P vs NP: Solutions of NP Problems

Abstract

The simplest solution is usually the best solution---Albert Einstein

Best news. After over 30 years of debating, the debate is over. Yes, P is equal to NP.
For the first time, NP problems have been solved in this paper. Techniques and formulas were developed and used to solve these problems as well as produce simple equations to help programmers apply the techniques. The techniques and formulas are based on an extended Ashanti fairness wisdom as exemplified below. If two people A and B are to divide items of different sizes which are arranged from the largest size to the smallest size, the procedure will be as follows. In the first round, A chooses the largest size, followed by B choosing the next largest size. In the second round, B chooses first, followed by A. In the third round, A chooses first, followed by B and the process continues up to the last item. To abbreviate the sequence in the above choices, one obtains the sequence "A, BB, AA, BB, AA". Let A and B divide the sum of the whole numbers, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 as equally as possible, by merely always choosing the largest number. Then A chooses 10, B chooses 9 and 8, followed by A choosing 7 and 6; followed by B choosing 5 and 4; followed by A choosing 3 an 2; and finally, B chooses 1. The sum of A's choices is 10 +7+ 6 + 3 + 2 = 28; and the sum of B's choices is 9 + 8 + 5 + 4 + 1 = 27, with error, plus or minus 0.5 . Observe the sequence "A, BB, AA, BB, AA". Observe also that the sequence is **not** "AB, AB, AB, AB, AB as one might think. If one were to use AB, AB, AB, AB, AB, the sum for A would be 10 + 8 + 6 + 4 + 2 = 30 and the sum for B would be 9 +7 + 5 +3 + 1 = 25, with error, plus or minus 2.5. The reason why the sequence is "A, BB, AA, BB, AA", and **not** "AB, AB, AB, AB, AB" is as follows. In the first round, when A chooses first, followed by B, A has the advantage of choosing the larger number and B has the disadvantage of choosing the smaller number. In the second round, if A were to choose first, A would have had two consecutive advantages, and therefore, in the second round, B will choose first to produce the sequence AB, B or ABB. In the third round, A chooses first, because B chose first in the second round. After three rounds, the sequence would be A, BB, AA. When this technique was applied to 100 items of different masses, by mere combinations, the total mass of A's items was equal to the total mass of B's items. Similarly, for 1000 items of different masses, the total mass of A's items was equal to the total mass of B's items.
By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses. School secretaries, office assistants can learn and apply the techniques covered.
The author also confirmed the notion that a method that solves one of these problems can also solve other NP problems. Consider the following two problems which are modifications of suggested NP problems from the Wikipedia (Simple English) website.

Problem 1: A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market.

Problem 2: A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for.

After solving Problem 1, one was able to solve Problem 2 by mere inspection of the solutions to Problem 1.

Since a method that solves a single NP problem can solve other NP problems, and six problems have been solved in this paper, all NP problems (well-posed problems) can be solved. If all NP problems can be solved, then all NP problems are P problems and therefore, P is equal to NP. The CMI Millennium Prize requirements have been satisfied.

Solutions of NP Problems

The following sample problems will be solved and analyzed. They are based on the suggested sample problems from the Wikipedia (Simple English) website. Many Thanks to Wikipedia.

Basis of the method used in solving the NP problems: **Ratios** To Page 3

Example 1 (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally.
14,13,12,11,10,9,8,7, 6,5,4,3,2,1.
To Page 4

Example 2a Consider the existence of dollar bills with denominations $100, $99, $98,...$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, divide the total value of these dollar bills equally between A and B. To Page 6

Example 2b Consider the existence of dollar bills with denominations $100, $99, $98,...$2, down to $1. Suppose the bills are on a table with the $100 bill at the top, followed by the $99, $98, $97 bills, and so on with the $1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question: (a) If a computer costs $2,000, can A afford to buy this computer?
(b) If a computer costs $3,000, can A afford to buy this computer? To Page 8

Example 3 Let one randomly delete some of the bills in Example 2a, a previous example, and divide the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining. To Page 8

Example 4 A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market. To Page 10

Example 5 A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for. To Page 12

Example 6 A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B. To Page 14

To Overall Conclusion

Basis of the method used in solving the NP problems: Ratios

Method 2 below is the method used for the solutions of the **NP** problems.

Example 1: Divide \$12 between A and B in the ratio 1: 2

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = $\frac{1}{1+2} = \frac{1}{3}$

Fraction of the money B receives = $\frac{2}{1+2} = \frac{2}{3}$

Step 2: Amount A receives = $\frac{1}{3} \times \frac{12}{1} = 4$

Amount B receives = $\frac{2}{3} \times \frac{12}{1} = 8$

Therefore, A receives \$4, and B receives \$8

(**Method 1** above is from the author's book entitled "Power of Ratios" by A. A. Frempong, and published by Yellowtextbooks.com.)

Method 2 (The process method)

The ratio 1:2 means whenever A receives \$1, B receives \$2.

Step 1: In the first round, A receives \$1, and B receives \$2.
After the first round, the amount of money remaining is \$12 - (\$1 + \$2) = \$9.

Step 2: In the second round, from this \$9, A receives \$1 and B receives \$2.
After the second round, the amount of money remaining = \$9 - (\$1+ \$2) = \$6

Step 3: In the third round, A receives \$1 and B receives \$2.
The amount remaining = \$6 - (\$1 + \$2) = \$3

Step 4: In the fourth and final round, A receives \$1 and B receives \$2.
The amount remaining = \$3 - (\$1 + \$2) = 0

Step 5: A's total = \$1 + \$1 + \$1 + \$1 = \$4
B's total = \$2 + \$2 + \$2 + \$2 = \$8

Example 2 Divide \$12 between A and B in the ratio 1: 1

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = $\frac{1}{1+1} = \frac{1}{2}$

Fraction of the money B receives = $\frac{1}{1+1} = \frac{1}{2}$

Step 2: Amount A receives = $\frac{1}{2} \times \frac{12}{1} = 6$

Amount B receives = $\frac{1}{2} \times \frac{12}{1} = 6$

Therefore, A receives \$6, and B receives \$6.

Method 2 (The process method)

The ratio 1:1 means whenever A receives \$1, B receives \$1.

Step 1: In the first round, A receives \$1, and B receives \$1.
After the first round, the amount of money remaining is \$12 - (\$1 + \$1) = \$10

Step 2: In the second round, from this \$10, A receives \$1 and B receives \$1.
After the second round, the amount of money remaining = \$10 - (\$1+ \$1) = \$8

Step 3: In the third round, A receives \$1, and B receives \$1.
The amount remaining = \$8 - (\$1 + \$1) = \$6

Step 4: In the fourth round, A receives \$1 and B receives \$1
The amount remaining = \$6 - (\$1 + \$1) = 4

Step 5: In the fifth round, A receives \$1 and B receives \$1.
The amount remaining = \$4 - (\$1 + \$1) = 2

Step 6: In the sixth and final round, A receives \$1 and B receives \$1.
The amount remaining = \$2 - (\$1 + \$1) = 0

Step 7: A's total = \$1 + \$1 + \$1 + \$1 + \$1 + \$1 = \$6.
B's total = \$1 + \$1 + \$1 + \$1 + \$1 + \$1 = \$6.

Case 1: Only two devisors A and B

Example 1 (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally.

14,13,12,11,10,9,8,7, 6,5,4,3,2,1.

Solution
For communication purposes, one will call the numbers to be divided the "dividends"; and one will call A and B the "divisors". Let the sum of A's choices be Q_A, and let the sum of B's choices be Q_B.

Step 1: Check to ensure that the numbers are arranged in decreasing order.
One will apply the wisdom method of the introduction.
That is, one applies "A, BB, AA, BB, AA, BB, AA, B

Method 1 Using braces
Step 2: A chooses the first element, 14
Step 3: B chooses the next two elements, 13 and 12.,
Step 4: A chooses the next two elements 11, and 10, and the alternating consecutive choices continue to the end.

$$\underbrace{14}_{A},\ \underbrace{13,\ 12}_{B},\ \underbrace{11,\ 10}_{A},\ \underbrace{9,\ 8}_{B},\ \underbrace{7,\ 6}_{A},\ \underbrace{5,\ 4}_{B},\ \underbrace{3,\ 2}_{A},\ \underbrace{1}_{B} \qquad (1)$$

Step 5: Add the choices for A and add the choices for B.

$$Q_A = 14 + 11 + 10 + 7 + 6 + 3 + 2$$
$$= 53$$
$$Q_B = 13 + 12 + 9 + 8 + 5 + 4 + 1$$
$$= 52$$

The sum for A = 53; and the sum for B = 52.

Method 2 (Tabular form)
Step 1: List the dividends as shown below

14	13	12	11	10	9	8	7	6	5	4	3	2	1

Step 2: Write the divisors A, BB, AA, BB, AA, etc, above the numbers, This is the choosing step.

A	B	B	A	A	B	B	A	A	B	B	A	A	B
14	13	12	11	10	9	8	7	6	5	4	3	2	1

Note: $\overset{A}{\boxed{14}}$ means A chooses 14. $\overset{B}{\boxed{13}}$ means B chooses 13.

Step 3: Collect and add the corresponding (dividends) choices

Q_A	Q_B
14	13
11	12
10	9
7	8
6	5
3	4
2	1
Total: 53	52

Mathematical formulas for choosing the elements

Let $a_1 = 14,\ a_2 = 13,\ a_3 = 12,\ a_4 = 11,\ a_5 = 10,$
$a_6 = 9,\ a_7 = 8,\ a_8 = 7,\ a_9 = 6,\ a_{10} = 5,\ a_{11} = 4,\ a_{12} = 3,\ a_{13} = 2,\ a_{14} = 1$

By experimentation, one obtains the following formulas for A and B.

$$Q_A = a_1 + \sum_{n=2,4,6,}^{6} a_{2n} + a_{2n+1} \qquad (Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13})$$

$$Q_B = \sum_{n=1,3,5,7}^{5} a_{2n} + a_{2n+1} + a_{14} \qquad (Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14})$$

Apply the formulas to above (The above formulas are valid for only **two** divisors.)

$$\begin{aligned} Q_A &= a_1 + \sum_{n=2,4,6}^{6} a_{2n} + a_{2n+1} \\ &= 14 + 11 + 10 + 7 + 6 + 3 + 2 \\ &= 53 \\ Q_B &= \sum_{n=1,3,5.}^{5} a_{2n} + a_{2n+1} + a_{14} \\ &= 13 + 12 + 9 + 8 + 5 + 4 + 1 \\ &= 52 \end{aligned}$$

Note that the above formulas using the sigma notation are valid for only two divisors, A and B. For three divisors A, B, and C, different formulas would have to be derived, based on the solutions of the problem.

Back to examples

Example 2a Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Method 2a: Using the numerical values and braces

Apply, A, BB, AA, BB, AA. (as in Method 1 of Example 1)

Step 1: A chooses the first \$100 bill. (Only a single item is removed).

Step 2: B chooses the next two bills, the \$99 and \$98 bills, (two items removed consecutively)

Step 3: A chooses the next two bills, the \$97 and \$96 bills, and the alternating removal continues to the end.

$\underbrace{100}_{A}, \underbrace{99\ 98}_{B}, \underbrace{97,\ 96}_{A}, \underbrace{95,\ 94}_{B}, \underbrace{93,\ 92}_{A}\ \underbrace{91,\ 90}_{B}, \underbrace{89,\ 88}_{A}, \underbrace{87\ 86}_{B}, \underbrace{85,\ 84}_{A}, \underbrace{83,\ 82}_{B}, \underbrace{81,\ 80}_{A}\ \underbrace{79,\ 78}_{B},$

$\underbrace{77,\ 76}_{A},, \underbrace{75\ 74}_{B}, \underbrace{73,\ 72}_{A}, \underbrace{71,\ 70}_{B}, \underbrace{69,\ 68}_{A}\ \underbrace{67,\ 66}_{B}, \underbrace{65,\ 64}_{A}, \underbrace{63\ 62}_{B}, \underbrace{61,\ 60}_{A}, \underbrace{59,\ 58}_{B}, \underbrace{57,\ 56}_{A}\ \underbrace{55,\ 54}_{B},$

$\underbrace{53,\ 52}_{A}, ,\underbrace{51,\ 50}_{B}, \underbrace{49\ 48}_{A}, \underbrace{47,\ 46}_{B}, \underbrace{45,\ 44}_{A}, \underbrace{43,\ 42}_{B}\ \underbrace{41,\ 40}_{A}, \underbrace{39,\ 38}_{B}, \underbrace{37\ 36}_{A}, \underbrace{35,\ 34}_{B}, \underbrace{33,\ 32}_{A}, \underbrace{31,\ 30}_{B}$

$\underbrace{29,\ 28}_{A},, \underbrace{27,\ 26}_{B}, \underbrace{25\ 24}_{A}, \underbrace{23,\ 22}_{B}, \underbrace{21,\ 20}_{A}, \underbrace{19,\ 18}_{B}, \underbrace{17,\ 16}_{A}, \underbrace{15,\ 14}_{B}, \underbrace{13,\ 12}_{A}, \underbrace{11,\ 10}_{B}, \underbrace{9,\ 8}_{A}, \underbrace{7,\ 6}_{B}, \underbrace{5,\ 4}_{A}$

$\underbrace{3,\ 2}_{B}, \underbrace{1}_{A},$

Step 4: Collect and add the choices (dividends) for A and B

$$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = \mathbf{2525}.$$

$$Q_B = 99 + 98 + 95 + 94 + 91 + 90 + 87 + 86 + 83 + 82 + 79 + 78 + 75 + 74 + 71 + 70 + 67 + 66 + 63 + 62 + 59 + 58 + 55 + 54\ 51 + 50 + 47 + 46 + 43 + 42 + 39 + 38 + 35 + 34 + 31 + 30 + 27 + 26 + 23 + 22 + 19 + 18 + 15 + 14 + 11 + 10 + 7 + 6 + 3 + 2 = \mathbf{2525}.$$

Conclusion: A receives \$2525 and B receives \$2525, Note **t**he zero error for A and B.

Method 2b: Using tabular form

Step 1: Write the divisors A and B above the numbers (as done in Method 2 of Example 1)

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Step 2: Collect and add the Choices (dividends)

$$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = \mathbf{2525.}$$

$Q_B = 99+98+95+94+91+90+87+86+83+82+79+78+75+74+71+70+67+66+$
$63+62+59+58+55+54+51+50+47+46+43+42+39+38+35+34+31+30+27+$
$26+23+22+19+18+15+14+11+10+7+6+3+2=\mathbf{2525}$.

The above results are pleasantly astonishing. Of the 2^{100} possible ways to divide the above bills, the above technique and consequently the derived formulas divided the above mixture of bills into exactly two equal parts in value. Why has this technique been hiding for nearly 30 years? Note that the ratio $Q_A : Q_B$ is $1:1$.

Equations for above: $Q_A = a_1 + \sum_{n=2,4,6,...}^{48} a_{2n} + a_{2n+1} + a_{100}$ and $Q_B = \sum_{n=1,3,5,...}^{49} a_{2n} + a_{2n+1}$

Method 1b: Using term numbers and braces Apply, A, BB, AA, BB, AA, ...

$\underbrace{a_1}_{A}, \underbrace{a_2\ a_3}_{B}, \underbrace{a_4, a_5}_{A}, \underbrace{a_6, a_7}_{B}, \underbrace{a_8, a_9}_{A}\ \underbrace{a_{10}, a_{11}}_{B}, \underbrace{a_{12}, a_{13}}_{A}, \underbrace{a_{14}\ a_{15}}_{B}, \underbrace{a_{16}, a_{17}}_{A}, \underbrace{a_{18}, a_{19}}_{B}, \underbrace{a_{20}, a_{21}}_{A}\ \underbrace{a_{22}, a_{23}}_{B},$

$\underbrace{a_{24}, a_{25}}_{A},, \underbrace{a_{26}, a_{27}}_{B}\ \underbrace{a_{28}, a_{29}}_{A}\ \underbrace{a_{30}, a_{31}}_{B}, \underbrace{a_{32}, a_{33}}_{A}\ \underbrace{a_{34}, a_{35}}_{B}, \underbrace{a_{36}, a_{37}}_{A}\ \underbrace{a_{38}\ a_{39}}_{B}\ \underbrace{a_{40}, a_{41}}_{A}, \underbrace{a_{42}, a_{43}}_{B}, \underbrace{a_{44}, a_{45}}_{A}$

$\underbrace{a_{46}, a_{47}}_{B}, \underbrace{a_{48}, a_{49}}_{A},, \underbrace{a_{50}, a_{51}}_{B}, \underbrace{a_{52}\ a_{53}}_{A}, \underbrace{a_{54}, a_{55}}_{B}\ \underbrace{a_{56}, a_{57}}_{A}\ \underbrace{a_{58}, a_{59}}_{B}\ \underbrace{a_{60}, a_{61}}_{A}, \underbrace{a_{62}, a_{63}}_{B}\ \underbrace{a_{64}\ a_{65}}_{A}, \underbrace{a_{66}, a_{67}}_{B}$

$\underbrace{a_{68}, a_{69}}_{A}\ \underbrace{a_{70}, a_{71}}_{B}\ \underbrace{a_{72}, a_{73}}_{A}, \underbrace{a_{74}, a_{75}}_{B}, \underbrace{a_{76}, a_{77}}_{A}\ \underbrace{a_{78}, a_{79}}_{B}\ \underbrace{a_{80}, a_{81}}_{A}\ \underbrace{a_{82}, a_{83}}_{B}, \underbrace{a_{84}, a_{85}}_{A}, \underbrace{a_{86}, a_{87}}_{B}, \underbrace{a_{88}, a_{89}}_{A},$

$\underbrace{a_{90}, a_{91}}_{B}, \underbrace{a_{92}, a_{93}}_{A}, \underbrace{a_{94}, a_{95}}_{B}, \underbrace{a_{96}, a_{97}}_{A}\ \underbrace{a_{98}, a_{99}}_{B}, \underbrace{a_{100}}_{A},$

Using the term numbers and tabular form

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Collect the terms for A and add them ; and similarly collect the terms for B and add them.

$Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33}$
$+ a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65}$
$+ a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77} + a_{80} + a_{81} + a_{84} + a_{85} + a_{88} + a_{89} + a_{92} + a_{93} + a_{96} + a_{97} + a_{100}$

$Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34}$
$+ a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66}$
$+ a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78} + a_{79} + a_{82} + a_{83} + a_{86} + a_{87} + a_{90} + a_{91} + + a_{94} + a_{95} + a_{98} + a_{99}$

Back to examples

Example 2b Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question: (a) If a computer costs \$2,000, can A afford to buy this computer?
(b) If a computer costs \$3,000, can A afford to buy this computer?

Answers: (a) From the solution of Example 2a, A received \$2,525, and therefore can afford to buy this computer.
Yes. A can afford to buy this \$2,000 computer.

(b) Since from the solution of Example 2a, A received \$2,525, and the computer costs \$3,000, A cannot afford to buy \$3,000 computer
No. A cannot afford to buy this \$3,000 computer.

Example 3

Let one randomly delete some of the bills in Example 2a, a previous example, and divide as equally as possible the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining.

Solution: Using the numerical values and braces

$\underbrace{98}_{A}.\underbrace{97,96}_{B},\underbrace{95,94}_{A},\underbrace{93,91}_{B},\underbrace{90,89}_{A},\underbrace{88,87}_{B},\underbrace{86.85}_{A},.\underbrace{81,80}_{B},\underbrace{79,78}_{A},\underbrace{77,76}_{B},$

$\underbrace{75,74}_{A},\underbrace{73,72}_{B},\underbrace{69.68}_{A},\underbrace{67,66}_{B},\underbrace{65,64}_{A},\underbrace{62,61}_{B},\underbrace{60,58}_{A},\underbrace{56,55}_{B},\underbrace{54,53}_{A}.\underbrace{51.50}_{B},$

$\underbrace{49,47}_{A},\underbrace{46,45}_{B},\underbrace{43.,41}_{A},\underbrace{40,39}_{B},\underbrace{37,36}_{A},\underbrace{35.34}_{B},\underbrace{33.32}_{A},\underbrace{31,30}_{B},\underbrace{29,26}_{A},\underbrace{25.24}_{B},$

$\underbrace{23,22}_{A},\underbrace{21,20}_{B},\underbrace{19,18}_{A},\underbrace{16,14}_{B},\underbrace{13,12}_{A},\underbrace{11,10}_{B},\underbrace{9,8}_{A},\underbrace{7,5}_{B},\underbrace{4,2}_{A},\underbrace{1}_{B}$

$$Q_A = 98+95+94+90+89+86+85+79+78+75+74+69+68+65+64+60+58+54+53 \\ +49+47+43+41+37+36+33+32+29+26+23+22+19+18+13+12+9+8+4+2 \\ = \mathbf{1937}$$

$$Q_B = 97+96+93+91+88+87+81+80+77+76+73+72+67+66+62+61+56+55+51 \\ +50+46+45+40+39+35+34+31+30+25+24+21+20+16+14+11+10+7+5+1 \\ = \mathbf{1933}$$

Total of Q_A and $Q_B = 3870$. Division by 2 yields 1935. For Q_A, relative error = $\frac{2}{1935} = 0.0010$ or about 0.1% For Q_B, relative error = $\frac{-2}{1935} = -0.0010$	For equality, interchange the 47 bill in Q_A and the 45 bill in Q_B. Thus A gives \$2 to B, resulting in equality of **\$1,935** each. Other bills can be interchanged.

Using term numbers

$$Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33} \\ + a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65} \\ + a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77}$$

$$Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34} \\ + a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66} \\ + a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78}$$

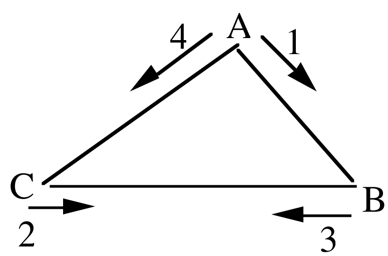
Observe above that the last term for Q_A is a_{77} and the last term foir Q_B is a_{78} (there are 78 terms)

Case 2: Three or more divisors

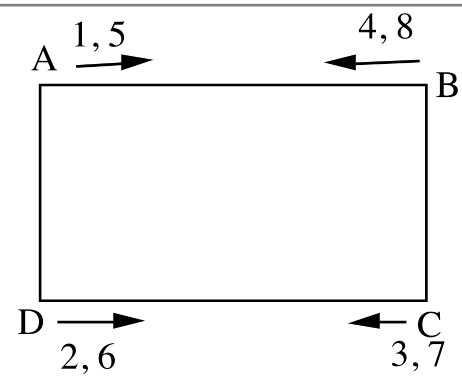
In the previous examples, for communication purposes, A and B were called the "divisors" and the numbers or terms to be divided were called "dividends". The concept of divisors A and B can be extended to three or more divisors such as A, B, C, or A, B, C, D, but in these cases, geometric figures will help keep track of the choices.

Geometric figures to keep track of the order and directions of the divisors
(For three or more divisors such as A, B, C; four divisors A, B, C,D)

The arrows are for directions



ABC / CBA / BCA / ACB



ABCD / DCBA / CDAB / BADC / ABCD

For ABC:

Step 1: Go Clockwise ABC (In the first round, A chooses first and C chooses last))
Step 2: Begin with C and reverse the direction in Step 1 and go CBA.
(Since C was at the largest disadvantage in the first round, by choosing last, C chooses chooses first in the second round followed by B)
Step 3: Begin with B and reverse previous direction (direction of C) and go clockwise BCA.
Step 4: Begin with A again, change previous direction (direction of B) and go counterclockwise ACB.

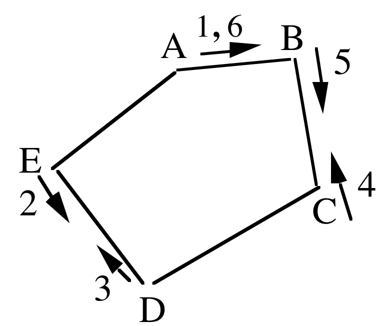
For ABCD

Step 1: Go Clockwise ABCD. (first round)
Step 2: Begin with D and reverse the direction in Step 1 (direction of A) and go DCBA......
Step 3: Begin with C and reverse previous direction (direction of D) and go clockwise CDAB.
Step 4: Begin with B reverse previous direction (direction of C) and go counterclockwise BADC.
Step 5: Beginning again, reverse the direction but by coincidence go clockwise ABCD. (5th round)

For five divisors A, B, C, D, E

ABCDE , *EDCBA*, *DEABC*, *CBAED* *BCDEA*

Step 1: Go Clockwise ABCDE
Step 2: Begin with E and reverse the direction in Step 1 and go EDCBA
Step 3: Begin with D and reverse previous direction and go clockwise DEABC
Step 4: Begin with C reverse previous direction and go counterclockwise CBAED
Step 5: Begin with B reverse previous direction and go clockwise BCDEA.
Step 6: Beginning again with A, reverse the direction but by coincidence, go clockwise ABCDE.



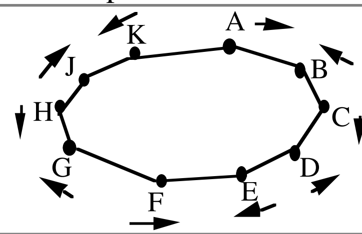
ABCDE , *EDCBA DEABC*
CBAED, *BCDEA*

Back to examples

Example 4: A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market.

Step 1: Arrange the items in decreasing order of their masses. Let the mass of the first item (largest) be 100 units, and let the masses of the rest of the items be respectively, 99, 98, 97, and so on down to smallest item of mass 1 unit. Let the 10 boxes be labeled A, B, C, D, E, F, G, H, J, and K The ten boxes are to divide the 100 items. **Imitate** Example 2 but with 10 divisors.

Guide1; *ABCDEFGHJK*
Guide 2; *KJHGFEDCBA*
Guide 3 *JKABCDEFGH*
Guide 4; *HGFEDCBAKJ*
Guide 5; *GHJKABCDEF*
Guide 6; *FEDCBAKJHG*
Guide 7: *EFGHJKABCD*
Guide 8; *DCBAKJHGFE*
Guide 9; *CDEFGHJKAB*
Guide10: *BAKJHGFEDC*



A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Step 2: **Collect the choices for A, B, C, D, E, F, G, H, J, K**

Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
100 a_1	99, a_2	98 a_3	97 a_4	96 a_5	95 a_6	94 a_7	93 a_8	92 a_9	91 a_{10}
81 a_{20}	82, a_{19}	83 a_{18}	84 a_{17}	85 a_{16}	86 a_{15}	87 a_{14}	88 a_{13}	89 a_{12}	90 a_{11}
78 a_{23}	77, a_{24}	76 a_{25}	75 a_{26}	74 a_{27}	73 a_{28}	72 a_{29}	71 a_{30}	80 a_{21}	79 a_{22}
63 a_{38}	64, a_{37}	65 a_{36}	66 a_{35}	67 a_{34}	68 a_{33}	69 a_{32}	70 a_{31}	61 a_{40}	62 a_{39}
56 a_{45}	55, a_{46}	54 a_{47}	53 a_{48}	52 a_{49}	51 a_{50}	60 a_{41}	59 a_{42}	58 a_{43}	57 a_{44}
45 a_{56}	46 a_{55}	47 a_{54}	48 a_{53}	49 a_{52}	50 a_{51}	41 a_{60}	42 a_{59}	43 a_{58}	44 a_{57}
34 a_{67}	33 a_{68}	32 a_{69}	31 a_{70}	40 a_{61}	39 a_{62}	38 a_{63}	37 a_{64}	36 a_{65}	35 a_{66}
27 a_{74}	28 a_{73}	29 a_{72}	30 a_{71}	21 a_{80}	22 a_{79}	23 a_{78}	24 a_{77}	25 a_{76}	26 a_{75}
12 a_{89}	11 a_{90}	20 a_{81}	19 a_{82}	18 a_{83}	17 a_{84}	16 a_{85}	15 a_{86}	14 a_{87}	13 a_{88}
9 a_{92}	10 a_{91}	1 a_{100}	2 a_{99}	3 a_{98}	4 a_{97}	5 a_{96}	6 a_{95}	7 a_{94}	8 a_{93}
Total: 505	**505**	**505**	**505**	**505**	**505**	**505**	**505**	**505**	**505**

Condition for sufficiency:
The 10 boxes would be sufficient to carry all the 100 items to the market if the mass of the contents of each box is equal to or less than 560 units. Since mass of the contents in each box is 505 units, which is less than 560 units, each box satisfies this sufficiency condition. Therefore, the 10 boxes would be sufficient to carry the 100 items to the market.
Note above that the ratio

$$Q_A : Q_B : Q_C : Q_D : Q_E : Q_F : Q_G : Q_H : Q_{J:} : Q_K = 1:1:1:1:1:1:1:1:1:1$$

Method 4b Using the term numbers

A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Collect the terms for A, B, C, D, E, F, G, H, J, K

$$\boxed{Q_A = a_1 + a_{20} + a_{23} + a_{38} + a_{45} + a_{56} + a_{67} + a_{74} + a_{89} + a_{92}}$$

$$\boxed{Q_B = a_2 + a_{19} + a_{24} + a_{37} + a_{46} + a_{55} + a_{68} + a_{73} + a_{90} + a_{91}}$$

$$\boxed{Q_C = a_3 + a_{18} + a_{25} + a_{36} + a_{47} + a_{54} + a_{69} + a_{72} + a_{81} + a_{100}}$$

$$\boxed{Q_D = a_4 + a_{17} + a_{26} + a_{35} + a_{48} + a_{53} + a_{70} + a_{71} + a_{82} + a_{99}}$$

$$\boxed{Q_E = a_5 + a_{16} + a_{27} + a_{34} + a_{49} + a_{52} + a_{61} + a_{80} + a_{83} + a_{98}}$$

$$\boxed{Q_F = a_6 + a_{15} + a_{28} + a_{33} + a_{50} + a_{51} + a_{62} + a_{79} + a_{84} + a_{97}}$$

$$\boxed{Q_G = a_7 + a_{14} + a_{29} + a_{32} + a_{41} + a_{60} + a_{63} + a_{78} + a_{85} + a_{96}}$$

$$\boxed{Q_H = a_8 + a_{13} + a_{30} + a_{31} + a_{42} + a_{59} + a_{64} + a_{77} + a_{86} + a_{95}}$$

$$\boxed{Q_J = a_9 + a_{12} + a_{21} + a_{40} + a_{43} + a_{58} + a_{65} + a_{76} + a_{87} + a_{94}}$$

$$\boxed{Q_K = a_{10} + a_{11} + a_{22} + a_{39} + a_{44} + a_{57} + a_{66} + a_{75} + a_{88} + a_{93}}$$

Sub-Conclusion
The fairness wisdom method has performed perfectly.
Observe above in Step 2 that the totals for Q_A, Q_B, Q_C, Q_D, Q_E, Q_F, Q_G, Q_H, Q_J, Q_K are all the same. The technique applied picked combinations to produce these equal totals.
Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order, a task a computer performs very fast..

In the next eample, Example 5, one will confirm the notion that a method that solves one of the NP problems can be used to solve other similar problems. One will use the results of the above example Example 4b to do the next problem.

Back to examples

Example 5: A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for.

Step1: Final Exams 8AM – 6PM $\begin{cases} A=8-9;\ B=9-10;\ C=10-11;\ D=11-12; E=12-1; \\ F=1-2;\ G=2-3;\ \ H=3-4;\quad J=4-5;\ \ K=5-6 \end{cases}$

Let the course numbers be $a_1,\ a_2,\ a_3,... a_{100}$ **Using the result of Example 4b**

A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Step 2: Collect the choices for A, B, C, D, E, F, G, H, J, K

Final Exam Schedule: 8 AM-6 PM

$Q_A=8-9;\ Q_B=9-10;\ Q_C=10-11;\ Q_D=11-12;\ Q_E=12-1;$
$Q_F=1-2;\ Q_G=2-3;\ Q_H=3-4;\ Q_J=4-5;\ Q_K=5-6$

8- 9	9-10	10-11	11-12	12-1	1-2	2-3	3-4	4-5	5-6
Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_{20}	a_{19}	a_{18}	a_{17}	a_{16}	a_{15}	a_{14}	a_{13}	a_{12}	a_{11}
a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{21}	a_{22}
a_{38}	a_{37}	a_{36}	a_{35}	a_{34}	a_{33}	a_{32}	a_{31}	a_{40}	a_{39}
a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{41}	a_{42}	a_{43}	a_{44}
a_{56}	a_{55}	a_{54}	a_{53}	a_{52}	a_{51}	a_{60}	a_{59}	a_{58}	a_{57}
a_{67}	a_{68}	a_{69}	a_{70}	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}
a_{74}	a_{73}	a_{72}	a_{71}	a_{80}	a_{79}	a_{78}	a_{77}	a_{76}	a_{75}
a_{89}	a_{90}	a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}
a_{92}	a_{91}	a_{100}	a_{99}	a_{98}	a_{97}	a_{96}	a_{95}	a_{94}	a_{93}

The final exam for every course has been scheduled. However, if a student takes for example, Course a_1 and course a_{20}, because the duration for the final exams for these two courses is 8-9 AM, the student cannot take the final exams for these two courses simultaneously. Therefore, it is **not** possible to prepare a schedule to allow every student to take the final exams for all registered courses on the same day. However, below is what is possible.

In order for every student to take the final exam for all courses registered for, ten days would be needed as shown below, where the course numbers are a_1, a_2, a_3,... a_{100} .

	8- 9	9-10	10-11	11-12	12-1	1-2	2-3	3-4	4-5	5-6
DAY	Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
1	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
2	a_{20}	a_{19}	a_{18}	a_{17}	a_{16}	a_{15}	a_{14}	a_{13}	a_{12}	a_{11}
3	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{21}	a_{22}
4	a_{38}	a_{37}	a_{36}	a_{35}	a_{34}	a_{33}	a_{32}	a_{31}	a_{40}	a_{39}
5	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{41}	a_{42}	a_{43}	a_{44}
6	a_{56}	a_{55}	a_{54}	a_{53}	a_{52}	a_{51}	a_{60}	a_{59}	a_{58}	a_{57}
7	a_{67}	a_{68}	a_{69}	a_{70}	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}
8	a_{74}	a_{73}	a_{72}	a_{71}	a_{80}	a_{79}	a_{78}	a_{77}	a_{76}	a_{75}
9	a_{89}	a_{90}	a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}
10	a_{92}	a_{91}	a_{100}	a_{99}	a_{98}	a_{97}	a_{96}	a_{95}	a_{94}	a_{93}

Observe how one used the results of the previous example (Example 4b) to solve the above problem, Example 5..

In the next example, one will cover an example involving 1000 items, which will be similar to Example 2a.

Back to examples

Example 6 A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B. Review Example 2a before proceeding

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
1000	999	998	997	996	995	994	993	992	991	990	989	988	987	986	985	984	983	982	981
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
980	979	978	977	976	975	974	973	972	971	970	969	968	967	966	965	964	963	962	961
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
960	959	958	957	956	955	954	953	952	951	950	949	948	947	946	945	944	943	942	941
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
940	939	938	937	936	935	934	933	932	931	930	929	928	927	926	925	924	923	922	921
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
920	919	918	917	916	915	914	913	912	911	910	909	908	907	906	905	904	903	902	901
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
900	899	898	897	896	895	894	893	892	891	890	889	888	887	886	885	884	883	882	881
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
880	879	878	877	876	875	874	873	872	871	870	869	868	867	866	865	864	863	862	861
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
860	859	858	857	856	855	854	853	852	851	850	849	848	847	846	845	844	843	842	841
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
840	839	838	837	836	835	834	833	832	831	830	829	828	827	826	825	824	823	822	821
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
820	819	818	817	816	815	814	813	812	811	810	809	808	807	806	805	804	803	802	801
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
800	799	798	797	796	795	794	793	792	791	790	789	788	787	786	785	784	783	782	781
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
780	779	778	777	776	775	774	773	772	771	770	769	768	767	766	765	764	763	762	761
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
760	759	758	757	756	755	754	753	752	751	750	749	748	747	746	745	744	743	742	741
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
740	739	738	737	736	735	734	733	732	731	730	729	728	727	726	725	724	723	722	721
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
720	719	718	717	716	715	714	713	712	711	710	709	708	707	706	705	704	703	702	701
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
700	699	698	697	696	695	694	693	692	691	690	689	688	687	686	685	684	683	682	681
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
680	679	678	677	676	675	674	673	672	671	670	669	668	667	666	665	664	663	662	661
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
660	659	658	657	656	655	654	653	652	651	650	649	648	647	646	645	644	643	642	641
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
640	639	638	637	636	635	634	633	632	631	630	629	628	627	626	625	624	623	622	621
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
620	619	618	617	616	615	614	613	612	611	610	609	608	607	606	605	604	603	602	601

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
600	599	598	597	596	595	594	593	592	591	590	589	588	587	586	585	584	583	582	581

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
580	579	578	577	576	575	574	573	572	571	570	569	568	567	566	565	564	563	562	561

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
560	559	558	557	556	555	554	553	552	551	550	549	548	547	546	545	544	543	542	541

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
540	539	538	537	536	535	534	533	532	531	530	529	528	527	526	525	524	523	522	521

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
520	519	518	517	516	515	514	513	512	511	510	509	508	507	506	505	504	503	502	501

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
500	499	498	497	496	495	494	493	492	491	490	489	488	487	486	485	484	483	482	481

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
480	479	478	477	476	475	474	473	472	471	470	469	468	467	466	465	464	463	462	461

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
460	459	458	457	456	455	454	453	452	451	450	449	448	447	446	445	444	443	442	441

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
440	439	438	437	436	435	434	433	432	431	430	429	428	427	426	425	424	423	422	421

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
420	419	418	417	416	415	414	413	412	411	410	409	408	407	406	405	404	403	402	401

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
400	399	398	397	396	395	394	393	392	391	390	389	388	387	386	385	384	383	382	381

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
380	379	378	377	376	375	374	373	372	371	370	369	368	367	366	365	364	363	362	361

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
360	359	358	357	356	355	354	353	352	351	350	349	348	347	346	345	344	343	342	341

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
340	339	338	337	336	335	334	333	332	331	330	329	328	327	326	325	324	323	322	321

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
320	319	318	317	316	315	314	313	312	311	310	309	308	307	306	305	304	303	302	301

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
300	299	298	297	296	295	294	293	292	291	290	289	288	287	286	285	284	283	282	281

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
280	279	278	277	276	275	274	273	272	271	270	269	268	267	266	265	264	263	262	261

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
260	259	258	257	256	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
240	239	238	237	236	235	234	233	232	231	230	229	228	227	226	225	224	223	222	221

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
220	219	218	217	216	215	214	213	212	211	210	209	208	207	206	205	204	203	202	201

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
200	199	198	197	196	195	194	193	192	191	190	189	188	187	186	185	184	183	182	181
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
180	179	178	177	176	175	174	173	172	171	170	169	168	167	166	165	164	163	162	161
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
160	159	158	157	156	155	154	153	152	151	150	149	148	147	146	145	144	143	142	141
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
140	139	138	137	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
120	119	118	117	116	115	114	113	112	111	110	109	108	107	106	105	104	103	102	101

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Concrete masses for Pile A

Step 2: Collect and add the Choices (dividends) :

$Q_{A1} = 1000 + 997 + 996 + 993 + 992 + 989 + 988 + 985 + 984 + 981 + 980 + 977 + 976 + 973$
$+ 972 + 969 + 968 + 965 + 964 + 961 + 960 + 957 + 956 + 953 + 952 + 949 + 948 + 945 +$
$944 + 941 + 940 + 937 + 936 + 933 + 932 + 929 + 928 + 925 + 924 + 921 + 920 + 917 +$
$916 + 913 + 912 + 909 + 908 + 905 + 904 + 901$ **= 47,525**

$Q_A = 900 + 897 + 896 + 893 + 892 + 889 + 888 + 885 + 884 + 881 + 880 + 877 + 876 + 873 + 872 +$
$869 + 868 + 865 + 864 + 861 + 860 + 857 + 856 + 853 + 852 + 849 + 848 + 845 + 844 + 841$
$+840 + 837 + 836 + 833 + 832 + 829 + 828 + 825 + 824 + 821 + 820 + 817 + 816 + 813 + 812$
$+809 + 808 + 805 + 804 + 801$ **= 42,525**

$Q_A = 800 + 797 + 796 + 793 + 792 + 789 + 788 + 785 + 784 + 781 + 780 + 777 + 776 + 773 + 772 +$
$769 + 768 + 765 + 764 + 761 + 760 + 757 + 756 + 753 + 752 + 749 + 748 + 745 + 744 + 741$
$+740 + 737 + 736 + 733 + 732 + 729 + 728 + 725 + 724 + 721 + 720 + 717 + 716 + 713 + 712$
$+709 + 708 + 705 + 704 + 701$ **= 37,525**

$Q_A = 700 + 697 + 696 + 693 + 692 + 689 + 688 + 685 + 684 + 681 + 680 + 677 + 676 + 673 + 672 +$
$669 + 668 + 665 + 664 + 661 + 660 + 657 + 656 + 653 + 652 + 649 + 648 + 645 + 644 + 641$
$+640 + 637 + 636 + 633 + 632 + 629 + 628 + 625 + 624 + 621 + 620 + 617 + 616 + 613 + 612$
$+609 + 608 + 605 + 604 + 601 =$ **32,525**

$Q_A = 600 + 597 + 596 + 593 + 592 + 589 + 588 + 585 + 584 + 581 + 580 + 577 + 576 + 573 + 572 +$
$569 + 568 + 565 + 564 + 561 + 560 + 557 + 556 + 553 + 552 + 549 + 548 + 545 + 544 + 541$
$+540 + 537 + 536 + 533 + 532 + 529 + 528 + 525 + 524 + 521 + 520 + 517 + 516 + 513 + 512$
$+509 + 508 + 505 + 504 + 501$ **= 27,525**

$Q_A = 500 + 497 + 496 + 493 + 492 + 489 + 488 + 485 + 484 + 481 + 480 + 477 + 476 + 473 + 472 +$
$469 + 468 + 465 + 464 + 461 + 460 + 457 + 456 + 453 + 452 + 449 + 448 + 445 + 444 + 441$
$+440 + 437 + 436 + 433 + 432 + 429 + 428 + 425 + 424 + 421 + 420 + 417 + 416 + 413 + 412$
$409 + 408 + 405 + 404 + 401$ **= 22,525**

$Q_A = 400 + 397 + 396 + 393 + 392 + 389 + 388 + 385 + 384 + 381 + 380 + 377 + 376 + 373 + 372 +$
$369 + 368 + 365 + 364 + 361 + 360 + 357 + 356 + 353 + 352 + 349 + 348 + 345 + 344 + 341$
$+340 + 337 + 336 + 333 + 332 + 329 + 328 + 325 + 324 + 321 + 320 + 317 + 316 + 313 + 312$
$309 + 308 + 305 + 304 + 301$ **= 17,525**

$Q_A = 300 + 297 + 296 + 293 + 292 + 289 + 288 + 285 + 284 + 281 + 280 + 277 + 276 + 273 + 272 +$
$269 + 268 + 265 + 264 + 261 + 260 + 257 + 256 + 253 + 252 + 249 + 248 + 245 + 244 + 241$
$+240 + 237 + 236 + 233 + 232 + 229 + 228 + 225 + 224 + 221 + 220 + 217 + 216 + 213 + 212$
$209 + 208 + 205 + 204 + 201$ **= 12,525**

$Q_A = 200 + 197 + 196 + 193 + 192 + 189 + 188 + 185 + 184 + 181 + 180 + 177 + 176 + 173 + 172 +$
$169 + 168 + 165 + 164 + 161 + 160 + 157 + 156 + 153 + 152 + 149 + 148 + 145 + 144 + 141$
$+140 + 137 + 136 + 133 + 132 + 129 + 128 + 125 + 124 + 121 + 120 + 117 + 116 + 113 + 112$
$109 + 108 + 105 + 104 + 101$ **= 7,525**

$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 +$
$69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41$
$+40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12$
$9 + 8 + 5 + 4 + 1$ **= 2,525**

Total for Q_A = 250,250 units

Concrete masses for Pile B

$$Q_B = 999+998+995+994+991+990+987+986+983+982+979+978+975+974+971+ 970+967+966+963+962+959+958+955+954+951+950+947+946+943+942+ 939+938+935+934+931+930+927+926+923+922+919+918+915+914+911+ 910+907+906+903+902 = \mathbf{47,525}$$

$$Q_B = 899+898+895+894+891+890+887+886+883+882+879+878+875+874+871+ 870+867+866+863+862+859+858+855+854+851+850+847+846+843+842+ 839+838+835+834+831+830+827+826+823+822+819+818+815+814+811+ 810+807+806+803+802 = \mathbf{42,525}$$

$$Q_B = 799+798+795+794+791+790+787+786+783+782+779+778+775+774+771+ 770+767+766+763+762+759+758+755+754+751+750+747+746+743+742+ 739+738+735+734+731+730+727+726+723+722+719+718+715+714+711+ 710+707+706+703+702 = \mathbf{37,525}$$

$$Q_B = 699+698+695+694+691+690+687+686+683+682+679+678+675+674+671+ 670+667+666+663+662+659+658+655+654+651+650+647+646+643+642+ 639+638+635+634+631+630+627+626+623+622+619+618+615+614+611+ 610+607+606+603+602 = \mathbf{32,525}$$

$$Q_B = 599+598+595+594+591+590+587+586+583+582+579+578+575+574+571+ 570+567+566+563+562+559+558+555+554+551+550+547+546+543+542+ 539+538+535+534+531+530+527+526+523+522+519+518+515+514+511+ 510+507+506+503+502 = \mathbf{27,525}$$

$$Q_B = 499+498+495+494+491+490+487+486+483+482+479+478+475+474+471+ 470+467+466+463+462+459+458+455+454+451+450+447+446+443+442+ 439+438+435+434+431+430+427+426+423+422+419+418+415+414+411+ 410+407+406+403+402 = \mathbf{22,525}$$

$$Q_B = 399+398+395+394+391+390+387+386+383+382+379+378+375+374+371+ 370+367+366+363+362+359+358+355+354+351+350+347+346+343+342+ 339+338+335+334+331+330+327+326+323+322+319+318+315+314+311+ 310+307+306+303+302 = \mathbf{17,525}$$

$$Q_B = 299+298+295+294+291+290+287+286+283+282+279+278+275+274+271+ 270+267+266+263+262+259+258+255+254+251+250+247+246+243+242+ 239+238+235+234+231+230+227+226+223+222+219+218+215+214+211+ 210+207+206+203+202 = \mathbf{12,525}$$

$$Q_B = 199+198+195+194+191+190+187+186+183+182+179+178+175+174+171+ 170+167+166+163+162+159+158+155+154+151+150+147+146+143+142+ 139+138+135+134+131+130+127+126+123+122+119+118+115+114+111+ 110+107+106+103+102 = \mathbf{7,525}$$

$$Q_B = 99+98+95+94+91+90+87+86+83+82+79+78+75+74+71+ 70+67+66+63+62+59+58+55+54+51+50+47+46+43+42+ 39+38+35+34+31+30+27+26+23+22+19+18+15+14+11+ 10+7+6+3+2 = \mathbf{2,525}$$

Total for Q_B = 250,250 units

Overall Conclusion

An extended Ashanti fairness wisdom technique was applied to solve six NP problems. In the first step, one did not think about mathematical models to apply, but rather thought about the process involved, and this led to the development of the fairness wisdom technique which was applied to a set of 100 items of different values or masses. Two people A and B were able to divide the items equally by merely choosing in turns from a set of ordered items. The total value or mass of A's items was found to be equal total value or mass of B's items, and these results are combinations of the items of different values or masses. This problem was followed by similar problems in which in addition to dividing the masses, certain conditions were to be satisfied before definite answers could be given to the problems. It is very pleasing that such a simple technique can produce desired combinations. Even though the solutions are combinations, no knowledge of combinatorial mathematics was involved or required. In fact, if one was asked to name the mathematics involved, the answer would be arithmetic. Thus, high school and middle school graduates could be taught the technique involved. From the solutions, formulas or simple equations were produced to help programmers apply the techniques. Confirmed was the notion that a method that solves one of these problems can also solve other NP problems.
Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order, a task a computer performs very fast. The technique was also applied to 1000 items; and the results were perfect, just like the results for the 100 items.

For 100 items, out of the possible 2^{100} combinations, the technique produced the desired combinations, and similarly, for 1000 items, out of 2^{1000} possible combinations, the technique produced the desired combinations. Therefore, the technique covered does not care whether there are 2^{100} or 2^{1000} possibilities. The desired and correct combinations are always produced. This technique can divide a set of items of different lengths, masses, volumes, value, or sizes into equal parts by combinations only.

There are social consequences of the method and principles used to divide the set of items into equal totals. The results can be applied by government agencies in the distribution of goods and services. Management personnel should be aware of the principles involved in the above technique. From the elementary school, through high school, and perhaps college, students should be taught the principles in the above wisdom technique, since throughout life, one is going to encounter situations in which two or more people are asked to choose in turns, from items of different values or sizes, and in this case, the sequence by which the choices are made matters; one may be either a participant or one may be in charge of the distribution process.

By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses. School secretaries and office assistants can learn and apply the techniques covered.

Finally, if a method can solve one NP problem, that method can also solve other NP problems. Since six NP problems have been solved in this paper, the formerly NP problems are now P problems, and therefore, it is concluded that P is equal to NP.

Perhaps, one may make the following statements.
1. NP plus human ability equals P. **2**. NP plus human inability is **not** equal to P.
3. NP minus human inability equals P.

References:

For paper edition of the above paper, see Appendix 6 of the book entitled "Power of Ratios" by A. A. Frempong, published by Yellowtextbooks.com. After solving the NP problems and reviewing the solutions, the author realized that a ratio process had been applied in solving the NP solutions, and in the beginning, did not consider including the solutions of NP problems in the "Power of Ratios" book which contains also the author's previous solutions of the Navier-Stokes equations plus solutions of the magnetohydrodynamic equations, (viXra.org). The " Power of Ratios" book covers definition of ratio and applications of ratios in mathematics, science, pharmacology, engineering, economics and business fields.

Back to examples