1 = 31$ are both primes;

: For $[p_1, q_1] = [5, 11]$ there exist the pair [m, n] = [3, 9] such that $p_2 = 5*11 - 9 + 1 = 47$ and $q_2 = 5*11 - 3 + 1 = 53$ are both primes;

: For $[p_1, q_1] = [5, 13]$ there exist the pair [m, n] = [5, 13] such that $p_2 = 5*13 - 13 + 1 = 53$ and $q_2 = 5*13 - 5 + 1 = 61$ are both primes;

: For $[p_1, q_1] = [7, 11]$ there exist the pair [m, n] = [7, 11] such that $p_2 = 7*11 - 11 + 1 = 67$ and $q_2 = 7*11 - 7 + 1 = 71$ are both primes;

: For $[p_1, q_1] = [7, 13]$ there exist the pair [m, n] = [7, 11] such that $p_2 = 7*11 - 11 + 1 = 67$ and $q_2 = 7*11 - 7 + 1 = 71$ are both primes;

: For $[p_1, q_1] = [11, 13]$ there exist the pair [m, n] = [5, 7] such that $p_2 = 11*13 - 5 + 1 = 137$ and $q_2 = 11*13 - 7 + 1 = 139$ are both primes.

Conjecture 2:

For any even number d there exist an infinity of pairs of primes $[p_1, q_1]$, where $q_1 - p_1 = d$, such that the numbers $p_2 = p_1*q_1 - p_1 + 1$ and $q_2 = p_1*q_1 - q_1 + 1$ are both primes.

Note: See, for instance, from the examples to the Conjecture 1 from above, the pair [5, 7] for d = 2, the pair [7, 11] for d = 4, the pair [5, 13] for d = 8.

29. Five conjectures on a diophantine equation involving two primes and a square of prime

Abstract. In this paper I make five conjectures about the primes q, r and the square of prime p^2, which appears as solutions in the diophantine equation 120*n*q*r + 1 = p^2, where n is non-null positive integer.

Conjecture 1:

For any n non-null positive integer there exist q, r primes such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 2:

For any q odd prime there exist n non-null positive integer and r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 3:

For any q, r odd primes there exist n non-null positive integer such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Conjecture 4:

For any n non-null positive integer and any q prime there exist r prime such that 120*n*q*r + 1 = p^2, where p is prime or a power of prime.

Examples:

: For [n, q] = [1, 5] there exist r = 17 such that p = 101 prime; also r = 37 such that p = 149 prime;
: For [n, q] = [1, 7] there exist r = 23 such that p = 139 prime; also r = 53 such that p = 211 prime;
: For [n, q] = [1, 11] there exist r = 13 such that p = 131 prime; also r = 83 such that p = 331 prime;

: For [n, q] = [2, 5] there exist r = 19 such that p = 151 prime;
: For [n, q] = [2, 7] there exist r = 3 such that p = 71 prime; also r = 17 such that p = 169 square of prime;
: For [n, q] = [2, 11] there exist r = 3 such that p = 89 prime;

: For [n, q] = [3, 7] there exist r = 13 such that p = 181 prime;
: For [n, q] = [3, 11] there exist r = 3 such that p = 109 prime;

: For [n, q] = [4, 5] there exist r = 67 such that p = 401 prime;
: For [n, q] = [4, 7] there exist r = 17 such that p = 239 prime;
: For [n, q] = [4, 11] there exist r = 11 such that p = 241 prime.

Conjecture 5:

For any n non-null positive integer there exist q prime such that 120*n*q^2 + 1 = p^2, where p is prime or a power of prime.
Note, for instance, the case from the examples below: 480*11^2 + 1 = 241^2.

30. An amazing formula for producing big primes based on the numbers 25 and 906304

Abstract. In this paper I present a formula for generating big primes and products of very few primes, based on the numbers 25 and 906304, formula equally extremely interesting and extremely simple, id est 25^n + 906304. This formula produces for n from 1 to 30 (and for n = 30 is obtained a number p with not less than 42 digits) only primes or products of maximum four prime factors.

Observation:

The number p = 25^n + 906304 is often a prime or a product of very few primes.

Note:

I came to this formula more or less by chance, but the number 906304 has at least one other special property: 906304 = 952^2 = 1105^2 – 561^2, where 561 and 1105 are the first and the second Carmichael numbers.

Examples:

: p = 25^1 + 906304 = 906329 prime;
: p = 25^2 + 906304 = 906929 prime;
: p = 25^3 + 906304 = 921929 = 37*24917;
: p = 25^4 + 906304 = 1296929 prime;
: p = 25^5 + 906304 = 10671929 = 421*25349;
: p = 25^6 + 906304 = 245046929 = 97*2526257;
: p = 25^7 + 906304 = 245046929 = 113*2957*18269;
: p = 25^8 + 906304 = 152588796929 = 36269*4207141;
: p = 25^9 + 906304 = 3814698171929 prime;
: p = 25^10 + 906304 = 95367432546929 = 41*2326034940169;
: p = 25^11 + 906304 = 2384185791921929 = 5573*427810118773;
: p = 25^12 + 906304 = 59604644776296929 = 61*139361*7011468949;
: p = 25^13 + 906304 = 1490116119385671929 = 1097*84389*16096358813;:
: p = 25^14 + 906304 = 37252902984620046929 prime;
: p = 25^15 + 906304 = 931322574615479421929 = 671477*1386976135616677;
: p = 25^16 + 906304 = 23283064365386963796929
= 1609*1830341*7905914013541;
: p = 25^17 + 906304 = 582076609134674073171929 prime;
: p = 25^18 + 906304 = 14551915228366851807546929
= 53^2*5180461099454201426681;
: p = 25^19 + 906304 = 363797880709171295166921929 prime;
: p = 25^20 + 906304 = 9094947017729282379151296929
= 41*237776289649*932927233281481;

Notes:

For n from 1 to 20, were obtained for p seven values which are primes, seven values which are semiprimes and six values which are products of three prime factors! Note also that the larger prime obtained in the examples above, p = 25^19 + 906304 = 363797880709171295166921929, has 27 digits!

For n from 21 to 30 were also obtained products of maximum four primes; these are the following values of p:

: n = 21, p = 227373675443232059478760671929;
: n = 22, p = 5684341886080801486968995046929;
: n = 23, p = 142108547152020037174224854421929;
: n = 24, p = 3552713678800500929355621338796929;
: n = 25, p = 88817841970012523233890533448171929;
: n = 26, p = 2220446049250313080847263336182546929;
: n = 27, p = 55511151231257827021181583404541921929;
: n = 28, p = 1387778780781445675529539585113526296929;
: n = 29, p = 34694469519536141888238489627838135671929;
: n = 30, p = 867361737988403547205962240695953370046929.

For n from 31 to 37 were also obtained products of maximum five primes; these are the following values of p:

: n = 31, p = 21684043449710088680149056017398834229421929
: n = 32, p = 542101086242752217003726400434970855713796929
: n = 33, p = 13552527156068805425093160010874271392823171929
: n = 34, p = 338813178901720135627329000271856784820557546929
: n = 35, p = 8470329472543003390683225006796419620513916921929
: n = 36, p = 211758236813575084767080625169910490512847901296929
: n = 37, p = 5293955920339377119177015629247762262821197510671929

Note that the number 25^34 + 906304 is a prime with 48 digits!

Conjecture:

There exist an infinity of primes p of the form p = 25^n + 906304.

31. Four unusual conjectures on primes involving Egyptian fractions

Abstract. In this paper I make four conjectures on primes, conjectures which involve the sums of distinct unit fractions such as 1/p(1) + 1/p(2) + (...), where p(1), p(2), (...) are distinct primes, more specifically the periods of the rational numbers which are the results of the sums mentioned above.

Conjecture 1:

There exist an infinity of infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Examples:

: the period of a(1) = 1/3 + 1/7 is equal to 6;
: the period of a(2) = 1/3 + 1/7 + 1/19 is equal to 18;
: the period of a(3) = 1/3 + 1/7 + 1/19 + 73 is equal to 72.
(...)
The sequence of p(1), p(2), p(3)... is 3, 7, 19, 72...
: the period of a(1) = 1/5 + 1/29 is equal to 28;
: the period of a(2) = 1/5 + 1/29 + 1/113 is equal to 112;
: the period of a(3) = 1/5 + 1/29 + 1/113 + 1/337 is equal to 336.
(...)
The sequence of p(1), p(2), p(3)... is 5, 29, 113, 337...

Conjecture 2:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is equal to p(2) - 1, the period of the rational number a(2) is equal to p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Conjecture 3:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) is a multiple of p(2) - 1, the period of the rational number a(2) is a multiple of p(3) – 1, the period of the rational number a(n) is equal to a(n) – 1.

Example:

: the period of a(1) = 1/7 + 1/17 is equal to 48 which is a multiple of 16;
: the period of a(2) = 1/7 + 1/17 + 1/19 is equal to 144 which is a multiple of 18;
: the period of a(3) = 1/7 + 1/17 + 1/19 + 1/23 is equal to 1584 a multiple of 22.
(...)
The sequence of p(1), p(2), p(3)... is 7, 17, 19, 23...

Conjecture 4:

For any p(1) odd prime there exist infinite sequences of the form a(1) = 1/p(1) + 1/p(2), a(2) = 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1) < p(2) < p(3)..., such that the period of the rational number a(1) divides p(2) - 1, the period of the rational number a(2) divides p(3) – 1, the period of the rational number a(n) divides a(n) – 1.

Example:

: the period of a(1) = 1/3 + 1/11 is equal to 2 which divides 10;
: the period of a(2) = 1/3 + 1/11 + 1/13 is equal to 6 which divides 12;
: the period of a(3) = 1/3 + 1/11 + 1/13 + 1/37 is equal to 6 which divides 36.
(...)

The sequence of p(1), p(2), p(3)... is 3, 11, 13, 37...

Conjecture 5:

For any Poulet number P there exist a rational number r equal to a sum of unit fractions 1/p(1) + 1/p(2) + 1/p(3), ..., where p(1), p(2), p(3)... are distinct odd primes, such that the period of r is equal to P – 1.

Example: the period of r = 1/5 + 1/29 + 1/113 + 1/271 is equal to 560, while 561 the second Poulet number and the first Carmichael number.

32. Three formulas that generate easily certain types of triplets of primes

Abstract. In this paper I present three formulas, each of them with the following property: starting from a given prime p, are obtained in many cases two other primes, q and r. I met the triplets of primes [p, q, r] obtained with these formulas in the study of Carmichael numbers; the three primes mentioned are often the three prime factors of a 3-Carmichael number.

Note:

To refer to the three formulas easily I will name them the formula alpha, beta or gama and the triplets obtained the triplet alpha, beta or gama.

Formula alpha:

The formula alpha is 30*a*n – (a*p + a – 1). The first prime of a triplet alpha is p and the other two ones are obtained giving to n values of integers, under the condition that a*p + a – 1 is prime.

Examples:

: For p = 11 and a = 2 the condition that a*p + a – 1 is prime is met because 2*11 + 2 – 1 = 23 which is prime; the formula alpha becomes 60*n – 23; it can be seen that for n = 1 is obtained 47 (prime) and for n = 2 is obtained 97 (prime) so we have the triplet alpha [11, 47, 97]; also for n = 3 is obtained 157 (prime) so other two triplets alpha are [11, 47, 157] and [11, 97, 157];

: For p = 7 and a = 3 the condition that a*p + a – 1 is prime is met because 3*7 + 3 – 1 = 23 which is prime; the formula alpha becomes 90*n – 23; it can be seen that for n = 1 is obtained 67 (prime) and for n = 2 is obtained 157 (prime) so we have the triplet alpha [7, 67, 157]; also for n = 4 is obtained 337 (prime) so other two triplets alpha are [7, 67, 337] and [7, 157, 337].

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula alpha.

Formula beta:

The formula beta is 30*a*n + (a*p + a – 1). The first prime of a triplet beta is p and the other two ones are obtained giving to n values of integers, under the condition that a*p + a – 1 is prime.

Examples:

: For p = 11 and a = 2 the condition that a*p + a – 1 is prime is met because 2*11 + 2 – 1 = 23 which is prime; the formula beta becomes 60*n + 23; it can be seen that for n = 1 is obtained 83 (prime) and for n = 4 is obtained 263 (prime) so we have the triplet beta [11, 83, 263]; also for n = 6 is obtained 383 (prime) so other two triplets beta are [11, 83, 383] and [11, 263, 383];

: For p = 19 and a = 3 the condition that a*p + a – 1 is prime is met because 3*19 + 3 – 1 = 59 which is prime; the formula beta becomes 90*n + 59; it can be seen that for n = 1 is obtained 149 (prime) and for n = 2 is obtained 239 (prime) so we have the triplet beta [59, 149, 239]; also for n = 4 is obtained 419 (prime) so other two triplets beta are [59, 149, 419] and [59, 239, 419].

Note: see the sequence A182416 in OEIS for the connection between Carmichael numbers and formula beta.

Formula gama:

The formula gama is 2*p*n – 2*n + p. The first prime of a triplet gama is p and the other two ones are obtained giving to n values of integers, under the condition that 2*p – 1 is prime.

Example:

: For p = 7 the condition that 2*p – 1 is prime is met; the formula gama becomes 12*n + 7; for n = 1 is obtained 19 (prime) and for n = 2 is obtained 31 so we have the triplet gama [7, 19, 31]; also for n = 3 is obtained 43 so other two triplets gama are [7, 19, 43] and [7, 31, 43].

Note: see the sequence A182207 in OEIS for the connection between Carmichael numbers and formula gama.

33. A new bold conjecture about a way in which any prime can be written

Abstract. In this paper I make a conjecture which states that any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones: 10k + 1, 10k + 3, 10k + 7 or 10k + 9.

Conjecture:

Any prime greater than or equal to 53 can be written at least in one way as a sum of three odd primes, not necessarily distinct, of the same form from the following four ones: 10k + 1, 10k + 3, 10k + 7 or 10k + 9.

Verifying the conjecture:
(For the first few primes greater than or equal to 53)

(Note that we will not show all ways in which a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

: 53 = 11 + 11 + 31;
: 59 = 13 + 23 + 23;
: 61 = 7 + 17 + 37;
: 67 = 19 + 19 + 29;
: 71 = 17 + 17 + 37;
: 73 = 11 + 31 + 31;
: 79 = 13 + 23 + 43;
: 83 = 11 + 31 + 41;
: 89 = 23 + 23 + 23;
: 97 = 19 + 19 + 59;
: 101 = 17 + 17 + 67;
: 103 = 11 + 31 + 61;
: 107 = 19 + 29 + 59;
: 109 = 13 + 13 + 83;
: 113 = 11 + 31 + 71;
: 127 = 19 + 29 + 79;
: 131 = 7 + 17 + 107;
: 137 = 19 + 29 + 89;
: 139 = 13 + 13 + 113;
: 149 = 13 + 23 + 113;
: 151 = 7 + 7 + 137.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 1.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 3.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 7.

Conjecture:

There exist an infinity of primes p that can be written as p = 2*m + n, where m and n are distinct primes of the form 10k + 9.

34. A bold conjecture about a way in which any square of prime can be written

Abstract. In this paper I make a conjecture which states that any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form 10k + 3 or all three of the form 10k + 7.

Conjecture:

Any square of a prime greater than or equal to 7 can be written at least in one way as a sum of three odd primes, not necessarily distinct, but all three of the form 10k + 3 or all three of the form 10k + 7.

Verifying the conjecture:
(For the first few primes greater than or equal to 7)

(Note that we will not show all ways in which a square of a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

: 7^2 = 49 = 13 + 13 + 23;
: 11^2 = 121 = 37 + 37 + 47;
: 13^2 = 169 = 13 + 43 + 113;
: 17^2 = 289 = 13 + 13 + 263;
: 19^2 = 361 = 7 + 17 + 337;
: 23^ = 529 = 13 + 53 + 563.

Conjecture:

Any square of a prime p^2, where p is greater than or equal to 7, can be written as p^2 = 2*m + n, where m and n are distinct primes, both of the form 10k + 3 or both of the form 10k + 7.

Verifying the conjecture:
(For the first few primes greater than or equal to 7)

(Note that we will not show all ways in which a square of a prime can be written in the way mentioned but only one way, enough to confirm the conjecture)

: 7^2 = 49 = 2*13 + 23;
: 11^2 = 121 = 2*37 + 47;
: 13^2 = 169 = 2*43 + 83;
: 17^2 = 289 = 2*13 + 263;
: 19^2 = 361 = 2*7 + 347;
: 23^2 = 529 = 2*13 + 503.

35. Statements on the infinity of few sequences or types of duplets or triplets of primes

Abstract. In this paper I make few statements on the infinity of few sequences or types of duplets and triplets of primes which, though could appear heterogenous, are all based on the observation of the prime factors of absolute Fermat pseudoprimes, Carmichael numbers, or of relative Fermat pseudoprimes to base two, Poulet numbers.

Note:

See, in my book "Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes", Part Four, "One hundred and fifty open problems regarding Fermat pseudoprimes".

Conjecture 1:

There exist an infinity of positive integers k such that 6*k – 1 and 18*k – 5 are both primes.

Conjecture 2:

There exist an infinity of positive integers k such that 6*k + 1 and 12*k + 1 are both primes.

Conjecture 3:

There exist an infinity of positive integers k such that 6*k + 1 and 18*k + 1 are both primes.

Conjecture 4:

There exist an infinity of positive integers k such that 6*k - 5 and 24*k - 5 are both primes.

Conjecture 5:

There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 1 are all three primes.

Conjecture 6:

There exist an infinity of positive integers k such that 6*k + 1, 12*k + 1 and 18*k + 13 are all three primes.

Conjecture 7:

There exist an infinity of positive integers k such that k, 2*k - 1 and 5*k - 4 are all three primes.

Conjecture 8:

There exist an infinity of positive integers k such that k, 2*k - 1 and 3*k - 2 are all three primes.

Conjecture 9:

There exist an infinity of positive integers k such that k, 3*k - 2 and 4*k - 3 are all three primes.

Conjecture 10:

There exist an infinity of positive integers k such that 40*k + 1, 60*k + 1 and 100*k + 1 are all three primes.

Conjecture 11:

There exist an infinity of positive integers k such that k, 2*k – 1, 7*k - 6 and 14*k - 13 are all four primes.

Conjecture 12:

There exist an infinity of positive integers k such that k, 2*k – 1, 6*k - 5 and 12*k - 11 are all four primes.

Conjecture 13:

There exist an infinity of pairs of distinct non-null positive integers m, n such that 60*m*n – 29 and 60*m*n – (60*m + 29) are both primes.

Conjecture 14:

There exist an infinity of pairs of distinct non-null positive integers [m, n] such that 40*m – 10*n - 29 and 40*m – 10*n – 129 are both primes.

Conjecture 15:

For any pair of twin primes [q, r] there exist an infinity of primes p of the form p = 7200*q*r*n + 1, where n is positive integer.

Examples:

: for [q, r] = [5, 7], p is prime for n = 1, 2, 4, 7 (...)

: for [q, r] = [11, 13], p is prime for n = 1, 3, 8 (...)

Conjecture 16:

There exist an infinity of primes p of the form p = n*s(p) – n + 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 17:

There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 18:

There exist an infinity of primes p of the form p = n*s(p) + n - 1, where n is positive integer and s(p) is the sum of the digits of p.

Conjecture 19:

There exist an infinity of primes p of the form p = m*n + m - n, where m and n are distinct odd primes.

Conjecture 20:

There exist an infinity of primes p of the form p = m^2 – m*n + n, where m and n are distinct odd primes.

Conjecture 21:

There exist an infinity of primes p of the form p = (q + 5^k)/ 10, where q is prime and k positive integer.

Conjecture 22:

There exist an infinity of primes p of the form p = (q + 5^k)/ 30, where q is prime and k positive integer.

Conjecture 23:

There exist an infinity of primes p of the form p = q^3 + 60, where q is prime.

Conjecture 24:

There exist an infinity of primes p, for n positive integer, of the following forms:

: 20*n^2 + 12*n + 1;
: 1800*n^2 + 840*n + 1;
: 3*n^2 + 6*n + 4;
: 4*n^2 + 172*n + 529;
: 20*n^2 + 364*n + 177;
: n^2 + 81*n + 39;
: n^2 + 10*n + 10.

36. An interesting relation between the squares of primes and the number 96 and two conjectures

Abstract. In this paper I make two conjectures based on the observation of an interesting relation between the squares of primes and the number 96.

Conjecture 1:

If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of numbers which are primes or squares of primes.

Example:

: for p = 5 are obtained the primes q = 313, 409, 601 (...) for k = 3, 4, 6 (...) and the squares of primes q = 11^2, 37^2 (...) for k = 1, 14 (...).

Conjecture 2:

If p is a prime greater than or equal to 5, then the sequence q = p^2 + 96*k, where k is positive integer, contains an infinity of semiprimes q = m*n, where m < n, with the following property: the number n – m + 1 is a prime or a square of a prime.

Example:

: for p = 5 are obtained the semiprimes q = 217 = 7*31 (and 31 – 7 + 1 = 5^2) for k = 2, q = 505 = 5*101 (and 101 – 5 + 1 = 97, prime) for k = 5, q = 697 = 17*41 (and 41 – 17 + 1 = 5^2) for k = 7, q = 793 = 13*61 (and 61 – 13 + 1 = 7^2) for k = 8, q = 889 = 7*127 (and 127 – 7 + 1 = 11^2) for k = 9, q = 985 = 5*197 (and 197 – 5 + 1 = 193, prime) for k = 10, q = 1081 = 23*47 (and 47 – 23 + 1 = 5^2) for k = 11, q = 1177 = 11*107 (and 107 – 11 + 1 = 97, prime) for k = 12, q = 1273 = 19*67 (and 67 – 19 + 1 = 7^2) for k = 13, q = 1465 = 5*293 (and 293 – 5 + 1 = 17^2) for k = 15.

Note that, for p = 5, were obtained for $1 \leq k \leq 15$ only primes, squares of primes and semiprimes with the property mention above.

Taking randomly a prime, id est 233, is obtained:
: for k = 1, the semiprime q = 329 = 7*47 (47 – 7 + 1 = 41);
: for k = 3, the prime q = 521;
: for k = 4, the prime q = 617;
: for k = 5, the semiprime q = 713 = 23*31 (31 – 23 + 1 = 3^2);
: for k = 6, the prime q = 809.

Taking randomly another prime, id est 769, is obtained:
: for k = 1, the semiprime q = 865 = 5*173 (173 – 5 + 1 = 13^2);
: for k = 2, the square of prime q = 31^2;
: for k = 4, the prime q = 1153;

: for k = 5, the prime q = 1249;
: for k = 7, the semiprime q = 1441 = 11*131 (131 – 11 + 1 = 11^2).

Conclusion:

It is clear from these examples that the formula p^2 + 96*k, where p is prime and k is positive integer, has the property to generate primes, squares of primes and semiprimes with the property shown.

37. A formula that seems to generate easily big numbers that are primes or products of very few primes

Abstract. The formula N = (p^4 – 2*p^2 + m)/(m – 1), where p is an odd prime and m is a positive integer greater than 1, seems to generate easily primes or products of very few primes.

Observation:

The formula N = (p^4 – 2*p^2 + m)/(m – 1), where p is an odd prime and m is a positive integer greater than 1, seems to generate easily primes or products of very few primes.

Examples:

For m = 2 the formula becomes N = p^4 – 2*p^2 + 2 and were obtained the following results for the sequence of the first five consecutive primes of the form 10*k + 1:

: for p = 11, N = 14401 prime;
: for p = 31, N = 921601 prime;
: for p = 41, N = 2822401 = 113*24977;
: for p = 61, N = 13838401 = 3313*4177;
: for p = 71, N = 25401601 = 101*251501.

For a larger prime of the same form, p = 961752931, is obtained N = 855855567096510789934200845104477377601, a semiprime with 39 digits.

For m = 3 the formula becomes N = (p^4 – 2*p^2 + 3)/2 and were obtained the following results for the sequence of the first five consecutive primes of the form 20*k + 9:

: for p = 29, N = 352801 = 17*20753;
: for p = 89, N = 31363201 prime;
: for p = 109, N = 70567201 = 2659*26539;
: for p = 149, N = 246420001 prime;
: for p = 229, N = 1374976801 = 11*124997891.

For m = 4 the formula becomes N = (p^4 – 2*p^2 + 4)/3 and were obtained the following results for the sequence of the first eight consecutive primes of the form 30*k + 13:

: for p = 13, N = 9409 = 97^2;
: for p = 43, N = 1138369 prime;
: for p = 73, N = 9462529 = 1609*5881;
: for p = 103, N = 37509889 = 43*872323;
: for p = 163, N = 235286209 prime;
: for p = 193, N = 462471169 prime;
: for p = 223, N = 824291329 prime;
: for p = 283, N = 2138029249 prime.

For two larger primes of the same form is obtained:

: for p = 1299763, N = 951339271160353903881409 prime;
: for p = 1299853, N = 951602794365121103901889 prime.

Taking randomly a prime, id est p = 29, are obtained the following results:

: for m = 2, N = 705601, a semiprime;
: for m = 3, N = 352801, a semiprime;
: for m = 4, N = 235201, a semiprime;
: for m = 5, N = 176401, a prime;
: for m = 6, N = 141121, a prime;
: for m = 7, N = 117601, a semiprime;
: for m = 8, N = 100801, a prime;
: for m = 9, N = 88201, a semiprime;
: for m = 10, N = 78401, a prime;
: for m = 11, N = 70561, a semiprime;
: for m = 12, N is not integer;
: for m = 13, N = 58801, a semiprime;
: for m = 14, N is not integer;
: for m = 15, N = 50401, a semiprime;
: for m = 16, N = 47041, a prime;
: for m = 17, N = 44101, a prime (...)

38. Four conjectures based on the observation of a type of recurrent sequences involving semiprimes

Abstract. In this paper I make four conjectures starting from the observation of the following recurrent relations: (((p*q – p)*2 – p)*2 – p)...), respectively (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes.

Observation:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes, p < q. Very interesting patterns can be observed between a(i) and b(i) in the case of the same semiprime p*q or between the terms of this recurrence relation for different semiprimes:

Let p*q = 7*13 = 91; then:

: a(1) = 2*91 – 7 = 175;
: a(2) = 2*175 – 7 = 343;
: a(3) = 2*343 – 7 = 679 = 7*97;
: a(4) = 2*679 – 7 = 1351 = 7*193;
: a(5) = 2*1351 – 7 = 2695;
: a(6) = 2*2695 – 7 = 5383 = 7*769
(...)
: b(1) = 2*91 – 13 = 169;
: b(2) = 2*169 – 13 = 325;
: b(3) = 2*325 – 13 = 637;
: b(4) = 2*637 – 13 = 1261 = 13*97;
: b(5) = 2*1261 – 13 = 2509 = 13*193;
: b(6) = 2*2509 – 13 = 5005;
: b(7) = 2*5005 – 13 = 9997 = 13*769
(...)

Note that a(3)/p = b(4)/q = 97, a(4)/p = b(5)/q = 193 and a(6)/p = a(7)/q = 769.

Let p*q = 11*13 = 143; then:

: a(1) = 2*143 – 11 = 275;
: a(2) = 2*275 – 11 = 539;
: a(3) = 2*539 – 11 = 1067 = 11*97;
: a(4) = 2*1067 – 11 = 2123 = 11*193;
: a(5) = 2*2123 – 11 = 4235;
: a(6) = 2*4235 – 11 = 8459 = 11*769;
: a(7) = 2*8459 – 11 = 16907;
: a(8) = 2*16907 – 11 = 33803;
: a(9) = 2*33803 – 11 = 67595;
: a(10)= 2*8459 – 11 = 135179 = 11*12289
(...)
: b(1) = 2*143 – 13 = 273;
: b(2) = 2*273 – 13 = 533 = 13*41;

: b(3) = 2*533 – 13 = 1053;
: b(4) = 2*1053 – 13 = 2093;
: b(5) = 2*2093 – 13 = 4173;
: b(6) = 2*4173 – 13 = 8333 = 13*641;
: b(7) = 2*8333 – 13 = 16653;
: b(8) = 2*16653 – 13 = 33293;
: b(9) = 2*33293 – 13 = 66573;
: b(10) = 2*66573 – 13 = 133133;
: b(11) = 2*66573 – 13 = 266253;
: b(12) = 2*266253 – 13 = 532493 = 13*40961
(...)

Note that, in the case of this semiprime, were obtained for a(i)/p the primes obtained for the first semiprime, id est 97, 193, 769, 12289 (which are primes of the form 6*2^n + 1, see the sequence A039687 in OEIS) but for b(i)/q other primes, id est 41, 641, 40961 ((which are primes of the form 5*2^n + 1, see the sequence A050526 in OEIS).

Let p*q = 7*11 = 77; then:
: a(1) = 2*77 – 7 = 147;
: a(2) = 2*147 – 7 = 287 = 7*41;
: a(3) = 2*287 – 7 = 567;
: a(4) = 2*567 – 7 = 1127;
: a(5) = 2*1127 – 7 = 2247;
: a(6) = 2*2247 – 7 = 4487 = 7*641.
(...)
: b(1) = 2*77 – 11 = 143 then for the following terms see a(i) in the first example of p*q = 11*13.

Let p*q = 193*199; then we obtain, as b(i)/q, the primes 769, 12289 (which are primes of the form 6*2^n + 1, obtained above) but for a(i)/p other set of primes not met before: 397, 3169, 6337 (...). To make things even more complicated, for p*q = 197*199 we obtain, for a(i)/p, the set of primes 397, 3169, 6337 mentioned above but for b(i)/q other set of primes not met before: 3137, 50177 (...), which are primes of the form 49*2^n + 1 (see the sequence A077498 in OEIS). Note also the interesting thing that 397, 3169 and 6337 are all three primes of the form 99*2^n + 1.

Let p*q = 13*233; then we obtain, as a(i)/p, the primes 929, 59393, which are primes of the form 29*2^n + 1. Seems amazing how many possible infinite sequences of primes can be obtained starting from a simple recurrence relation and a randomly chosen pair of distinct odd primes.

Conjecture 1:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of primes of the form a(i)/p as well as an infinity of primes of the form b(i)/q for any pair [p, q].

Conjecture 2:

Let a(i) be the general term of the sequence formed in the following way: a(i) = (((p*q – p)*2 – p)*2 – p)...) and b(i) be the general term of the sequence formed in the following way: b(i) = (((p*q – q)*2 – q)*2 – q)...), where p, q are distinct odd primes. Then there exist an infinity of pairs [p, q] such that the sequence of primes a(i)/p is the same with the sequence of primes b(i)/q.

Conjecture 3:

There exist an infinity of primes, for k positive integer, of the form n*2^k + 1, for n equal to 5, 6, 29, 49 or 99 (note that this conjecture is a consequence of Conjecture 1 and the examples observed above).

Conjecture 4:

There exist an infinity of positive integers n such that the sequence n*2^k + 1, where k is positive integer, contains an infinity of primes.

39. Conjecture that states that a Mersenne number with odd exponent is either prime either divisible by a 2-Poulet number

Abstract. In this paper I make a conjecture which states that any Mersenne number (number of the form 2^n – 1, where n is natural) with odd exponent n, where n is greater than or equal to 3, also n is not a power of 3, is either prime either divisible by a 2-Poulet number. I also generalize this conjecture stating that any number of the form P = ((2^m)^n – 1)/3^k, where m is non-null positive integer, n is odd, greater than or equal to 5, also n is not a power of 3, and k is equal to 0 or is equal to the greatest positive integer such that P is integer, is either a prime either divisible by at least a 2-Poulet number (I will name this latter numbers Mersenne-Coman numbers) and I finally enunciate yet another related conjecture.

Note:

For a list of 2-Poulet numbers see the sequence A214305 which I posted on OEIS. For a list of Mersenne numbers see the sequence A000225 in OEIS.

Conjecture 1:

Any Mersenne number 2^n – 1 with odd exponent n, where n is greater than or equal to 3, also n is not a power of 3, is either prime either divisible by a 2-Poulet number.

Verifying the conjecture:
(For the first thirteen such n)

: 2^3 – 1 = 7, prime;
: 2^5 – 1 = 31, prime;
: 2^7 – 1 = 127, prime;
: 2^11 – 1 = 2047 = 23*89, a 2-Poulet number;
: 2^13 – 1 = 8191, prime;
: 2^15 – 1 = 32767 = 7*31*151, which is divisibe by 4681 = 31*151, a 2-Poulet number;
: 2^17 – 1 = 131071, prime;
: 2^19 – 1 = 524287, prime;
: 2^21 – 1 = 2097151 = 7^2*127*337, which is divisibe by 42799 = 127*337, a 2-Poulet number;
: 2^23 – 1 = 8388607 = 47*178481, a 2-Poulet number;
: 2^25 – 1 = 33554431 = 31*601*1801, which which is divisibe by 1082401 = 601*1801, a 2-Poulet number;
: 2^29 – 1 = 536870911 = 233*1103*2089, which which is divisibe by 256999 = 233*1103, a 2-Poulet number;
: 2^31 – 1 = 2147483647, prime.

Conjecture 2:

Any Mersenne-Coman number of the form P = ((2^m)^n – 1)/3^k, where m is non-null positive integer, n is odd, greater than or equal to 5, also n is not a power of 3, and k is equal to 0 or is equal to the greatest positive integer such that P is integer, is either a prime either divisible by at least a 2-Poulet number.

Verifying the conjecture:
(For m = 2 and the first twelve such n)

: (4^5 – 1)/3 = 341 = 11*31, a 2-Poulet number;
: (4^7 – 1)/3 = 5461 = 43*127, a 2-Poulet number;
: (4^11 – 1)/3 = 1398101 = 23*89*683, which is divisibe by:
    : 2047 = 23*89, a 2-Poulet number;
    : 15709 = 23*683, a 2-Poulet number;
    : 60787 = 89*683, a 2-Poulet number.
: (4^13 – 1)/3 = 22369621 = 2731*8191, a 2-Poulet number;
: (4^15 – 1)/3^2 is divisibe by:
    : 341 = 11*31, a 2-Poulet number;
    : 4681 = 31*151, a 2-Poulet number;
    : 10261 = 31*331, a 2-Poulet number;
    : 49981 = 151*331, a 2-Poulet number.
: (4^17 – 1)/3 = 5726623061 = 43691*131071, a 2-Poulet number;
: (4^19 – 1)/3 = 91625968981 = 174763*52487, a 2-Poulet number;
: (4^21 – 1)/3^2 divides 5461, 14491, 233017, 42799, 688213 and 1826203, all of them 2-Poulet numbers;
: (4^23 – 1)/3 = 23456248059221 = 47*178481*2796203, which is divisibe by**:**
    : 8388607 = 47*178481, a 2-Poulet number;
    : 131421541 = 47*2796203, a 2-Poulet number;
    : 499069107643 = 178481*2796203, a 2-Poulet number.
: (4^29 – 1)/3 = 96076792050570581 = 59*233*1103*2089*3033169, which is divisibe by**:**
    : 13747 = 59*233, a 2-Poulet number;
    : 65077 = 59*1103, a 2-Poulet number;
    : 123251 = 59*2089, a 2-Poulet number;
    : 178956971 = 59*3033169, a 2-Poulet number;
    : 256999 = 233*1103, a 2-Poulet number;
    : 486737 = 233*2089, a 2-Poulet number;
    : 706728377 = 233*3033169, a 2-Poulet number;
    : 2304167 = 1103*2089, a 2-Poulet number;
    : 3345585407 = 1103*3033169, a 2-Poulet number;
    : 6336290041 = 2089*3033169, a 2-Poulet number.
: (4^31 – 1)/3 = 1537228672809129301= 715827883*2147483647, a 2-Poulet number;
: (4^37 – 1)/3 = 6296488643826193618261= 223*1777*25781083*616318177, which is divisibe by 396271 = 223*1777 and other 2-Poulet numbers.

Verifying the conjecture:
(For m = 3 and the first four such n)

: 8^5 – 1 = 32767 = 7*31*151, which is divisibe by 4681 = 31*151, a 2-Poulet number;
: 8^7 – 1 = 2097151 = 7^2*127*337, which is divisibe by 42799 = 127*337, a 2-Poulet number;

: 8^11 – 1 = 8589934591 = 7*23*89*599479, which is divisibe by 2047 = 23*89, a 2-Poulet number;
: 8^13 – 1 = 549755813887 = 7*79*8191*121369, which is divisibe by 647089 = 79*8191, a 2-Poulet number.

Verifying the conjecture:
(For m = 4 and the first four such n)

: (16^5 – 1)/3 divides 341 = 11*31, a 2-Poulet number;
: (16^7 – 1)/3 divides 5461 = 43*127, a 2-Poulet number;
: (16^11 – 1)/3 divides 2047 = 23*89, a 2-Poulet number;
: (16^13 – 1)/3 divides 8321 = 53*157, a 2-Poulet number.

Note:

The Mersenne-Coman primes (Mersenne-Coman numbers which are primes) seems to be very rare. For m = 2 (*i.e.* 4^n – 1, where n is odd, n ≥ 5) there is no such a prime up to n = 107.

Conjecture 3:

For any prime p greater than or equal to 5 the number (4^p – 1)/3 is either prime either a product of primes $p_1*p_2*...p_n$ such that all the numbers p_i*p_j are 2-Poulet numbers for $1 \leq i < j \leq n$.

Note:

This Conjecture is verified for p up to 31 (see the Conjecture 2 above).

40. Conjecture that states that a Fermat number is either prime either divisible by a 2-Poulet number

Abstract. In this paper I make a conjecture which states that any Fermat number (number of the form 2^(2^n) + 1, where n is natural) is either prime either divisible by a 2-Poulet number. I also generalize this conjecture stating that any number of the form N = ((2^m)^p + 1)/3^k, where m is non-null positive integer, p is prime, greater than or equal to 7, and k is equal to 0 or is equal to the greatest positive integer such that N is integer, is either a prime either divisible by at least a 2-Poulet number (I will name this latter numbers Fermat-Coman numbers) and I finally enunciate yet another related conjecture.

Note:

For a list of 2-Poulet numbers see the sequence A214305 which I posted on OEIS. For a list of Fermat numbers see the sequence A000215 in OEIS.

Conjecture 1:

Any Fermat number F = 2^(2^n) + 1 is either prime either divisible by a 2-Poulet number.

Note:

It is known that the first 5 Fermat numbers (3, 5, 17, 257, 65537) are primes. Also, for n = 5 is obtained F = 4294967297 = 641*6700417, which is, indeed, a 2-Poulet number (for the next two (composite) Fermat numbers, 18446744073709551617 340282366920938463463374607431768211457, semiprimes, I couldn't verify if they are 2-Poulet numbers).

Conjecture 2:

Any Fermat-Coman number of the form N = ((2^m)^p + 1)/3^k, where m is non-null positive integer, p is prime, greater than or equal to 7, and k is equal to 0 or is equal to the greatest positive integer such that N is integer, is either a prime either divisible by at least a 2-Poulet number.

Verifying the conjecture:
(For m = 1 and the first eight such p)

: (2^7 + 1)/3 = 43, prime;
: (2^11 + 1)/3 = 683, prime;
: (2^13 + 1)/3 = 2731, prime;
: (2^17 + 1)/3 = 43691, prime;
: (2^19 + 1)/3 = 174763, prime;
: (2^23 + 1)/3 = 2796203, prime;
: (2^29 + 1)/3 = 178956971 = 59*3033169, a 2-Poulet number;
: (2^31 + 1)/3 = 715827883, prime;

Verifying the conjecture:

(For m = 2 and the first four such p)

: 4^7 + 1 = 16385 = 5*29*113, which is divisibe by 3277 = 29*113, a 2-Poulet number;
: 4^11 + 1 = 4194305 = 5*397*2113, which is divisibe by 838861 = 397*2113, a 2-Poulet number;
: 4^13 + 1 = 67108865 = 5*53*157*1613, which is divisibe by:
  : 8321 = 53*157, a 2-Poulet number;
  : 85489 = 53*1613, a 2-Poulet number;
  : 253241 = 157*1613, a 2-Poulet number;
: 4^17 + 1 = 17179869185 = 5*137*953*26317, which is divisibe by:
  : 130561 = 137*953, a 2-Poulet number;
  : 3605429 = 137*26317, a 2-Poulet number;
  : 25080101 = 953*26317, a 2-Poulet number.

Verifying the conjecture:
(For m = 3 and the first two such p)

: (8^7 + 1)/3^2 = 233017 = 43*5419, a 2-Poulet number;
: (8^11 + 1)/3^2 = 954437177 = 67*683*20857, which is divisibe by 1397419 = 67*20857, a 2-Poulet number.

Note:

The Fermat-Coman primes (Fermat-Coman numbers which are primes) seems to be very rare.

Conjecture 3:

For any prime p greater than or equal to 7 the number (4^p + 1)/5 is either prime either a product of primes $p_1*p_2*...p_n$ such that all the numbers p_i*p_j are 2-Poulet numbers for $1 \le i < j \le n$ (this Conjecture is verified for p up to 17 (see the Conjecture 2 above).