The idea of the Arithmetica

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May 21, 2014

Abstract

During the 360 years of Fermat's last theorem is to be proved, this proposition was the presence appear full-length novel in "The Lord of the Rings", such as the "One Ring". And finally in 1994, it was proved completely by Andrew Wiles. However interesting proof is Fermat has been is still unknown. This will be assumed in the category of algebra probably.

introduction

Natural number X,Y and Z solution of 3 or more that this equation holds $X^n + Y^n = Z^n$ does not exist. Fermat is proven for the conditions of n = 4. It is sufficient if n is examining the conditions of prime numbers greater than or equal to 3 for this.

Theorem 1 *Triangle the hypotenuse of Pythagorean theorem is z, can be expressed by the following relation by using the l and m.*

$$\left(l^2 - m^2\right)^2 + 2^2 \left(lm\right)^2 = \left(l^2 + m^2\right)^2$$

$$\begin{aligned} x^2 &= \left(l^2 - m^2\right)^2 \\ y^2 &= 2^2 \left(lm\right)^2 \\ z^2 &= \left(l^2 + m^2\right)^2 \end{aligned}$$

$(xyz \neq 0)$

Put $\quad X, Y, Z \in \mathbb{N} \quad$ *prime number* $= p \geq 3$

Conditions. $\quad X, Y, Z \in$ *even number*

$$
\begin{aligned}
X^p &= 2^p X_1^p \\
Y^p &= 2^p Y_1^p \\
Z^p &= 2^p Z_1^p
\end{aligned}
$$

$(X_1, Y_1, Z_1 \in \mathbb{N})$

Add the following conditions.

$X_1^p = \left(X_{II}^2\right)^p \qquad X_{II} \in \mathbb{N}$

Corollary 2 $\quad (s+t)^2 + (s-t)^2 = 2\left(s^2+t^2\right)$*:The sum of the squares of two*

$(s, t \in \mathbb{R} \quad s > t)$

$$
\begin{aligned}
X^p &= (s+t)^2 \\
Y^p &= (s-t)^2 \\
Z^p &= 2\left(s^2+t^2\right)
\end{aligned}
$$

Put $\quad s = 2^{\frac{p-1}{2}} S_1 \quad t = 2^{\frac{p-1}{2}} T_1$

$$
\begin{aligned}
Z^p &= 2\left(s^2+t^2\right) \\
&= 2\left(\left(2^{\frac{p-1}{2}} S_1\right)^2 + \left(2^{\frac{p-1}{2}} T_1\right)^2\right) \\
&= 2\left(2^{p-1} S_1^2 + 2^{p-1} T_1^2\right) \\
2^p Z_1^p &= 2^p \left(S_1^2 + T_1^2\right) \\
Z_1^p &= S_1^2 + T_1^2
\end{aligned}
$$

$$
\begin{aligned}
X^p &= (s+t)^2 \\
&= \left(2^{\frac{p-1}{2}} S_1 + 2^{\frac{p-1}{2}} T_1\right)^2 \\
2^p X_1^p &= 2^{p-1} (S_1 + T_1)^2 \\
X_1^p &= 2^{-1} (S_1 + T_1)^2 \\
&\text{Similarly,} \\
Y_1^p &= 2^{-1} (S_1 - T_1)^2
\end{aligned}
$$

Put $\quad S_1 = 2^{\frac{p+1}{2}} S_2 \quad T_1 = 2^{\frac{p+1}{2}} T_2$

$$\begin{aligned}
Z_1^p &= S_1^2 + T_1^2 \\
&= \left(2^{\frac{p+1}{2}} S_2\right)^2 + \left(2^{\frac{p+1}{2}} T_2\right)^2 \\
&= 2^{p+1} S_2^2 + 2^{p+1} T_2^2 \\
&= 2^{p+1}\left(S_2^2 + T_2^2\right) \qquad \cdots ①
\end{aligned}$$

$$\begin{aligned}
X_1^p &= 2^{-1}\left(S_1 + T_1\right)^2 \\
&= \left(2^{-\frac{1}{2}}\right)^2 \left(2^{\frac{p+1}{2}} S_2 + 2^{\frac{p+1}{2}} T_2\right)^2 \\
&= \left(2^{\frac{p}{2}} S_2 + 2^{\frac{p}{2}} T_2\right)^2 \qquad \text{So } X_1^p = \left(X_{II}^p\right)^2, \\
X_{II}^p &= 2^{\frac{p}{2}} S_2 + 2^{\frac{p}{2}} T_2 \qquad \cdots ②
\end{aligned}$$

$$\begin{aligned}
Y_1^p &= 2^{-1}\left(S_1 - T_1\right)^2 \\
&= 2^{-1}\left(2^{\frac{p+1}{2}} S_2 - 2^{\frac{p+1}{2}} T_2\right)^2 \\
&= 2^p \left(S_2 - T_2\right)^2 \\
&= 2^p \left(S_2^2 + T_2^2 - 2S_2T_2\right) \\
&= 2^p \left(S_2^2 + T_2^2\right) - 2^{p+1} S_2 T_2 \quad \text{And multiplied by 2to both sides.} \\
2Y_1^p &= 2^{p+1}\left(S_2^2 + T_2^2\right) - 2^{p+2} S_2 T_2
\end{aligned}$$

Referring to ①,

$$2Y_1^p = Z_1^p - 2^{p+2} S_2 T_2 \qquad \cdots ③$$

$$\begin{aligned}
2^{p+1}\left(S_2^2 + T_2^2\right) &= 2Y_1^p + 2^{p+2} S_2 T_2 \qquad \text{And multiplied by } 2^{\frac{p}{2}} \text{to both sides.} \\
2^{p+1}\left(2^{\frac{p}{2}} S_2 + 2^{\frac{p}{2}} T_2\right) &= 2^{\frac{p}{2}+1} Y_1^p + 2^{\frac{p}{2}+p+2} S_2 T_2 \qquad \text{Referring to ②,} \\
2^{p+1} X_{II}^p &= 2^{\frac{p}{2}+1} Y_1^p + 2^{\frac{p}{2}+p+2} S_2 T_2 \\
2^{\frac{p}{2}+1} Y_1^p &= 2^{p+1} X_{II}^p - 2^{\frac{p}{2}+p+2} S_2 T_2 \quad \text{And multiplied by } 2^{-\frac{p}{2}} \text{to both sides.} \\
2Y_1^p &= 2^{\frac{p}{2}+1} X_{II}^p - 2^{p+2} S_2 T_2
\end{aligned}$$

In comparison with the ③,

$$\begin{aligned}
Z_1^p &= 2^{\frac{p}{2}+1} X_{II}^p \\
&\left(Z_1^p \;,\;\; X_{II}^p \in \mathbb{N} \qquad p \in odd \text{ number}\right) \\
Z_1^p &\neq 2^{\frac{p}{2}+1} X_{II}^p
\end{aligned}$$

Therefore, $\qquad x^n + y^n \neq z^n \qquad (xyz \neq 0 \quad p \geq 3)$