AN INEQUALITY CONCERNING THE SMARANDACHE FUNCTION

Maohua Le
Zhanjiang Normal College, Zhanjang, Guangdong, P.R.China

Abstract. For any positive integer n, let $S(n)$ denote the Smarandache function of n. In this paper we prove that $S(mn) \le S(m)+S(n)$.

Let N be the set of all positive integers. For any positive integer n, let $S(n)$ denote the Smarandache function of n. By [2], we have

$$S(n)=\min\{k \mid k \in N,\ n \mid k!\}. \tag{1}$$

Recently, Jozsef[1] proved that

$$S(mn) \le mS(n),\quad m,n \in N. \tag{2}$$

In this paper we give a considerable improvement for the upper bound (2). We prove the following result.

Theorem. For any positive integers m,n, we have
$$S(mn) \le S(m)+S(n).$$

Proof. Let $a=S(m)$ and $b=S(n)$. Then we have

$$n \mid b!, \tag{3}$$

by (1). Let x be a positive integer with $x \ge a$, and let

$$\binom{x}{a} = \frac{x(x-1)\dots(x-a+1)}{a!} \tag{4}$$

be a binomial coeficient. It is a well known fact that $\binom{x}{a}$ is a positive integer. So we have

$$a! \mid x(x-1)\dots(x-a+1), \tag{5}$$

by (4). Further, since $m \mid a!$, we get from (5) that

$$m \mid x(x-1)\dots(x-a+1), \tag{6}$$

for any positive integer x with $x \ge a$. Put $x=a+b$. We see from (3) and (6) that

$$mn \mid b!(b+1)\dots(b+a)=(a+b)!. \tag{7}$$

Thus we get from (7) that $S(mn) \leq a+b=S(m)+S(n)$. The theorem is proved.

References

1. S.Jozsef, On certain inegualites involving the Smarandache function, Smarandache Notce J. 7(1996), No.1-3, 3-6.
2. F.Smarandache "A function in the number theory"," Smarandache Function J". 1(1990), No.1, 3-17.