The Concept of Gravitorotational Acceleration Field and its Consequences for Compact Stellar Objects

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Abstract: In a previous series of papers relating to the Combined Gravitational Action (CGA), we have exclusively studied orbital motion without spin. In the present paper we apply CGA to any self-rotating material body, *i.e.*, an axially spinning massive object, which itself may be locally seen as a gravitorotational source because it is capable of generating the gravitorotational acceleration field, which seems to be unknown to previously existing theories of gravity. The consequences of such an acceleration field are very interesting, particularly for Compact Stellar Objects.

Keywords: CGA, gravitorotational acceleration field, gravitorotational energy, neutron stars, pulsars

1. Introduction

-*A brief summary of the CGA*: We have previously shown in a series of articles [1,2,3,4] that the Combined Gravitational Action (CGA) as an alternative gravity theory is very capable of investigating, explaining and predicting, in its framework, some old and new gravitational phenomena. Conceptually, the CGA is basically founded on the concept of the combined gravitational potential energy (CGPE) which is actually a new form of velocity-dependent-GPE defined by the expression

$$U \equiv U(r,v) = -\frac{k}{r}\left(1+\frac{v^2}{w^2}\right), \tag{1}$$

where $k = GMm$; G being Newton's gravitational constant; M and m are the masses of the gravitational source A and the moving test-body B; $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ is the relative distance between A and B; $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ is the velocity of the test-body B relative to the inertial reference frame of source A; and w is a specific kinematical parameter having the physical dimensions of a constant velocity defined by

$$w = \begin{cases} c_0, \text{ if } B \text{ is in relative motion inside the vicinity of } A \\ v_{\text{esc}} = \sqrt{2GM/R}, \text{ if } B \text{ is in relative motion outside the vicinity of } A \end{cases} \tag{2}$$

where c_0 is the light speed in local vacuum and v_{esc} is the escape velocity at the surface of the gravitational source A.

In the CGA-context, the velocity-dependent-GPE (1) is simply called CGPE because it is, in fact, a combination of the static-GPE $V(r) = k\,r^{-1}$ and the dynamic-GPE $W(r,v) = k\,r^{-1}(v/w)^2$.

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The main difference between the CGPE (1) as a generalization of classical GPE and the previously well-known velocity-dependent-GPEs is clearly situated in the originality and simplicity of the expression (1). For example, the originality of CGPE is reflected by the fact that the CGPE is explicitly depending on r and v but also is implicitly depending on w since the latter is, by definition, – a specific kinematical parameter having the physical dimensions of a constant velocity – .The implicit dependence of CGPE on w is expressed in terms of '*inside the vicinity of A*' and '*outside the vicinity of A*' in (2). Furthermore, the CGPE may be reduced to the static-GPE when $v \ll w$ or $v = 0$.

Hence, starting from the CGPE and using only the very familiar tools of classical gravitomechanics and the Euler-Lagrange equations, we have established the CGA-formalism [1,2,3]. The main consequence of CGA is the dynamic gravitational field (DGF), $\mathbf{\Lambda}$, which is phenomenologically an induced field that is more precisely a sort of gravitational induction due to the relative motion of material body inside the vicinity of the gravitational source [1,2,3]. In general, the magnitude of DGF is of the form

$$\Lambda = \pm \frac{GM}{r^2}\left(\frac{v}{w}\right)^2 . \tag{3}$$

Eq.(3) means that DGF may play a double role, that is to say, when perceived/interpreted as an extra-gravitational acceleration, $\Lambda > 0$, or an extra-gravitational deceleration, $\Lambda < 0$, (see Ref. [3] for a detailed discussion).

In the papers [1,2,3,4] we have exclusively focused our interest on the orbital motion and gravitational two-body problem. In the present paper, we shall apply CGA to any self-rotating (spinning) material body, *i.e.*, axially rotating massive object that itself may be locally seen as a gravitorotational source since it is capable of generating the gravitorotational acceleration field $\boldsymbol{\lambda}$, which seems to be unknown to previous theories of gravity.

2. Concept of the gravitorotational acceleration field

Phenomenologically speaking, the concept of the gravitorotational acceleration field vector (GRA), $\boldsymbol{\lambda}$, is very similar to DGF, that is if $\mathbf{\Lambda}$ is mainly induced by the relative motion of the massive test body in the vicinity of the principal gravitational source, the GRA is intrinsically generated by any massive body in a state of rotational motion, independently of the principal gravitational source, which itself may be characterized by its proper GRA during its axial-rotation, and therefore the gravitorotational acceleration field is, in fact, a combination of *gravity* and *rotation*.

3. Expression of GRA

In order to derive an explicit expression for GRA, let us first rewrite Eq.(3) for the case when $\Lambda > 0$, that is

$$\Lambda = \frac{GM}{r^2}\left(\frac{v}{w}\right)^2 , \tag{4}$$

and consider a massive body of mass M and radius R, which is intrinsically in a state of axial-rotation in its proper reference frame at rotational velocity of magnitude $v_{\text{rot}} = \Omega R$ independently of the presence of any other gravitational source. Therefore, according to the concept of GRA, in such a case, the rotating massive body should be locally seen as a gravitorotational source when $\|\Lambda\| \rightarrow \|\lambda\| \equiv \lambda$ as $r \rightarrow R$, $v \rightarrow v_{\text{rot}}$ and $w \rightarrow c_0$, thus (4) becomes after substitution

$$\lambda = \frac{GM}{R^2}\left(\frac{\Omega R}{c_0}\right)^2 . \tag{5}$$

Since $\Omega = 2\pi P^{-1}$, where P is the rotational period, hence we get after substitution into (5), the expected expression of GRA

$$\lambda = GM\left(\frac{2\pi}{c_0 P}\right)^2 . \tag{6}$$

It is clear from Eq.(6), GRA λ depends exclusively on the mass and rotational period, therefore, mathematically may be treated as a function of the form $\lambda \equiv \lambda(M, P)$.

The structure of Eq.(6) allows us to affirm that for any astrophysical massive object, the magnitude of λ should be infinitesimally small for slowly rotating massive stellar objects and enormous for rapidly rotating ones. Furthermore, in order to confirm our assertion numerically, we have selected seven well-known (binary) pulsars and calculated their GRAs, and compared them with the Sun's GRA. The values are listed in Table 1.

OBJECT Sun + PRS	P (s)	M (M_{sun})	λ (m s^{-2})	REF.
Sun	2.358720×10^{6}	1	1.047211×10^{-8}	
B 1913+16	5.903000×10^{-2}	1.4410	2.409380×10^{7}	a
B 1534+12	3.790000×10^{-2}	1.3400	5.435171×10^{7}	b,c
B 2127+11C	3.053000×10^{-2}	1.3600	8.501044×10^{7}	d
B 1257+12	6.200000×10^{-3}	1.4000	2.121932×10^{9}	e
J 0737-3039	2.280000×10^{-2}	1.3381	1.500000×10^{8}	f
B 1937+21	1.557800×10^{-3}	1.4000	3.364000×10^{10}	g
J 1748-2446ad	1.395000×10^{-3}	1.4000	4.194982×10^{10}	h

Table 1: The values of GRA for seven well-known (binary) pulsars compared with the Sun's GRA value.
Ref.: a) Taylor and Weisberg [5]; b) Arzoumanian [6]; c) Wolszcan [7]; d) Deich and Kulkarni [8]; e) Konacki and Wolszcan [9]; f) Kramer and Wex [10]; g) Takahashi *et al.* [11]; h) Hessels *et al.* [12].
Note: To calculate these values, we have used $G = 6.67384 \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$, $c_0 = 299792458\ \text{m s}^{-1}$, $M_{\text{sun}} = 1.9891 \times 10^{30}$ kg and $P_{\text{sun}} = 27.30\,\text{d}$.

Analysis of Table1 gives us the following results: 1) The magnitude of the Sun's GRA, $\lambda_{\text{sun}} = 1.047211 \times 10^{-8}\ \text{ms}^{-2}$, is extremely weak that's why its effect on the solar system is unobservable, but perhaps it is only the Sun's immediate vicinity that should be affected by it. Since GRA is explicitly independent of the radius of the rotating massive object, the extreme weakness of the Sun's GRA is mainly due to the huge value of the rotational period, $P_{\text{sun}} = 2.358720 \times 10^{6}\ \text{s}$, compared with those of the pulsars. 2) In spite of the fact that the pulsars' masses are nearly equal, the pulsars' rotational periods show a neat inequality between them. Also, the different values of GRA for each celestial object show us how sensitive GRA is to variation in rotational period.

4. Mutual dependence between the mass and the rotational period

Since GRA may be treated as a function of the form $\lambda \equiv \lambda(M, P)$ hence we can show more clearly the existence of the mutual dependence between the mass and rotational period of the same rotating body *via* GRA. For this purpose, we deduce from Eq.(6) the following expression

$$\frac{M}{P^2} = \left(\frac{c_0^2}{4\pi^2 G}\right)\lambda . \tag{7}$$

Obviously, Eq.(7) shows us the expected mutual dependence between the mass and rotational period *via* GRA. Moreover, because the rotational period is an intrinsic physical quantity, here, according to Eq.(7), the spin of any massive celestial body should vary with mass independently of cosmic time.

5. Link between GRA and rotational acceleration

Now, returning to Eq.(6) and showing that GRA and the rotational acceleration

$$a_{\text{rot}} = \Omega^2 R , \tag{8}$$

are in fact proportional, $\lambda \propto a_{\text{rot}}$, and the constant of proportionality is precisely the compactness factor $\varepsilon = GM/c_0^2 R$ that characterizes any massive celestial body. To this end, it suffices to multiply and divide by the radius R the right hand side of Eq.(6) to get the expected expression

$$\lambda = \varepsilon\, a_{\text{rot}} . \tag{9}$$

According to the expression (9), GRA is at the same time an *old* and a *new* natural physical quantity that should play a crucial role, especially for compact stellar objects, *e.g.*, the rotating neutron stars and pulsars for which the compactness ε has a large value compared to that of normal stellar objects. By way of illustration, the Sun's compactness has the value $\varepsilon_{\text{sun}} = 4.926858 \times 10^{-6}$.

6. Consequences of GRA

In what follows we will show that, in the CGA-context , the transitional state, dynamical stability and instability of a uniformly rotating neutron star (NS) depend on the *'antagonism'* between centrifugal force and gravitational force, or in energetic terms, between rotational kinetic energy (RKE) and gravitational binding energy (GBE).

Usually the physics of NS considers that the source of the emitted energy is essentially the RKE, however, such a consideration should immediately imply that, at least in the medium term, the gravitational binding energy should absolutely dominate RKE and as a result the NS should be prematurely in a state of gravitational collapse. Hence, as we will see, the main source of the emitted energy is not the RKE but the gravitorotational energy (GRE), a sort of *new* physical quantity which is a direct consequence of GRA.

Let us now determine the conditions of transitional state, dynamical stability and instability that may be characterized any NS at least in the medium term. With this aim , we assume a uniformly rotating NS as a homogeneous rigid spherical body of mass M, radius R and rotational velocity $\Omega = 2\pi / P$, where P is the rotational period. It is RKE and GBE that are, respectively, defined by the well-known formulae:

$$E_{\mathrm{rot}} = I\Omega^2/2, \tag{10}$$

and

$$E_{\mathrm{G}} = -\frac{3}{5}\frac{GM^2}{R}, \tag{11}$$

where $I = 2MR^2/5$ is the moment of inertia of NS under consideration. Hence, the total energy is

$$W = E_{\mathrm{rot}} + E_{\mathrm{G}}, \tag{12}$$

which presents the following conditions:

a) $W < 0$, NS is in a state of dynamical stability,
b) $W = 0$, NS is in a state of transition,
c) $W > 0$, NS is in a state of dynamical instability.

It is worth noting that the three suggested conditions a), b) and c) are taken in the medium term because NS may be suddenly in a state of dynamical perturbation or in a state of transition from stability to instability and vice versa.

7. Critical rotational period

Knowing the critical rotational period (CRP) of NS is very important because CRP should be treated as a parameter of reference on which the temporal evolution of NS depends. Furthermore, since the change from stability to instability and vice versa should pass obligatorily *via* the transitional state, we therefore, from the transitional state, deduce an expression for the CRP, thus after performing a simple algebraic calculation, we get the following expected expression

$$P_c = 2\pi R\sqrt{\frac{R}{3GM}}\,. \tag{13}$$

We can numerically evaluate the CRP by taking, through this paper, the standard NS mass and radius, namely, $M = 1.4M_{sun}$ and $R = 10\,\mathrm{km}$, thus by substituting these values into (13), we find

$$P_c = 2.660963\times 10^{-4} \cong 0.2661\,\mathrm{ms}\,, \tag{14}$$

which is a tiny fraction of the smallest yet observed rotational period, $P = 1.3950\,\mathrm{ms}$, of PRS J1748-2446ab [13]. Further, according to (14), the critical value of GRA for a standard NS should be

$$\lambda_c = 1.153\times 10^{12}\,\mathrm{ms}^{-2}\,. \tag{15}$$

8. Gravitorotational energy

Now we approach the most important consequence of GRA, that is, the gravitorotational energy (GRE), which should qualitatively and quantitatively characterize any massive rotating body. As we will see, GRE is quantitatively very comparable to the amount of RKE, particularly for NS and pulsars. Since GRE is a direct consequence of GRA, hence GRE should be proportional to GRA, *i.e.*, $\mathcal{E} \propto \lambda$ or equivalently

$$\mathcal{E} = \kappa\lambda\,. \tag{16}$$

Let us determine the constant of the proportionality κ by using dimensional analysis, as follows:

$$[\kappa] = \frac{[\mathcal{E}]}{[\lambda]} = \frac{\mathrm{ML^2T^{-2}}}{\mathrm{LT^{-2}}} = \mathrm{ML}.$$

we can remark that the dimensional quantity ML has the physical dimensions of the product of mass and length, therefore, for our case κ should take the form $\kappa = MR = 5I/2R$ and by substituting into (16), we find the required expression for GRE

$$\mathcal{E} = \frac{5}{2}\frac{\lambda I}{R}\,. \tag{17}$$

In order to show that the amount of GRE $\mathcal{E}$ is quantitatively very comparable to that of RKE, particularly for the compact stellar objects, we can use Table 1. The numerical values of E_{rot} and $\mathcal{E}$ are listed in Table 2.

OBJECT Sun + PRS	E_{rot} (J)	$\mathcal{E}$ (J)
Sun	1.3655×10^{36}	5.7950×10^{30}
B 1913+16	6.4948×10^{41}	6.9180×10^{41}
B 1534+12	1.4651×10^{42}	1.4500×10^{42}
B 2127+11C	2.2915×10^{42}	2.3010×10^{42}
B 1257+12	5.7200×10^{43}	5.9140×10^{43}
J 0737-3039	4.0426×10^{42}	4.0000×10^{42}
B 1937+21	9.0604×10^{44}	9.3680×10^{44}
J 1748-2446ad	1.1300×10^{45}	1.1680×10^{45}

Table 2: comparison of the numerical values of E_{rot} and $\mathcal{E}$ for the Sun and seven well known (binary) pulsars.

Analysis of Table 2: The numerical values listed in Table 2 show us, excepting the Sun's values, that all the values of E_{rot} and $\mathcal{E}$ are very comparable for the seven (binary) pulsars. This fact is mainly due, at the same time, to the rotational period and the compactness ε. To illustrate this fact, let us return to the expression (17) which may be written as follows:

$$\mathcal{E} = 5\varepsilon E_{\mathrm{rot}} \ . \tag{18}$$

And as $5\varepsilon \cong 1$ for the NSs hence that's why $\mathcal{E} \cong E_{\mathrm{rot}}$ as it is well illustrated in Table 2. From all this we arrive at the following result: In the CGA-context, the RKE cannot be considered as the main source of the emitted energy for rotating neutron stars and pulsars because-in energetic terms- its own role is to balance, approximately, the GBE, at least in the medium term. Therefore, the veritable principal source of the emitted energy should undoubtedly be GRE, as illustrated by the GRE numerical values listed in Table 2, which are quantitatively very comparable to those of RKE for pulsars. Moreover, if we take into account the critical value of GRA (15), we get the following critical value for GRE

$$\mathcal{E}_{\mathrm{c}} = 3.210\times10^{46}\,\mathrm{J} = 3.210\times10^{53}\,\mathrm{erg}\ . \tag{19}$$

9. Rotating magnetars

Rotating magnetized neutron stars (magnetars) are also important compact stellar objects. That's why it is possible, in the CGA-context, to exploit GRE as an energetic reservoir for rotating magnetars by assuming that there is a certain physical mechanism that can convert all or at least a significant part of GRE into an extreme internal magnetic energy:

$$E_B = B^2 R^3 = \mathcal{E}\ , \tag{20}$$

which could, of course, produce an extreme *internal* magnetic field strength

$$B = \sqrt{\mathcal{E} R^{-3}} \quad \text{(T)}. \tag{21}$$

The observed surface (external) dipole magnetic field strength B_0 would be lower than the internal field strength B. As an illustration, let us evaluate the strength of internal magnetic field of radio pulsar B 1931+24. We have according to Ref. [13] the following parameters: $P = 0.813\,\mathrm{s}$ and $B_0 \cong 3\times 10^{12}\,\mathrm{G}$. By taking, as usual, $M = 1.4 M_{\mathrm{sun}}$ and $R = 10\,\mathrm{km}$, we find for GRE $\mathcal{E} = 3.440\times 10^{39}\,\mathrm{J}$, and after substitution into (21), we obtain

$$B = 5.865\times 10^{13}\,\mathrm{T} = 5.865\times 10^{17}\,\mathrm{G}. \tag{22}$$

Finally, let us evaluate the critical strength of internal magnetic field. We have according to (19) and (21), the following value

$$B_{\mathrm{c}} = 1.80\times 10^{17}\,\mathrm{T} = 1.80\times 10^{21}\,\mathrm{G}. \tag{23}$$

10. Conclusion

Basing on our gravity model, Combined Gravitational Action, we have derived an explicit expression for the concept of gravitorotational acceleration field (GRA), which is unknown to previously established gravity theories. The most significant result of GRA is the gravitorotational energy (GRE), which should qualitatively and quantitatively characterize any massive rotating body. Furthermore, GRE is exploited as an energetic reservoir, particularly for neutron stars and pulsars.

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