# Schrödinger's cat paradox resolution using GRW collapse model.

J.Foukzon[1], A.A.Potapov[2], S.A. Podosenov[3]

1. Israel Institute of Technology.
2. IRE RAS,
3. All-Russian Scientific-Research Institute

**Abstract**: Possible solution of the Schrödinger's cat paradox is considered.

## I. Introduction

As Weinberg recently reminded us [1], the measurement problem remains a fundamental conundrum. During measurement the state vector of the microscopic system collapses in a probabilistic way to one of a number of classical states, in a way that is unexplained, and cannot be described by the time-dependent Schrödinger equation [1].To review the essentials, it is sufficient to consider two-state systems. Suppose a nucleus $\mathbf{n}$, whose Hilbert space is spanned by orthonormal states $|s_i(t)\rangle$, $i = 1,2,$ where $|s_1(t)\rangle = \left|\text{undecayed nucleus at instant } t\right\rangle$ and $|s_2(t)\rangle = \left|\text{decayed nucleus at instant } t\right\rangle$ is in the superposition state,

$$|\Psi_t\rangle_{\mathbf{n}} = c_1|s_1(t)\rangle + c_2|s_2(t)\rangle, |c_1|^2 + |c_2|^2 = 1. \tag{1.1}$$

A measurement apparatus $A$, which may be micro- or macroscopic, is designed to distinguish between states $|s_i(t)\rangle$ by transitioning at each instant $t$ into state $|a_i(t)\rangle$ if it finds $\mathbf{n}$ is in $|s_i(t)\rangle$, $i = 1,2.$ Assume the detector is reliable, implying the $|a_1(t)\rangle$ and $|a_2(t)\rangle$ are orthonormal at each instant $t$-i.e., $\langle a_1(t)||a_2(t)\rangle = 0$ and that the measurement interaction does not disturb states $|s_i\rangle$ -i.e., the measurement is "ideal". When $A$ measures $|\Psi_t\rangle_{\mathbf{n}}$, the Schrödinger equation's unitary time evolution then leads to the "measurement state" $|\Psi_t\rangle_{\mathbf{n}A}$ :

$$|\Psi_t\rangle_{\mathbf{n}A} = c_1|a_1(t)\rangle + c_2|a_2(t)\rangle, |c_1|^2 + |c_2|^2 = 1. \tag{1.2}$$

of the composite system $\mathbf{n}A$ following the measurement.

Standard formalism of continuous quantum measurements [2],[3],[4],[5] leads to a definite but unpredictable measurement outcome, either $|a_1(t)\rangle$ or $|a_2(t)\rangle$ and that $|\Psi_t\rangle_{\mathbf{n}}$ suddenly "collapses" at instant $t'$ into the corresponding state $|s_i(t')\rangle$. But unfortunately equation (1.2) does not appear to resemble such a collapsed state at instant $t'$?.

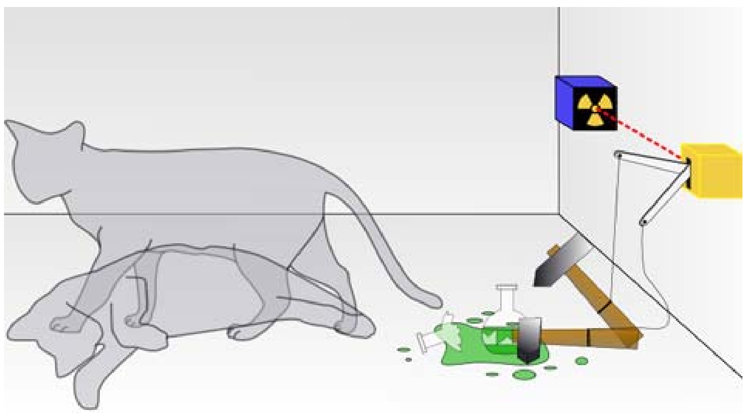
The measurement problem is as follows:

(**I**) How do we reconcile canonical collapse models postulate's

(**II**) How do we reconcile the measurement postulate's definite outcomes with the "measurement state" $|\Psi_t\rangle_{\mathbf{n}A}$ at each instant $t$ and

(**III**) how does the outcome become irreversibly recorded in light of the Schrödinger equation's unitary and, hence, reversible evolution?

This paper deals with only the special case of the measurement problem, known as Schrödinger's Cat paradox. For a good and complete explanation of this paradox see Leggett [6] and Hobson [7].



Pic.1.Schrödinger's cat.

**Schrödinger's cat**: a cat, a flask of poison, and a radioactive source are placed in a sealed box. If an internal monitor detects radioactivity (i.e. a single atom decaying), the flask is shattered, releasing the poison that kills the cat. The Copenhagen interpretation of quantum mechanics implies that after a while, the cat is simultaneously alive and dead. Yet, when one looks in the box, one sees the cat either alive or dead, not both alive and dead. This poses the question of when exactly quantum superposition ends and reality collapses into one possibility or the other.

## II. Canonical collapse models.

In order to appreciate how canonical collapse models work, and what they are able to achieve, we briefly review the GRW model. Let us consider a system of $n$ particles which, only for the sake of simplicity, we take to be scalar and spinless; the GRW model is defined by the following postulates: (1) The state of the system is represented by a wave function $\psi_t(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$ belonging to the Hilbert space $\mathcal{L}_2(\mathbb{R}^{3n})$. (2) At random times, the wave function experiences a sudden jump of the form:

$$\psi_t(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \rightarrow \frac{\Re_m(\mathbf{x})\psi_t(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)}{\|\Re_n(\mathbf{x})\psi_t(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)\|_2}, \tag{1.1}$$

where $\psi_t(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$ is the state vector of the whole system at time $t,$ immediately prior to the jump process and $\Re_n(\mathbf{x})$ is a linear operator which is conventionally chosen equal to:

$$\Re_m(\mathbf{x}) = (\pi r_c^2)^{-3/4} \exp\left[-\frac{(\widehat{\mathbf{q}}_m - \mathbf{x})^2}{2r_c^2}\right], \tag{1.2}$$

where $r_c$ is a new parameter of the model which sets the width of the localization process, and $\widehat{\mathbf{q}}_m$ is the position operator associated to the $m$-th particle of the system and the random variable $\mathbf{x}$ corresponds to the place where the jump occurs.(3) It is assumed that the jumps are distributed in time like a Poissonian process with frequency $\lambda = \lambda_{GRW}$ this is the second new parameter of the model. (4) Between two consecutive jumps, the state vector evolves according to the standard Schrödinger equation.

The 1-particle master equation of the GRW model takes the form

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}\left[\widehat{\mathbf{H}}, \rho(t)\right] - T[\rho(t)]. \tag{1.3}$$

Here $\widehat{\mathbf{H}}$ is the standard quantum Hamiltonian of the particle, and $T[\cdot]$ represents the effect of the spontaneous collapses on the particle's wave function. In the position representation, this operator becomes:

$$\langle \mathbf{x}|T[\rho(t)]|\mathbf{y}\rangle = \lambda\left\{1 - \exp\left[-\frac{(\mathbf{x}-\mathbf{y})^2}{4r_c^2}\right]\right\}\langle \mathbf{x}|\rho(t)|\mathbf{y}\rangle. \tag{1.4}$$

Another modern approach to stochastic reduction is to describe it using a stochastic nonlinear Schrödinger equation, an elegant simplied example of which is the following one particle case known as Quantum Mechanics with Universal Position Localization [QMUPL]:

$$d|\psi_t(x)\rangle = \left[-\frac{i}{\hbar}\widehat{\mathbf{H}} - k(\widehat{q} - \langle q_t\rangle)^2 dt\right]|\psi_t(x)\rangle dt + \sqrt{2k}\,(\widehat{q} - \langle q_t\rangle)dW_t|\psi_t(x)\rangle. \qquad (1.5)$$

Here $\widehat{q}$ is the position operator, $\langle q_t\rangle = \langle\psi_t|\widehat{q}|\psi_t\rangle$ it is its expectation value, and $\lambda$ is a constant, characteristic of the model, which sets the strength of the collapse mechanics, and it is chosen proportional to the mass $m$ of the particle according to the formula: $\lambda = (m/m_0)\lambda_0$, where $m_0$ is the nucleon's mass and $\lambda_0$ measures the collapse strength. It is easy to see that Eqn.(1.5) contains both non-linear and stochastic terms, which are necessary to induce the collapse of the wave function. For an examle let us consider a free particle ($\widehat{\mathbf{H}} = p^2/2m$), and a Gaussian state:

$$\psi_t(x) = \exp\left\{-a_t(x - \bar{x}_t)^2 + i\bar{k}_t x\right\}. \qquad (1.6)$$

It is easy to see that $\psi_t(x)$ given by Eqn.(1.6) is solution of Eqn.(1.5), where

$$\frac{da_t}{dt} = \lambda - \frac{2i\hbar}{m}a_t^2, \frac{d\bar{x}_t}{dt} = \frac{\hbar}{m}\bar{k}_t + \frac{\sqrt{\lambda}}{2\,\mathrm{Re}(a_t)}\dot{W}_t, \frac{d\bar{k}_t}{dt} = -\sqrt{\lambda}\,\frac{\mathrm{Im}(a_t)}{\mathrm{Re}(a_t)}\dot{W}_t. \qquad (1.7)$$

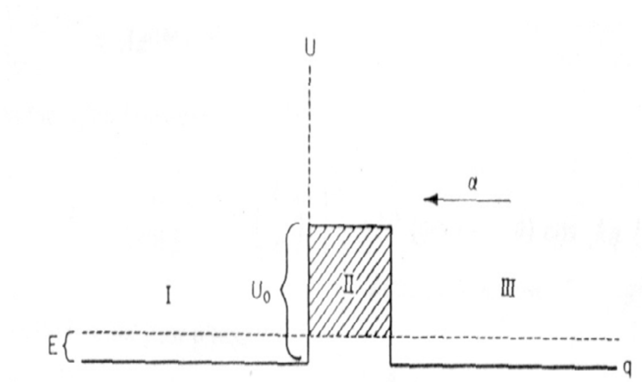
The CSL model is defined by the following stochastic differential equation in the Fock space:

$$d|\psi_t(\mathbf{x})\rangle = \left[-\frac{i}{\hbar}\widehat{\mathbf{H}} - k\left(\widehat{M}(\mathbf{x}) - \langle M_t(\mathbf{x})\rangle\right)^2 dt\right]|\psi_t(\mathbf{x})\rangle dt + \\ + \sqrt{2k}\left(\widehat{M}(\mathbf{x}) - \langle M_t(\mathbf{x})\rangle\right)dW_t(\mathbf{x})|\psi_t(\mathbf{x})\rangle. \qquad (1.8)$$

## II.Generalization Gamow theory of the alpha decay via tunneling using GRW collapse model.

By 1928, George Gamow had solved the theory of the alpha decay via tunneling [7]. The alpha particle is trapped in a potential well by the nucleus. Classically, it is forbidden to escape, but according to the (then) newly discovered principles of quantum mechanics, it has a tiny (but non-zero) probability of "tunneling" through the barrier and appearing on the other side to escape the nucleus. Gamow solved a model potential for the nucleus and derived, from first principles, a relationship between the half-life of the decay, and the energy of the emission.

The $\alpha$-particle has total energy $E$ and is incident on the barrier from the right to left.




Pic. 2.1.The particle has total energy $E$ and
is incident on the barrier $V(x)$ from right to left.

The Schrodinger equation in each of regions $\mathbf{I} = \{x|x < 0\}, \mathbf{II} = \{x|0 \le x \le l\}$ and $\mathbf{III} = \{x|x > l\}$ takes the folloving form

$$\frac{\partial^2 \Psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}[E - U(x)]\Psi(x) = 0, \qquad (2.1)$$

where

$$U(x) = \begin{cases} 0 \text{ for } x < 0 \\ U_0 \text{ for } 0 \le x \le l \\ 0 \text{ for } x > l \end{cases} \qquad (2.2)$$

The solutions reads [8]:

$$\Psi_{\mathbf{III}}(x) = C_+ \exp(ikx) + C_- \exp(-ikx),$$

$$\Psi_{\mathbf{II}}(x) = B_+ \exp(k'x) + B_- \exp(-k'x), \tag{2.3}$$

$$\Psi_{\mathbf{I}}(x) = A\cos(kx) = \frac{A}{2}[\exp(ikx) + \exp(-ikx)],$$

where

$$k = \frac{2\pi}{\hbar}\sqrt{2mE},$$

$$k' = \frac{2\pi}{\hbar}\sqrt{2m(U_0 - E)}. \tag{2.4}$$

At the boundary $x = 0$ we have the following boundary conditions:

$$\Psi_{\mathbf{I}}(0) = \Psi_{\mathbf{II}}(0), \left.\frac{\partial\Psi_{\mathbf{I}}(x)}{\partial x}\right|_{x=0} = \left.\frac{\partial\Psi_{\mathbf{II}}(x)}{\partial x}\right|_{x=0}. \tag{2.5}$$

At the boundary $x = l$ we have the following boundary conditions

$$\Psi_{\mathbf{I}}(l) = \Psi_{\mathbf{II}}(l), \left.\frac{\partial\Psi_{\mathbf{I}}(x)}{\partial x}\right|_{x=l} = \left.\frac{\partial\Psi_{\mathbf{II}}(x)}{\partial x}\right|_{x=l}. \tag{2.6}$$

From the boundary conditions (2.5)-(2.6) one obtain [8]:

$$B_+ = \frac{A}{2}\left(1 + i\frac{k}{k'}\right), B_- = \frac{A}{2}\left(1 - i\frac{k}{k'}\right),$$

$$C_+ = A[ch(k'l) + iDsh(k'l)], C_- = i(ASsh(k'l)\exp(ikl)), \tag{2.7}$$

$$D = \frac{1}{2}\left(\frac{k}{k'} - \frac{k'}{k}\right), S = \frac{1}{2}\left(\frac{k}{k'} + \frac{k'}{k}\right).$$

From (2.5) one obtain the conservation law

$$|A|^2 = |C_+|^2 - |C_-|^2. \tag{2.7}$$

Let us introduce now a function $E(x,l)$ :

$$E(x,l) = \begin{cases} (\pi r_c^2)^{-1/4} \exp\left(\frac{x^2}{2r_c^2}\right) \text{ for } -\infty < x < \frac{l}{2} \\ (\pi r_c^2)^{-1/4} \exp\left(\frac{(x-l)^2}{2r_c^2}\right) \text{ for } \frac{l}{2} \leq x < \infty \end{cases} \tag{2.8}$$

**Assumption 2.1**. We assume now that:
(i) at instant $t = 0$ the wave function $\Psi_{\mathbf{I}}(x)$ experiences a sudden jump of the form

$$\Psi_{\mathbf{I}}(x) \to \Psi_{\mathbf{I}}^{\#}(x) = \frac{\Re_{\mathbf{I}}(x)\Psi_{\mathbf{I}}(x)}{\|\Re_{\mathbf{I}}(x)\Psi_{\mathbf{I}}(x)\|_2}, \tag{2.9}$$

where $\Re_{\mathbf{I}}(x)$ is a linear operator which is chosen equal to:

$$\Re_{\mathbf{I}}(\mathbf{x}) = (\pi r_c^2)^{-1/4} \exp\left[-\frac{\widehat{\mathbf{q}}^2}{2r_c^2}\right]; \tag{2.10}$$

(ii) at instant $t = 0$ the wave function $\Psi_{\mathbf{II}}(x)$ experiences a sudden jump of the form

$$\Psi_{\mathbf{II}}(x) \to \Psi_{\mathbf{II}}^{\#}(x) = \frac{\Re_{\mathbf{II}}(x)\Psi_{\mathbf{II}}(x)}{\|\Re_{\mathbf{II}}(x)\Psi_{\mathbf{II}}(x)\|_2}, \tag{2.11}$$

where $\Re_{\mathbf{II}}(x)$ is a linear operator which is chosen equal to:

$$\Re_{\mathbf{II}}(\mathbf{x}) = E(x,l); \tag{2.12}$$

(iii) at instant $t = 0$ the wave function $\Psi_{\mathbf{III}}(x)$ experiences a sudden jump of the form

$$\Psi_{\mathbf{III}}(x) \to \Psi^{\#}_{\mathbf{III}}(x) = \frac{\Re_{\mathbf{III}}(x)\Psi_{\mathbf{III}}(x)}{\|\Re_{\mathbf{III}}(x)\Psi_{\mathbf{III}}(x)\|_2}, \tag{2.13}$$

where $\Re_{\mathbf{III}}(x)$ is a linear operator which is chosen equal to:

$$\Re_{\mathbf{III}}(\mathbf{x}) = (\pi r_c^2)^{-1/4}\exp\left[-\frac{(\widehat{\mathbf{q}}-l)^2}{2r_c^2}\right]. \tag{2.14}$$

**III.Resolution of the Schrödinger's Cat paradox.**

Let $|s_1(t)\rangle$ and $|s_2(t)\rangle$ be

$$\begin{aligned} |s_1(t)\rangle &= \left|\text{undecayed nucleus at instant } t\right\rangle, \\ |s_2(t)\rangle &= \left|\text{decayed nucleus at instant } t\right\rangle. \end{aligned} \tag{3.1}$$

In a good approximation we assume now that

$$|s_1(0)\rangle = \Psi^{\#}_{\mathbf{II}}(x) \tag{3.2}$$

and

$$|s_2(0)\rangle = \Psi^{\#}_{\mathbf{I}}(x). \tag{3.3}$$

**Remark 3.1**. Note that: (i) $|s_2(0)\rangle = \left|\text{decayed nucleus at instant } 0\right\rangle =$ $= \left|\text{free } \alpha\text{-particle at instant } 0\right\rangle$. (ii) Feynman propagator of a free $\alpha$-particle are [9]:

$$K_2(x,t,x_0) = \left(\frac{m}{2\pi i\hbar t}\right)^{1/2}\exp\left\{\frac{i}{\hbar}\left[\frac{m(x-x_0)^2}{2t}\right]\right\}. \tag{3.4}$$

Therefore from Eq.(3.3),Eq.(2.9) and Eq.(3.4) we obtain

$$|s_2(t)\rangle = \Psi_{\mathbf{I}}^{\#}(x,t) = \int_{-\infty}^{0} \Psi_{\mathbf{I}}^{\#}(x_0) K_2(x,t,x_0) dx_0 =$$

$$(\pi r_c^2)^{-1/4} \times \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \times \int_{-\infty}^{0} \exp\left(-\frac{x_0^2}{2r_c^2}\right) \exp\left(-i\frac{2\pi}{\hbar}\sqrt{2mE}\,x_0\right) \times$$

$$\times \exp\left\{\frac{i}{\hbar}\left[\frac{m(x-x_0)^2}{2t}\right]\right\} dx_0 =$$

$$(\pi r_c^2)^{-1/4} \times \left(\frac{m}{2\pi i\hbar_\varepsilon t}\right)^{1/2} \times \int_{-\infty}^{0} \exp\left(-\frac{x_0^2}{2r_c^2}\right) \times \tag{3.5}$$

$$\times \exp\left\{\frac{i}{\hbar}\left[\frac{m(x-x_0)^2}{2t} - \pi\sqrt{4mE}\,x_0\right]\right\} dx_0 =$$

$$(\pi r_c^2)^{-1/4} \times \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \times \int_{-\infty}^{0} \exp\left(-\frac{x_0^2}{2r_c^2}\right) \times \exp\left\{\frac{i}{\hbar}[S(t,x,x_0)]\right\} dx_0,$$

where

$$S(t,x,x_0) = \frac{m(x-x_0)^2}{2t} - \pi\sqrt{8mE}\,x_0. \tag{3.6}$$

We assume now that

$$\hbar \ll 2r_c^2 \ll 1. \tag{3.7}$$

Oscillatory integral in RHS of Eq.(3.5) is calculated now directly using stationary phase approximation. The phase term $S(x,x_0)$ given by Eq.(3.6) is stationary when

$$\frac{\partial S(t,x,x_0)}{\partial x_0} = -\frac{m(x-x_0)}{t} - \pi\sqrt{8mE} = 0. \tag{3.8}$$

Therefore

$$-\frac{m(x-x_0)}{t} - \pi\sqrt{8mE} = 0,$$
$$-(x-x_0) = \pi t\sqrt{8E/m}, \tag{3.9}$$

and thus stationary point $x_0(t,x)$ are

$$x_0(t,x) = \pi t\sqrt{8E/m} + x. \tag{3.10}$$

Thus from Eq.(3.5) and Eq.(3.10) using stationary phase approximation we obtain

$$|s_2(t)\rangle =$$
$$(\pi r_c^2)^{-1/4} \times \exp\left[-\frac{x_0^2(t,x)}{2r_c^2}\right] \times \exp\left\{\frac{i}{\hbar}[S(t,x,x_0(t,x))]\right\} + O(\hbar), \tag{3.11}$$

where

$$S(x,x_0(t,x)) = \frac{m(x-x_0(t,x))^2}{2t} - \pi\sqrt{8mE}\,x_0(t,x). \tag{3.12}$$

From Eq.(3.11) we obtain

$$\langle s_2(t)||s_2(t)\rangle \simeq (\pi r_c^2)^{-1/2} \times \exp\left[-\frac{\left(\pi t\sqrt{8E/m} + x\right)^2}{r_c^2}\right]. \tag{3.13}$$

**Remark 3.2**. From the inequality (3.7) follows that $\alpha$-particle at each instant $t \geq 0$ moves quasiclassically from right to left by the law

$$x(t) = -\pi t\sqrt{8E/m}, \tag{3.14}$$

i.e. estimating the position $\{x_i(t,\mathbf{x}_0,t_0;\hbar)\}_{i=1}^{d}$ at each instant $t \geq 0$ with final error $r_c$ gives $|\langle x_i\rangle(t,0,0;\hbar) - x_i(t,\mathbf{x}_0,t_0)| \leq r_c, i = 1,\ldots,d$ with a probability

$$\mathbf{P}\{|\langle x\rangle(t,0,0;\hbar) - x(t)| \leq r_c\} \simeq 1.$$

**Remark 3.3**. We assume now that a distance between radioactive source and internal monitor which detects a single atom decaying (see Pic.1) is equal to $L$.

**Proposition 3.1**. After $\alpha$-decay the collaps: $\left|\text{live cat}\right\rangle \to \left|\text{death cat}\right\rangle$ arises at instant

$$T = \frac{L}{\pi\sqrt{8E/m}}. \tag{3.15}$$

Suppose now that a nucleus $\mathbf{n}$, whose Hilbert space is spanned by orthonormal states $|s_i(t)\rangle,\ \ i = 1,2,$where $|s_1(t)\rangle = \left|\text{undecayed nucleus at instant } t\right\rangle$ and $|s_2(t)\rangle = \left|\text{decayed nucleus at instant } t\right\rangle$ is in the superposition state,

$$|\Psi_t\rangle_{\mathbf{n}} = c_1|s_1(t)\rangle + c_2|s_2(t)\rangle, |c_1|^2 + |c_2|^2 = 1. \tag{3.16}$$

**Remark 3.4**. Note that: (i) $|s_1(0)\rangle = \left|\text{undecayed nucleus at instant } 0\right\rangle =$ $= \left|\ \alpha\text{-particle iside region } (0,l] \text{ at instant } 0\right\rangle$. (ii) Feynman propagator of $\alpha$-particle iside region $(0,l]$ are [9]:

$$K_2(x,t,x_0) = \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \exp\left\{\frac{i}{\hbar}[S(t,x,x_0)]\right\}, \tag{3.17}$$

where

$$S(t,x,x_0) = \frac{m(x-x_0)^2}{2t} + mt(U_0 - E)(x + x_0) - mt^3(U_0 - E)/12. \tag{3.18}$$

Therefore from Eq.(2.11)-Eq.(2.12) and Eq.(3.17) we obtain

$$|s_1(t)\rangle = \Psi^{\#}_{\mathbf{II}}(x,t) = \int\limits_0^l \Psi^{\#}_{\mathbf{II}}(x_0) K_2(x,t,x_0)dx_0 =$$

$$\left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \int\limits_0^l E(x_0,l)\Psi_{\mathbf{II}}(x_0)\theta_l(x_0)\exp\left\{\frac{i}{\hbar}[S(t,x,x_0)]\right\}dx_0, \tag{3.19}$$

where

$$\theta_l(x) = \begin{cases} 0 \text{ for } x \in [0,l] \\ 0 \text{ for } x \notin [0,l] \end{cases}$$

**Remark 3.4**.We assume now that

$$l\hbar^{-1} \leq 1. \tag{3.20}$$

Oscillatory integral in RHS of Eq.(3.19) is calculated now directly using stationary phase approximation. The phase term $S(x,x_0)$ given by Eq.(3.18) is stationary when

$$\frac{\partial S(t,x,x_0)}{\partial x_0} = -\frac{m(x-x_0)}{t} + mt(U_0 - E) = 0. \tag{3.21}$$

and thus stationary point $x_0(t,x)$ are

$$x_0(t,x) = mt^2(U_0 - E) - x. \tag{3.22}$$

Thus from Eq.(3.19) and Eq.(3.22) using stationary phase approximation we obtain

$$|s_1(t)\rangle = \Psi^{\#}_{\mathbf{II}}(x,t) =$$

$$E(x_0(t,x),l)\Psi_{\mathbf{II}}(x_0(t,x))\theta_l(x_0(t,x))\exp\left\{\frac{i}{\hbar}[S(t,x,x_0(t,x))]\right\} + O\left(\sqrt{\hbar}\right). \tag{3.23}$$

**Proposition 3.2**. Suppose that a nucleus $\mathbf{n}$ is in the superposition state given by Eq.(3.16). the collaps: $|\text{live cat}\rangle \to |\text{death cat}\rangle$ arises at instant

$$T = \frac{L}{|c_2|^2\sqrt{8\pi^2 E/m}}. \tag{3.15}$$

**Proof**. Imediately follows by simple calculation from Eq.(A.13), Proposition 3.1 and Eq.(3.23).

**Thus is the collapsed state of the cat always shows definite outcomes even if cat also consists of a superposition**:

$$|\text{cat}\rangle = c_1|\text{live cat}\rangle + c_2|\text{death cat}\rangle.$$

Contrary to van Kampen's [10] and some others' opinions, "looking" at the outcome changes nothing, beyond informing the observer of what has already happened.

We remain: there are widespread claims that Schrödinger's cat is not in a definite alive or dead state but is, instead, in a superposition of the two. van Kampen, for example, writes "The whole system is in a superposition of two states: one in which no decay has occurred and. . .one in which it has occurred. Hence, the state of the cat also consists of a superposition: $|\text{cat}\rangle = c_1|\text{live cat}\rangle + c_2|\text{death cat}\rangle$. The state remains a superposition until an observer looks at the cat" [10].

**Appendix**. **A**.

The time-dependent Schrodinger equation governs the time evolution of a quantum mechanical system:

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = \widehat{\mathbf{H}}\Psi(\mathbf{x},t). \qquad (A.1)$$

The average, or expectation, value $\langle x_i\rangle$ of an observable $x_i$ corresponding to a quantum mechanical operator $\widehat{x}_i$ is given by:

$$\langle x_i\rangle(t,\mathbf{x}_0,t_0;\hbar) = \frac{\int\limits_{\mathbb{R}^d} x_i|\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)|^2 d^d x}{\int\limits_{\mathbb{R}^d}|\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)|^2 d^d x}. \qquad (A.2)$$

$$i = 1,\ldots,d.$$

**Remark A.1**. We assume now that: the solution $\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)$ of the time-dependent Schrödinger equation (A.1) has a good approximation by a delta function such that

$$|\Psi(\mathbf{x},t,\mathbf{x}_0,t_0;\hbar)|^2 \simeq \prod_{i=1}^{d}\delta(x_i - x_i(t,\mathbf{x}_0,t_0)), \tag{A.3}$$
$$x_i(t,\mathbf{x}_0,t_0) = x_{i,0},$$
$$i = 1,\ldots,d.$$

**Remark A.2**. Note that under conditions given by Eq.(A.3) QM-system which governed by Schrödinger equation Eq.(A.1) completely evolve quasiclassically i.e. estimating the position $\{x_i(t,\mathbf{x}_0,t_0;\hbar)\}_{i=1}^{d}$ at each instant $t$ with final error $\delta$ gives $|\langle x_i\rangle(t,\mathbf{x}_0,t_0;\hbar) - x_i(t,\mathbf{x}_0,t_0)| \leq \delta, i = 1,\ldots,d$ with a probability

$$\mathbf{P}\{|\langle x_i\rangle(t,\mathbf{x}_0,t_0;\hbar) - x_i(t,\mathbf{x}_0,t_0)| \leq \delta\} \simeq 1.$$

Thus from Eq.(A.2) and Eq.(A.3) we obtain

$$\langle x_i\rangle(t,\mathbf{x}_0,t_0;\hbar) \simeq$$

$$\simeq \frac{\int_{\mathbb{R}^d} x_i \prod_{i=1}^{d=1}\delta(x_i - x_i(t,\mathbf{x}_0,t_0))d^dx}{\int_{\mathbb{R}^d}\prod_{i=1}^{d=1}\delta(x_i - x_i(t,\mathbf{x}_0,t_0))d^dx} = x_i(t,\mathbf{x}_0,t_0). \tag{A.4}$$
$$i = 1,\ldots,d.$$

Thus under condition given by Eq.(A.3) one obtain

$$\langle x_{i,t}\rangle(t,\mathbf{x}_0,t_0;\hbar) \simeq x_i(t,\mathbf{x}_0,t_0), \tag{A.5}$$
$$i = 1,\ldots,d.$$

**Remark A.3**.Let $\Psi_i(\mathbf{x},t,\mathbf{x}_0,t_0), i = 1,2$ be the solutions of the time-dependent Schrödinger equation (A.1). We assume now that $\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)$ is a linear superposition such that

$$\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0) = c_1\Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0) + c_2\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0). \tag{A.6}$$
$$|c_1|^2 + |c_2|^2 = 1.$$

Then we obtain

$$|\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)|^2 = (\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)\Phi^*(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)) =$$

$$= ([c_1\Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0) + c_2\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)]) \times$$

$$\times([c_1^*\Psi_1^*(\mathbf{x},t,\mathbf{x}_0,t_0) + c_2^*\Psi_2^*(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)]) = \qquad (A.7)$$

$$= |c_1|^2\left(|\Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0)|^2\right) + c_1^*c_2(\Psi_1^*(\mathbf{x},t,\mathbf{x}_0)\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)) +$$

$$|c_2|^2\left(|\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)|^2\right) + c_1c_2^*(\Psi_1(\mathbf{x},t,\mathbf{x}_0)\Psi_2^*(\mathbf{x},t,\mathbf{y}_0,t_0)).$$

**Definition A.1**. Let $\langle\mathbf{x}\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0)$ be a vector-function

$$\langle\mathbf{x}\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0) : [0,T]\times\mathbb{R}^d\times\mathbb{R}^d\times[0,T]\to\mathbb{R}^d$$

$$\langle\mathbf{x}\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0) = \{\langle x_1\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0),\ldots,\langle x_d\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0)\}, \qquad (A.8)$$

where

$$\langle x_i\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0) = \int\limits_{\mathbb{R}^d} x_i|\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)|^2 d^d x =$$

$$= |c_1|^2 \int\limits_{\mathbb{R}^d} x_i|\Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0)|^2 d^d x +$$

$$+|c_2|^2 \int\limits_{\mathbb{R}^d} x_i|\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)|^2 d^d x + \qquad (A.9)$$

$$+c_1^* c_2 \int\limits_{\mathbb{R}^d} x_i \Psi_1^*(\mathbf{x},t,\mathbf{x}_0,t_0)\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0) d^d x +$$

$$+c_1 c_2^* \int\limits_{\mathbb{R}^d} x_i \Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0)\Psi_2^*(\mathbf{x},t,\mathbf{y}_0,t_0) d^d x.$$

**Definition A.2**. Let $\Delta(t,\mathbf{x}_0,\mathbf{y}_0,t_0)$ be a vector-function

$$\Delta(t,\mathbf{x}_0,\mathbf{y}_0,t_0) : [0,T] \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d$$

$$(\Delta(t,\mathbf{x}_0,\mathbf{y}_0,t_0)) = \{\delta_1(t,\mathbf{x}_0,\mathbf{y}_0,t_0),\ldots,\delta_d(t,\mathbf{x}_0,\mathbf{y}_0,t_0)\}, \qquad (A.10)$$

where

$$\delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0) = \delta[x_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0)] =$$

$$= c_1^* c_2 \int\limits_{\mathbb{R}^d} x_i \Psi_1^*(\mathbf{x},t,\mathbf{x}_0,t_0)\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)d^d x + \qquad (A.11)$$

$$+c_1 c_2^* \int\limits_{\mathbb{R}^d} x_i \Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0)\Psi_2^*(\mathbf{x},t,\mathbf{y}_0,t_0)d^d x.$$

Substituting Eqs.(A.11) into Eqs.(A.9) gives

$$\langle x_i\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0) = \int\limits_{\mathbb{R}^d} x_i |\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)|^2 d^d x =$$

$$= |c_1|^2 \int\limits_{\mathbb{R}^d} x_i |\Psi_1(\mathbf{x},t,\mathbf{x}_0,t_0)|^2 d^d x + \qquad (A.12)$$

$$+|c_2|^2 \int\limits_{\mathbb{R}^d} x_i |\Psi_2(\mathbf{x},t,\mathbf{y}_0,t_0)|^2 d^d x + \delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0) =$$

$$= |c_1|^2 \langle x_i\rangle(t,\mathbf{x}_0,t_0) + |c_2|^2 \langle x_i\rangle(t,\mathbf{y}_0,t_0) + \delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0).$$

Substitution equations (A.5) into equations (A.12) gives

$$\langle x_i\rangle(t,\mathbf{x}_0,\mathbf{y}_0,t_0) = \int\limits_{\mathbb{R}^d} x_i|\Phi(\mathbf{x},t,\mathbf{x}_0,\mathbf{y}_0,t_0)|^2 d^d x =$$

$$= |c_1|^2\langle x_i\rangle(t,\mathbf{x}_0,t_0) + |c_2|^2\langle x_i\rangle(t,\mathbf{y}_0,t_0) + \delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0) \quad (A.13)$$

$$\simeq |c_1|^2 x_i(t,\mathbf{x}_0,t_0) + |c_2|^2 x_i(t,\mathbf{y}_0,t_0) + \delta_i(t,\mathbf{x}_0,\mathbf{y}_0,t_0).$$

**Appendix. B.**

The Schrödinger equation in region $\mathbf{I} = \{x|x < 0\}$ has the folloving form

$$\hbar^2\frac{\partial^2\Psi_{\mathbf{I}}(x)}{\partial x^2} + 2mE\Psi_{\mathbf{I}}(x) = 0. \quad (B.1)$$

From Schrödinger equation (B.1) follows

$$\hbar^2\int\limits_{-\infty}^{0}\frac{\partial^2\Psi_{\mathbf{I}}(x)}{\partial x^2}dx + 2mE\int\limits_{-\infty}^{0}\Psi_{\mathbf{I}}(x)dx = 0. \quad (B.2)$$

Let $\Psi_{\mathbf{I}}^{\#}(x)$ be a function

$$\Psi_{\mathbf{I}}^{\#}(x) = \phi(x)\Psi_{\mathbf{I}}(x), \quad (B.3)$$

where

$$\phi(x) = (\pi r_c^2)^{-3/4}\exp\left(\frac{x^2}{2r_c^2}\right) \quad (B.4)$$

see Eq.(2.9). Note that

$$\frac{\partial^2[\phi(x)\Psi_{\mathbf{I}}(x)]}{\partial x^2} = \frac{\partial}{\partial x}\left[\Psi_{\mathbf{I}}(x)\frac{\partial \phi(x)}{\partial x} + \phi(x)\frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x}\right] =$$

$$2\frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x}\frac{\partial \phi(x)}{\partial x} + \Psi_{\mathbf{I}}(x)\frac{\partial^2 \phi(x)}{\partial x^2} + \phi(x)\frac{\partial^2 \Psi_{\mathbf{I}}(x)}{\partial x^2}. \quad (B.5)$$

Therefore substitution (B.2) into LHS of the Schrödinger equation (B.1) gives

$$\hbar^2 \int_{-\infty}^{0} \frac{\partial^2 \Psi^{\#}_{\mathbf{I}}(x)}{\partial x^2} dx + 2mE \int_{-\infty}^{0} \Psi^{\#}_{\mathbf{I}}(x) dx =$$

$$\hbar^2 \int_{-\infty}^{0} \frac{\partial^2 \phi(x)\Psi_{\mathbf{I}}(x)}{\partial x^2} dx + 2Em \int_{-\infty}^{0} \phi(x)\Psi_{\mathbf{I}}(x) dx =$$

$$2\hbar^2 \int_{-\infty}^{0} \frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x}\frac{\partial \phi(x)}{\partial x} dx + \hbar^2 \int_{-\infty}^{0} \Psi_{\mathbf{I}}(x)\frac{\partial^2 \phi(x)}{\partial x^2} dx + \quad (B.6)$$

$$+ \int_{-\infty}^{0} \phi(x)\left\{\hbar^2 \frac{\partial^2 \Psi_{\mathbf{I}}(x)}{\partial x^2} + 2Em \int_{-\infty}^{0} \Psi_{\mathbf{I}}(x)\right\} dx.$$

Note that

$$\int_{-\infty}^{0} \phi(x)\left\{\hbar^2 \frac{\partial^2 \Psi_{\mathbf{I}}(x)}{\partial x^2} + 2Em \int_{-\infty}^{0} \Psi_{\mathbf{I}}(x)\right\} dx = 0. \quad (B.7)$$

Therefore from Eq.(B.6) and Eq.(2.3)-Eq.(2.4) one obtain

$$\hbar^2 \int_{-\infty}^{0} \frac{\partial^2 \Psi_{\mathbf{I}}^{\#}(x)}{\partial x^2} dx + 2mE \int_{-\infty}^{0} \Psi_{\mathbf{I}}^{\#}(x) dx =$$

$$\hbar^2 \int_{-\infty}^{0} \frac{\partial^2 \phi(x) \Psi_{\mathbf{I}}(x)}{\partial x^2} dx + 2Em \int_{-\infty}^{0} \phi(x) \Psi_{\mathbf{I}}(x) dx = \qquad (B.8)$$

$$= 2\hbar^2 \int_{l}^{\infty} \frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x} \frac{\partial \phi_{\varepsilon}(x)}{\partial x} dx + \hbar^2 \int_{l}^{\infty} \Psi_{\mathbf{I}}(x) \frac{\partial^2 \phi(x)}{\partial x^2} dx.$$

From Eq.(B.6) one obtain

$$\frac{\partial \phi(x)}{\partial x} = (\pi r_c^2)^{-3/4} \frac{\partial}{\partial x} \exp\left[ -\frac{x^2}{2r_c^2} \right] = -(\pi r_c^2)^{-3/4} r_c^{-2} x \exp\left[ -\frac{x^2}{2r_c^2} \right],$$

$$\frac{\partial^2 \phi(x)}{\partial x^2} = -(\pi r_c^2)^{-3/4} r_c^{-2} \exp\left[ -\frac{x^2}{2r_c^2} \right] + \qquad (B.9)$$

$$+(\pi r_c^2)^{-3/4} r_c^{-4} x^2 \exp\left[ -\frac{x^2}{2r_c^2} \right].$$

From Eq.(B.9) and Eq.(2.3)-Eq.(2.4) one obtain

$$\hbar^2 \int\limits_{-\infty}^{0} \frac{\partial \Psi_{\mathbf{I}}(x)}{\partial x} \frac{\partial \phi(x)}{\partial x} dx =$$

$$-\frac{\hbar^2}{(\pi r_c^2)^{3/4} r_c^2} \int\limits_{-\infty}^{0} \frac{\partial \exp(ikx)}{\partial x} x \exp\left[-\frac{x^2}{2r_c^2}\right] dx = \qquad (B.10)$$

$$-\frac{2\pi\sqrt{2mE}\,\hbar}{(\pi r_c^2)^{3/4} r_c^2} \int\limits_{-\infty}^{0} x \exp\left(i \frac{2\pi\sqrt{2mE}}{\hbar} x\right) \exp\left[-\frac{x^2}{2r_c^2}\right] dx,$$

$$k = \frac{2\pi}{\hbar}\sqrt{2mE}\,.$$

and

$$\hbar^2 \int\limits_{-\infty}^{0} \Psi_{\mathbf{I}}(x) \frac{\partial^2 \phi(x)}{\partial x^2} dx = -\frac{\hbar^2}{(\pi r_c^2)^{3/4} r_c^2} \int\limits_{-\infty}^{0} \exp(ikx) \exp\left[-\frac{x^2}{2r_c^2}\right] dx + \qquad (B.11)$$

$$+\frac{\hbar^2}{(\pi r_c^2)^{3/4} r_c^2} \int\limits_{-\infty}^{0} x^2 \exp(ikx) \exp\left[-\frac{x^2}{2r_c^2}\right] dx.$$


[1] S. Weinberg, Phys. Rev. A 85, 062116 (2012).

[2] A. Bassi, K. Lochan, S. Satin, T. P. Singh, H. Ulbricht, Models of Wave-function Collapse, Underlying Theories, and Experimental Tests, Rev. Mod. Phys. 85, 471-527 (2013) http://arxiv.org/abs/1204.4325

[3] K.Jacobs, D. A. Steck, A Straightforward Introduction to Continuous Quantum Measurement, Contemporary Physics 47, 279 (2006) DOI:10.1080/00107510601101934

[4] M.B.Mensky,Continuous Quantum Measurements and Path Integrals, Hardcover: 188 pp. Publisher: CRC Press (January 1, 1993) ISBN-10: 0750302283 ISBN-13: 978-0750302289

[5] M.B.Mensky,Quantum Measurements and Decoherence: Models and Phenomenology,Series: Fundamental Theories of Physics, Vol. 110 2000, XVI, 231 pp. ISBN 978-94-015-9566-7
[6] A.J. Leggett, Schrödinger's Cat and Her Laboratory Cousins, Contemp. Phys.,1984, v.25, No.6, pp. 583-598.
[7] A. Hobson, Two-photon interferometry and quantum state collapse, Phys. Rev. A 88, 022105 – Published 8 August 2013
[8] G. Gamow,"Zur Quantentheorie des Atomkernes",Z. Physik 51, 204 (1928)
[9] R. Feinman, A. Hibbs, (2005), Quantum Mechanics and Path Integrals, Dover (Emended Edition).
[10] N. G. van Kampen, Physica A 153, 97 (1988).