Bell's Inequality Loophole: Precession

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Justifying a local hidden variable theory requires an explanation of Bell's inequality violation. Ever since Bell derived the inequality to test the classical prediction on the correlation of two spin-$\frac{1}{2}$ particles, many experiments have observed the violation, and thus concluded against the local realism, while validating the non-locality of quantum entanglement. Still, many scientists remain unconvinced of quantum entanglement because the experiments have loopholes that could potentially allow a local realistic explanation. Upholding the local realism, this paper introduces how a precession of the spin would produce a cosine-like correlation function, and furthermore how it would also contribute to a fair sampling loophole. Simulating the precession in Monte Carlo method reveals that it can explain the observed Bell's violation using only classical mechanics.

1 Introduction

In my previous paper, I introduced the physical mechanism that explains quantum entanglement via special relativity [1]. However, that was under a premise that quantum entanglement is a real physical phenomenon. In this paper, I take on an opposing premise that the entanglement is not real, but local hidden variable is real instead, and introduce a physical mechanism behind local hidden variable, namely a spin precession.

Monte Carlo simulation will show that the precession itself alone can produce a cosine-like function of the correlation, instead of the usual linear function predicted without the precession. And when the simulation takes into account of the coincidence-related fair sampling loophole, the cosine-like function resembles the theoretical cosine wave predicted by quantum mechanics. Also, the experiment by Rowe, et al. [2], which supposedly closed fair sampling loophole will be discussed.

1.1 Bell's Inequality

Bell's inequality is used to test local hidden variable theory of classical mechanics against quantum entanglement of quantum mechanics. Bell originally derived his inequality based on the system of two spin-$\frac{1}{2}$ particles, and later adapted the more generalized form, Clauser-Horne-Shimony-Holt (CHSH) inequality [3] (this paper refers to both Bell's original inequality and CHSH inequality as Bell's inequality for convenience). The inequality relies on the assumption that if a local hidden variable exists, then the correlation value of the two detectors can be expressed as shown below:

$$P(\hat{a},\hat{b}) = \int \overline{A}(\hat{a},\lambda)\ \overline{B}(\hat{b},\lambda)\ \rho(\lambda)\ \mathrm{d}\lambda \tag{1}$$

where $\overline{A}(\hat{a},\lambda)$ and $\overline{B}(\hat{b},\lambda)$ are the averaged measured spin outcomes by the detector A pointing at an angle a and the detector B at an angle b, respectively; and $\rho(\lambda)$ is a probability distribution of a hidden variable [3]. Equation (1) leads to the following CHSH inequality:

$$-2 \leq P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}') + P(\hat{a}',\hat{b}) + P(\hat{a}',\hat{b}') \leq 2 \tag{2}$$

which defines the limit on the classical local realism [3].

On the other hand, the correlation value predicted by quantum mechanics is:

$$P(\hat{a},\hat{b}) = -cos\theta \tag{3}$$

where θ is the angle between a and b [3]. And unlike the classical limit, the quantum correlation can have the maximum violation of $2\sqrt{2}$ [4].

1.2 Bell's Inequality Loopholes

There are many loopholes in Bell's inequality experiments, but I will mention only two of them that are relevant to this paper: (1) fair sampling and (2) time dependent parameters.

1.2.1 Fair Sampling Loophole

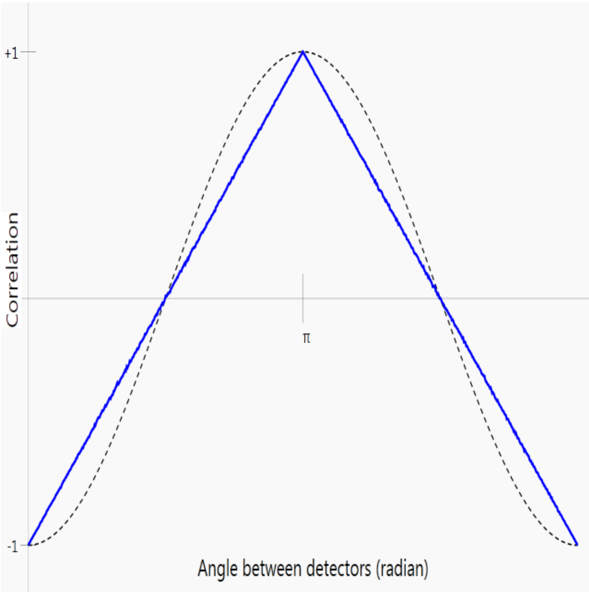
Fair sampling loophole basically means that an experiment does not record all the event data for various reasons and these missing data can lead to a local realism explanation of Bell's inequality violation [5]. Practically all experiments except for a few have this loophole [5], with one of them being carried out by Rowe, et al. [2], which will be discussed later. One of the reasons is coincidence, where both detectors must detect their particles within a close time range in order to record a detection event of a spin pair [5].

1.2.2 Time Dependent Parameters

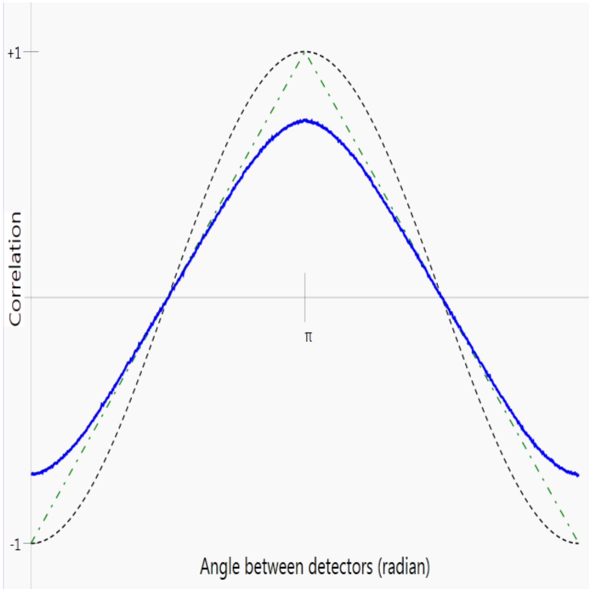
Hess and Philipp described how time dependent parameters in a local hidden variable would invalidate the very basis of all Bell-type inequalities [6]. Although they do not claim to know of any mathematical properties of these parameters nor do they claim that their existence in nature [6], the study by Marshall, et al., on atomic cascade experiments match this description because they attribute wave amplitude of photon as being the reason to refute the Bell's violation [7]. In this paper, I introduce precession as being a physical explanation for the time dependency of local hidden variable for spin particles.

2 Precession

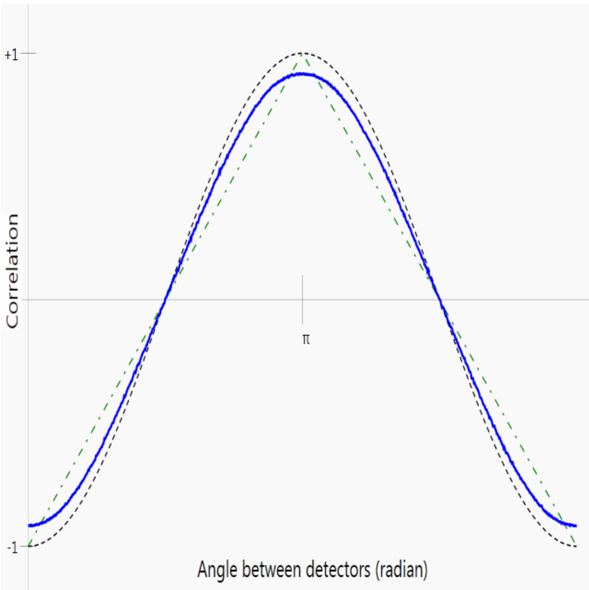
Let us suppose that a spin of each entangled particle undergoes a precession. The precession can be introduced without violating the law of conservation of energy. What impact would the precession have on Bell's inequality? Here are the results of my Monte Carlo simulations of the system of two entangled spin particles undergoing precession.



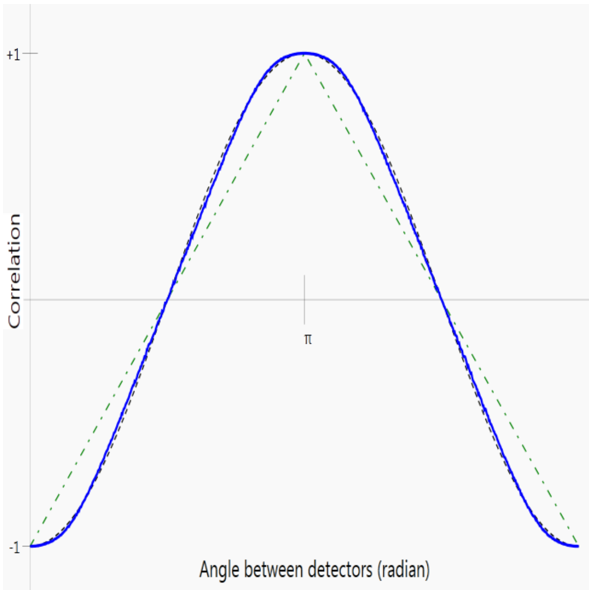
(a) The correlation of the simulated spin precessing at any arbitrary angle (perfectly synced precession phases & no fair sampling loophole)



(b) The correlation of the simulated spin precessing at 20° (not synced & no fair sampling loophole)



(c) The correlation of the simulated spin precessing at 20° (not synced & exists a fair sampling loophole = no-coincidence under 10°)



(d) The correlation of the simulated spin precessing at 10° (not synced & exists a fair sampling loophole = no-coincidence under 10°)

Figure 1: Monte Carlo simulations of anti-parallel spin pair with precession - the solid line/curve is the simulated prediction with precession; the dotted straight lines are the original classical prediction without precession; the dotted cosine is the quantum mechanical prediction. (the angle between detectors was incremented by 0.5° with 100,000 simulation per each step.)

Figure 1a shows the simulated correlation function when both detectors measure their anti-parallel spin particles at the exact same moment, in which case the two precession phases are perfectly synchronized in the opposite direction, as assumed by Bell's inequality. In such case, the linear correlation function originally predicted by the inequality is still accurate, and thus the simulated prediction matches exactly what the original classical prediction looks like, as expected. This validates that the Monte Carlo simulation is working correctly.

However, Figure 1b shows the cosine-like function of the correlation simulated with the spin precession in 20° (arbitrarily chosen) where both detectors do not measure the particles at the exact same moment, e.g. the detector A measures its particle first, and the detector B measures its particle a split second later, as would be the case with virtually all measurements made in a real world experiment. Then, those two detectors would most likely make their measurements when one spin has a randomly different precession phase than the other.

In such case, the simulation shows that the precession would cause the correlation function to deviate from the usual classical prediction of the straight lines. The amount of deviation would depend on the angle of precession from its rotational axis, with an increasing angle shifting the minima and maxima points vertically toward the horizontal axis and making the function look cosine-like with a decreasing amplitude (until the angle reaches 90° at which point the function would look like the flat horizontal axis).

To summarize, there exists a range of the precession angles that would produce a cosine-like function of the correlation.

2.1 Precession Impact on Bell's Inequality

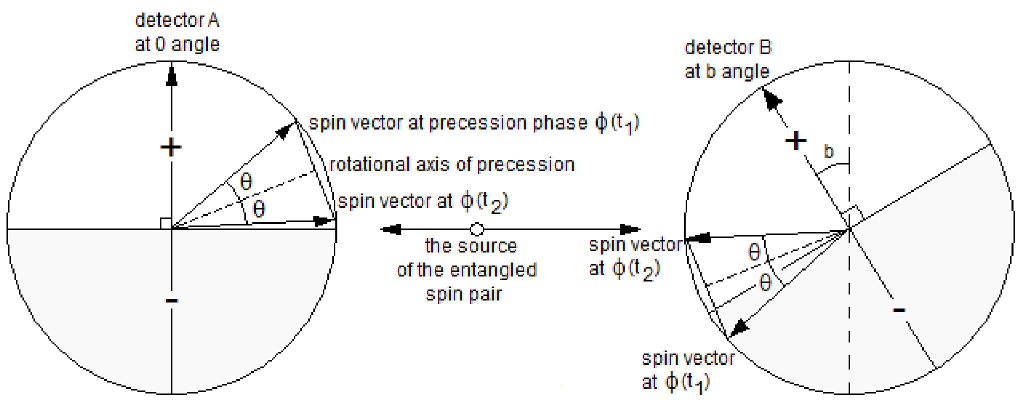


Figure 2: Detection of two anti-parallel spin particles with precession

Let us analyze in details what impact this phase difference of precession has on the correlation, by understanding how precession can change the measured outcomes as shown in Figure 2. The source emits a pair of anti-parallel spin particles. The detector A measures a spin particle first. In this particular case, the detector A would measure the spin outcome of $+1$ no matter what the precession phase is – let us assume that it made the detection when the precession angle was at $\Phi(t_1)$. Next, if the detector B were to measure its particle at the exact same moment, then it would measure the spin outcome of -1 as expected because both spin vectors are perfectly synchronized at $\Phi(t_1)$. However, that is an unrealistic scenario because the detector A and B are not located at the exact same distance away from the source, and thus the second particle has to travel an extra distance to reach the detector B. During this travel, its precession phase would change. So, for example, if the detector B were to measure the spin when its precession phase was at $\Phi(t_2)$, then it would measure a different spin outcome of $+1$, instead of -1.

This time dependent nature of precession makes the assumption behind Equation 1 of Bell's inequality to be no longer accurate. If the local hidden variable, λ, is taken to be a precession, then the equation would become as follows:

$$P(\hat{a}, \hat{b}) = \int \overline{A}(\hat{a}, \lambda(\hat{r}_A, \theta_A, \Phi_A))\ \overline{B}(\hat{b}, \lambda(\hat{r}_B, \theta_B, \Phi_B))\ \rho(\lambda)\ \mathrm{d}\lambda \tag{4}$$

where $\hat{r}$ is a unit vector of the rotational axis of a spin precession; θ is the precession angle from the rotational axis; and Φ is the precession phase.

Although $\hat{r}$ and θ are synchronized between both detectors, Φ_A and Φ_B are likely be different in a real world experiment. This means that experimentally measured outcomes of $\overline{A}$ and $\overline{B}$, technically, do not represent the theoretical outcomes of $\overline{A}$ and $\overline{B}$ coupled together in the equation, because it expects the same local hidden variable to be shared by both $\overline{A}$ and $\overline{B}$.

Nevertheless, despite this inaccuracy, I think the classical limit derived by Bell's inequality remains valid in this case. That is because it calculates with the averaged measured outcomes, which fortunately can compensate for the randomness of many phase differences. As long as $\overline{A}$ and $\overline{B}$ produce their averaged outcomes in a deterministic manner to $\hat{r}$ and θ, the validity of the inequality would hold.

But, the correlation value, $P(\hat{a}, \hat{b})$, would now get reduced and have a smaller absolute value due to the introduction of precession. The reason is because an original detection outcomes of $(+1, -1)$ of an anti-parallel pair can now be divided into four sub-outcomes: $(+1, -1)$, $(+1, +1)$, $(-1, -1)$, $(-1, +1)$, which clearly reduces the number of anti-correlated outcomes and adds to the number of correlated outcomes. The result of Monte Carlo simulation in Figure 1b validates this prediction.

2.2 Precession and Fair Sampling Loophole

Although it is a progress to be able to explain the cosine-shaped correlation observed by the experiments, it is not yet enough because a precession-based correlation still falls under the limit of Bell's inequality. Many experiments have observed the violation of Bell's inequality with a larger wave amplitude, as seen by the cosine wave in Figure 1b.

However, this amplitude issue can be solved by fair sampling loophole [5], more specifically by how a coincidence is detected.

Let us analyze how a precession would contribute to this coincidence loophole by looking at Figure 2. Each detector has two hemispheres where one hemisphere would measure a spin as $+1$ outcome and the other as -1 outcome. These two hemispheres are divided by a plane orthogonal to the measuring direction of the detector. Now, if the spin happens to lie on this orthogonal plane or if it is very close to the plane, then the detector would not be able to determine if the spin is $+1$ or -1 because the spin vector component onto the direction of the detector would be too small to be distinguished from background noises. If so, either both detectors would not be able to detect any of the spin pair, or even if the other detector makes a detection, the coincidence monitor would discard the measurement out from the calculation. And since a precession makes it more likely for a spin at one of the detectors to get closer to the orthogonal plane than if there were no precession, it would increase the count of no-coincidence.

This kind of filtering in the sample data creates a bias in the calculated correlation; it favors the detection of those spins that point more in parallel direction to the detector, which in turn, alters the calculated correlation value toward $+1$ and -1, while disregarding those spins close to the orthogonal plane that would have contributed toward the value of 0. In other words, this bias from no-coincidence would result in the correlation function with the larger wave amplitude than what it should have been normally.

Figure 1c and Figure 1d show the results of Monte Carlo simulation that validate this prediction. The increase in the wave amplitude depends on how far away from the orthogonal plane a spin can be in order for a detection to occur - the further away, the bigger the amplitude (with the limit at ± 1).

For example, Figure 1c shows the correlation function when the precession angle is $20°$ and the detection threshold angle of no-coincidence is $10°$ from the orthogonal plane. This closely matches the experimentally observed correlation function of the proton-proton scattering within the reported uncertainty [8].

Furthermore, if the precession angle is $10°$, then the simulated correlation function matches almost exactly the cosine wave of quantum prediction as shown in Figure 1d.

2.3 Precession and CHSH Violation

Monte Carlo simulation was carried out to calculate the value of the CHSH inequality in Equation (2). Using the precession angle of $20°$ and the detection threshold angle of $10°$, the violation of CHSH inequality in the amount of -2.206 ± 0.001 occurs for a set of detection angles of $0°$, $45°$, $22.5°$, and $67.5°$, as shown in Table 1.

Even a higher violation can be achieved with a different set of detection angles of, e.g. $-22.5°$, $67.5°$, $22.5°$, and $112.5°$ resulting in the CHSH violation of -2.530 ± 0.002. Changing the precession angle or the detection threshold angle can result in a higher violation, too.

Detector settings				
detection angle (degrees)	$\hat{a}$	$\hat{a}'$	$\hat{b}$	$\hat{b}'$
	0°	45°	22.5°	67.5°

CHSH calculation						
	Run #	$P(\hat{a},\hat{b})$	$P(\hat{a},\hat{b}')$	$P(\hat{a}',\hat{b})$	$P(\hat{a}',\hat{b}')$	CHSH value
	1	-0.846	-0.332	-0.846	-0.846	-2.206
	2	-0.846	-0.333	-0.846	-0.847	-2.206
	3	-0.846	-0.332	-0.845	-0.846	-2.205
	4	-0.846	-0.333	-0.846	-0.847	-2.206
correlation	5	-0.847	-0.334	-0.846	-0.846	-2.205
	6	-0.847	-0.334	-0.847	-0.846	-2.206
	7	-0.847	-0.332	-0.848	-0.846	-2.209
	8	-0.847	-0.331	-0.845	-0.848	-2.209
	9	-0.845	-0.331	-0.846	-0.846	-2.206
	10	-0.847	-0.333	-0.847	-0.845	-2.206
$\text{CHSH} = P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}') + P(\hat{a}',\hat{b}) + P(\hat{a}',\hat{b}') = -2.206 \pm 0.001$						

- precession angle: 20°
- detection threshold angle: 10°
- simulation count per correlation: 1,000,000

Table 1: Monte Carlo simulation result of CHSH inequality violation by anti-parallel spin pair with precession and no-coincidence loophole

To summarize, the experimentally observed violations of Bell's inequality can be explained by taking into account of a spin precession and the fair sampling loophole.

3 Methods

Monte Carlo simulation was used to calculate the correlations shown in Figure 1 and to calculate the CHSH violation in Table 1. The computer program was written to simulate a pair of anti-parallel spin particles with precession, measured by two detectors at their detection angles.

For each simulated pair, a set of four random rotation transformations were generated to simulate a precessing spin vector pointing at a random direction for the first detector A. Then, those same rotation transformations were re-used for the second detector B to generate an anti-parallel spin vector, except for the rotational transformation of precession phase, which was re-randomized to simulate a random phase difference. The two rotation transformations were used to control a polar angle (precession angle) and an azimuthal angle (precession phase) of a precession. The two additional rotation transformations were used to control a polar angle and an azimuthal angle of a random spin orientation. The care was taken to generate truly random polar angles in order to represent an evenly distributed points on a spherical surface [9].

Each detector measures an outcome of either $+1$ or -1 depending on the final spin orientation onto the direction of the detection as shown in Figure 2. These two outcomes were compared against each other to determine a single correlation of $+1$ or -1 for this one pair. Many more new spin pairs were generated and measured, in order to calculate the final correlation measured by the simulation, from which the average value of all single correlations was calculated as shown in Equation (5).

$$P_{measured}(\hat{a}, \hat{b}) = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{++} + N_{--} + N_{+-} + N_{-+}} \tag{5}$$

where N_{++}, N_{--}, N_{+-}, N_{-+} are the total number of outcomes for $(+1, +1)$, $(-1, -1)$, $(+1, -1)$, $(-1, +1)$, respectively [8].

For the simulation run with the no-coincidence loophole, an outcome of 0 is measured if a spin vector falls below the specified threshold angle from the orthogonal plane to the detection direction; then, both outcomes from this no-coincidence pair are disregarded from the calculation.

It was possible to produce a violation of Bell's inequality by varying the precession angle and no-coincidence threshold angle, in the simulation utilizing only classical mechanics. Although the violation can occur with a different variation, it was not possible to produce a violation when a precession angle was taken to be random for each spin pair. For the case of no precession, although no-coincidence alone could violate the inequality with even a bigger value, its correlation function reverted back to being linear with the straight lines overlaying on the cosine wave, with a short plateau at the minima and maxima points.

4 Conclusions

The local hidden variable of a spin precession, in conjunction with fair sampling loophole, can produce a violation of Bell's inequality. The Monte Carlo simulations reveal that it can generate a cosine-like function of the correlation that closely matches the cosine function predicted by quantum mechanics.

In addition, a CHSH inequality violation of -2.530 ± 0.002 could be achieved from Bell's angles of $-22.5°$, $67.5°$, $22.5°$, and $112.5°$ when simulated with the precession angle of $20°$ and the no-coincidence threshold angle of $10°$. The different sets of Bell's angles, precession angle, and detection threshold angle could produce a different CHSH violation, too.

5 Discussion

Rowe, et al., carried out an experiment with two entangled $^9Be^+$ ions in a trap; this experiment was supposed to have closed fair sampling loophole [2]. Although its highly efficient detection closed the detection aspect of fair sampling loophole [5], I believe it may have opened a new kind of fair sampling loophole.

The experimental setup used by Rowe, et al., is based on the experiment carried out by Sackett, et al [2]. Sackett, et al., describes how Raman transition is used to entangle two $^9Be^+$ ions in $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ states with the transitional states of $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ [10].

My main concern is that $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are transitional states that are kept small in the population in order to achieve the entangled states, $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ [10]. So, the system has a predisposition for $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ states over $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ states. Thus, the detector would detect more $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ states than it would in a normal, e.g. proton-proton scattering experiment, which consequently, would raise the correlation value higher than the actual.

For example, let us ask a question: would $P_{measured}(0, 180°)$ populate the system with all anti-correlated states, $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$? If the system were to follow the predicted behavior of Bell's inequality, then it should. But, the transitional states would not exist alone without the entangled states; otherwise, it indicates that the system being tested was not entangled to begin with.

Similarly, Table 1 in the paper of Sackett, et al., shows that the starting condition of the experiment has non-zero number of ions in the transitional states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ [10]. This, again, does not follow the predicted behavior of Bell's inequality.

In addition, Sackett, et al., mentions that it is difficult to quantify the amount of entanglement present in the system [10]. They estimate that the value of "entanglement of formation" is roughly 0.5 [10], which implies that the half of ions in the system are not entangled. However, I reckon those non-entangled ions would also get detected along with the entangled ions and thus affect the overall data.

These factors may be the reason why the correlation values reported in Table 2 in the paper of Rowe, et al., do not look like those usual values expected from Bell's inequality. Normally, $P(\hat{a}, \hat{a}) = \pm 1$. But, Rowe, et al., reports $P(-\frac{\pi}{8}, -\frac{\pi}{8})$ and $P(\frac{3\pi}{8}, \frac{3\pi}{8}) \approx \pm 0.5$ [2]; Bell's inequality permits this to happen if the detector fails to register some detections [3], but that cannot be true because the detection efficiency in this experiment was high. Thus, it could be that the non-zero $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ states, in conjunction with non-entangled ions, may have lowered this correlation value.

Also, normally, $P(0, \pm\frac{\pi}{2}) = 0$. But, Rowe, et al., reports $P(-\frac{\pi}{8}, \frac{3\pi}{8})$ and $P(\frac{3\pi}{8}, -\frac{\pi}{8}) \approx 0.5$ [2]. It could be that the predisposition for $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ may have raised this correlation value.

Therefore, although the transitional system used in the experiment produces the entangled states, and can be used to close the detection efficiency loophole, the such system with a predisposition for particular states may not be a good candidate for testing Bell's inequality.

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