

Exploring the possibility of New Physics: Quantum Mechanics and the Warping of Spacetime

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Abstract

This paper suggests new physics. In a different approach, it proposes fundamental particles formed from infinite superpositions with mass borrowed from a Higgs type scalar field. However energy is also borrowed from zero point vector fields. Just as the Standard Model divides the fundamental particles into two types...those with mass and those without, with the Higgs mechanism providing the difference...infinite superpositions seem also to divide naturally into two sets: (a) those with “infinitesimal” mass, and (b) those with significant mass (from micro electron volts upwards). In the infinitesimal set (a), photons, gluons and gravitons (to fit with cosmology and the expansion of the cosmos) all have $\approx 10^{-34}$ eV mass, approximately the inverse of the causally connected horizon radius. These values are so close to zero the symmetry breaking of the Standard Model remains essentially valid. These particles travel so close to the speed of light they have virtually fixed helicity, with the Higgs mechanism increasing their mass from infinitesimal type (a) to significant or measurable type (b) values. Also the energy in the zero point fields (borrowed to build the fundamental particles) is limited, particularly at the extreme wavelengths of virtual gravitons interacting at near horizon radii. Any causally connected region grows with time after the big bang and the number of virtual gravitons with wavelengths similar to the size of the causally connected region increases approximately as the square of the causally connected mass. Space has to expand exponentially with time in an accelerating manner after the big bang to make available the zero point energy to meet this increased requirement. For similar reasons the extra gravitons near mass concentrations change the metric in proportion to m/r , in accordance with the Schwarzschild solution of Einstein’s equations. It suggests possible spacetime boundaries at the event horizons of black holes in line with one of the current Firewall Paradox implications. Approximately the first two thirds of this paper look at building and analysing the fundamental particles formed from infinite virtual superpositions. The final portion looks at the expanding Universe and connections with General Relativity; but only after attempting to show that infinite superpositions can be equivalent to the Standard Model fundamental particles, apart from infinitesimal differences.

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1 Introduction

Since the weak and electromagnetic forces were unified in the 1960's physicists have wished to somehow unite all the fundamental forces. Initially it seemed that three of the forces (excluding gravity) could unite near Planck scale. High energy experiments however ruled out the possibility of single energy unification. Some type of Supersymmetry is seen by many as the solution to several current Standard Model problems; one version modifying the high energy running constants of the three forces in such a manner they unite near the Planck scale. The particles predicted however have yet to be seen. String theory is also seen by many as the future path, but not all physicists are comfortable with its non-testability and need for 10 or 11 dimensions. The enormous landscape of different universes or multiverse it proposes is also widely regarded as the solution to the minute amount of dark energy proposed to explain the current accelerating expansion of the universe. Some recent cosmological surveys [1] however have not so far supported dark energy as the cause of this acceleration. This all suggests some important and relevant questions, for example:

1. Is it possible that the fundamental forces may connect in some different way?
2. Are the extra dimensions of String Theory really necessary?
3. Is "The Multiverse" the only explanation of accelerating cosmic expansion?
4. Can the problems these theories solve be addressed differently?

Approaching all this in a completely new direction, this paper explores possible solutions to these puzzles in a different way, but still using basic principles of quantum mechanics and relativity. Apart from infinitesimal differences it seems to be consistent with the Standard Model. It requires the universe to expand exponentially after the big bang in an accelerating manner. It changes the metric around mass concentrations in accordance with General Relativity. It requires photons, gluons and gravitons to have mass of $\approx 10^{-34} eV$, close to some recent proposals [2] [3] giving gravitons a mass of $< 10^{-33} eV$ to explain the accelerating expansion of the universe. It argues General Relativity cuts off at Black Hole event horizons, one of the possible implications of the current Firewall paradox.[14] [15] [16] [17] [18].

1.1 Summary

Papers modifying the Standard Model are too numerous to list, however we briefly touch on a small number of some early versions of these in section 1.1.2. The approach in this paper is very different from that in most of these earlier papers. The main differences are summarized below.

1.1.1 General Relativity as our starting point

General Relativity tells us that all forms of mass, energy and pressure are sources of the gravitational field. Thus to create gravitational fields all spin $\frac{1}{2}$ leptons & quarks, spin 1 gluons, photons, W^\pm & Z^0 particles etc. emit virtual gravitons, except possibly gravitons themselves (section 6.2.3), as gravitational energy is not part of the Einstein tensor.

The starting point of this paper assumes there is a common thread uniting these fundamental particles making this possible. Equations are developed that unite the amplitudes of the colour and electromagnetic coupling constants with that of gravity. The precision required by quantum mechanics for half integral and integral angular momentum allows gravity to be included, despite the vast disparity in magnitude between gravity and the other two. This combination of colour, electromagnetic and gravitational amplitudes in the same equation is possible because of a radically different approach taken in this paper: An approach using infinite superpositions of positive and negative integral \hbar angular momentum virtual wavefunctions for spin $\frac{1}{2}$, spin 1 and spin 2 particles. The final result is almost identical to the Standard Model, with infinitesimal but important differences.

The total angular momentum can be summed over all wavenumbers k ; from $k=0$ to some cutoff value k_{cutoff} . We will assume (as with many unification theories) that the cutoff for these infinite superpositions is somewhere near Planck scale. Firstly imagine a universe where the gravitational constant $G \rightarrow 0$. As $G \rightarrow 0$, the Planck length $L_p \rightarrow 0$, the Planck energy $E_p \rightarrow \infty$ and $k_{cutoff} \rightarrow \infty$ also. If we sum the angular momentum of these infinite superpositions when $G \rightarrow 0$ (i.e. from $k=0$ to $k_{cutoff} \rightarrow \infty$) we get precisely half integral or integral \hbar for the fundamental spin $\frac{1}{2}$, spin 1 & spin 2 particles in appropriate m states. If we now put $G > 0$ the infinitesimal effect of including gravity can be balanced by an equal but opposite effect due to the non-infinite cutoff value in k . A near Planck scale superposition cutoff requires gravity to be included to get precisely half integral or integral \hbar . (Section 4.2)

These infinite superpositions have another very relevant property relating to the fact that all experiments indicate that fundamental particles such as electrons behave as point particles. Each wavefunction with wavenumber k , which we label as ψ_k , has a maximum radial probability at $r \approx 1/k$ and they all look the same (Figure 1.1. 1.) Every wavefunction ψ_k of these infinite superpositions, interacts only with virtual photons (for example) of the same k ; if superpositions representing say an electron are probed with such photons (that interact only with wavefunction ψ_k) the resolution possible is of the same order as the dimensions of ψ_k , both have $r \approx 1/k$. The higher the energy of the probing particle the smaller the ψ_k it

interacts with, the resolution of an observing photon can never be fine enough to see any ψ_k dimensions. Even if this energy approaches the Planck value, with a matching ψ_k radius near the Planck length it is still not possible to resolve it. This behaviour is consistent with the quantum mechanical properties of point particles.

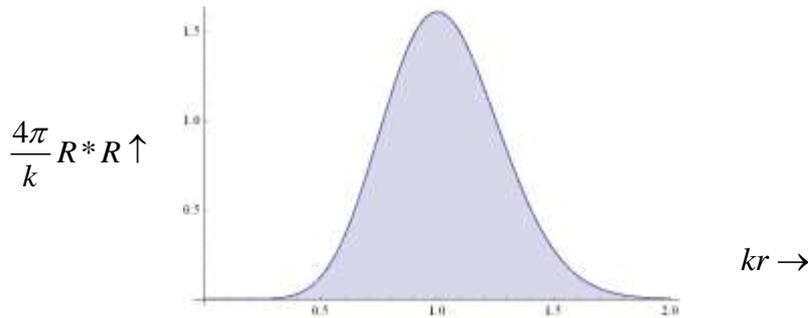


Figure 1.1. 1 The radial probability of the dominant $n = 6$ for spin $\frac{1}{2}$ wavefunction ψ_{6k} .

1.1.2 Primary interactions and Secondary interactions

Supposing that superpositions can in fact build the fundamental spin $\frac{1}{2}$, spin 1, and spin 2 particles, then what builds the superpositions? Before answering that question, this paper can only make sense if we divide the world of all interactions into two categories.

Secondary Interactions are those we are familiar with and are covered by the Standard Model, but with the addition of gravity, which is not included in the Standard Model. They take place between the fundamental spin $\frac{1}{2}$, spin 1 and spin 2 particles formed from infinite superpositions. They are the QED/QCD etc, interactions of all real world experiments.

Primary Interactions we conjecture on the other hand are those that build infinite superpositions and are hidden to the real world of experiments.

The majority of this paper is about these primary interactions, and the superpositions they build representing the fundamental spin $\frac{1}{2}$, spin 1 and spin 2 particles. Primary interactions are between spin zero particles borrowed from a Higgs type scalar field and the zero point vector fields. In the 1970's models were proposed with preons as common building blocks of leptons and quarks [4] [5] [6] [7]. In contrast with the virtual particles in this paper some of these earlier models used real spin $\frac{1}{2}$ building blocks. Real substructure has difficulties with large masses if compressed into the small volumes required to approach point particle behaviour. On the other hand with *virtual substructure borrowing energy from zero point fields the mass contribution at high k values can be cancelled* (section 3.2.1). As in earlier models this paper also calls the common building blocks preons, *but here the preons are both*

virtual and spin zero. They also now build all spin $\frac{1}{2}$ leptons and quarks, spin 1 gluons, photons, W & Z particles, plus spin 2 gravitons in contrast to only the leptons and quarks in the earlier models. (See Table 2.2. 1.)

As these preons have zero spin they possess no weak charge, primary interactions (section 2.2.1) can take place only with the zero point colour, electromagnetic and gravitational fields. The three primary coupling constants for each of these three zero point fields are different from, (but related to) the secondary coupling constants. The behaviour of primary coupling is also entirely different from secondary coupling. Secondary coupling strengths vary (or run) with wavenumber k (the electromagnetic increasing with k and colour decreasing with k). In contrast, we conjecture primary coupling strengths (or constants) do not run. In this paper virtual preons are continually born with mass out of a Higgs type scalar field, existing only for time $\Delta t \approx \hbar / E$. At their birth, they interact while still bare with zero point vector fields at this instant of birth $t = 0$. The primary coupling constants consequently are fixed for all k : there is no time for charge canceling or reinforcing, which in secondary interactions forms around the bare charge progressively after its birth. The equations work only if this is true, and they also work only if the primary colour coupling constant is 1. This does not seem implausible as it simply means that primary colour coupling is certain (sections 2.2.2). The ratio between the primary and secondary colour coupling constants labelled χ_c is thus (*if primary colour coupling is 1*) the inverse of the secondary (or usual α_3^{-1} of QCD) colour coupling constant *at the superposition cutoff @ Planck Energy.* (Sections 3.3 & 4.2.2)

To enable the primary coupling to colour, electromagnetic and gravitational zero point fields, preons need colour, electric charge and mass. Red green or blue coloured preons have positive electric charge; anticolour red, green or blue preons have negative electric charge. Their mass which is borrowed from some type of scalar Higg's field must always be non-zero, which is discussed further in section 1.1.3. As there are 8 gluon fields, superpositions are built with 8 virtual preons for each virtual wavefunction ψ_k . The nett sum of these 8 electric charges is $0, \pm 2, \pm 4, \pm 6$, and never $> \pm 6$. This leads to the usual $0, \pm 1/3, \pm 2/3, \pm 1$ electric charge seen in the real world. Various combinations of these 8 preons in appropriate superpositions can build leptons and quarks, colour changing and neutral gluons, neutral photons, neutral massive Z^0 photons and the charged massive W^\pm photons. (Table 2.2. 1)

1.1.3 Photons, gluons and gravitons with infinitesimal mass ($\approx 10^{-34} eV$).

For many decades after the discovery of the neutrino in the 1930s it was thought to be massless, and to travel at velocity c . Towards the end of last century however evidence slowly accumulated that this may not in fact be true, and that the family of 3 neutrinos have masses in the electron volt range. Due to this very low mass, and their normal emitted energies, they invariably travel at virtually the velocity of light c . Photons also have always been seen as massless traveling precisely at velocity c , except in the case of the massive W^\pm & Z^0 . Massless virtual photons have an infinite range, which has always been seen as an absolute requirement of the electromagnetic field. On the other hand, this paper requires some rest frame (even if this frame normally moves virtually at c) in which to build all the fundamental particles. Table 6.2 1 suggests photons, gluons and gravitons have $\approx 10^{-34} eV$ mass with *a range of approximately the inverse of the causally connected horizon radius*, and velocities sufficiently close to that of light their helicity remains essentially fixed. This allows some form of Higgs mechanism to increase this infinitesimal mass to the various values in the massive set. These infinitesimal masses are in line with some recent proposals [2] [3] where gravitons have a mass of $< 10^{-33} eV$ to explain accelerating expansion.

The virtual wavefunction we use is $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta, \varphi)$ an $l=3$ wavefunction. This virtual $l=3$ property is normally hidden. In the same way as scattering experiments on spin 0 pions show spin 0 properties, and not the properties of the two canceling spin $1/2$ component particles, this $l=3$ property of the virtual components of superpositions is not visible in the real world. Scattering experiments can exhibit only the spin properties of the resulting particle. The individual angular momentum vectors $|\mathbf{L}| = 2\sqrt{3}\hbar$ of the infinite superposition all sum to a resulting: $|\mathbf{L}_{Total}| = (\sqrt{3}/2)\hbar, \sqrt{2}\hbar$ or $\sqrt{6}\hbar$ for spin $1/2$, spin 1 or spin 2 respectively, in a similar way to two spin $1/2$ particles forming spin 0 or spin 1 states.

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta, \varphi)$ has Eigenvalues $\mathbf{P}_{nk}^2 = n^2 \hbar^2 k^2$ with $|\mathbf{P}_{nk}| = n\hbar k$, suggesting it borrows n parallel $|\hbar\mathbf{k}|$ quanta from zero point vector fields provided n is integral. We can see this by letting $k \rightarrow \infty$ allowing energy $E \rightarrow n\hbar\omega$ by absorbing n quanta $\hbar\omega$ from the zero point vector fields (section 2.3.2). As spin 3 needs at least 3 spin 1 particles to create it, the lowest integral number n can be is 3. The virtual $l=3$ property can however be used to derive the magnetic moment of a charged spin $1/2$, $m = \pm 1/2$ state as a function of n . Section 3.5 shows $g=2$ Dirac electrons need an average (over integral n states) of $\bar{n} \approx 6.0135$. Three member superpositions $\psi_k = \sum c_n \psi_{nk}$ with $n=5, 6, \& 7$ achieve

this, creating Dirac spin $\frac{1}{2}$ states. We also find that $n=6$ is the dominant member and each superposition ψ_k needs at least 3 members to make all the equations consistent for Dirac particles. Secondary interactions at any wavenumber k can occur with ψ_k if integers n change by ± 1 , thus changing the Eigenvalues $|\mathbf{P}| = n\hbar k$ by $\pm\hbar k$ where this can be only a temporary rearrangement of the triplets of values of n . This is true, whether the interaction is with leptons, quarks, photons, gluons, W & Z particles, or gravitons. (Section 3.3)

1.1.4 Superpositions require only squared vector potentials

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta, \varphi)$ also requires a squared vector potential to create it: $Q^2 A^2 = n^4 \hbar^2 k^4 r^2 / 81$. There are no linear potential terms in contrast with secondary interactions. The primary interaction operator is $\hat{P}^2 = -\hbar^2 \nabla^2 + Q^2 A^2$, with no linear potential terms included and Q simply represents a collective symbol for all the effective charges concerned. As an example, the dominant $n=6$ wavefunction of a spin $\frac{1}{2}$ Dirac ψ_k requires a squared vector potential of $Q^2 A^2 = n^4 \hbar^2 k^4 r^2 / 81 = 16 \hbar^2 k^4 r^2$ (section 2.3.1). Primary coupling between the 8 virtual preons and the colour, electromagnetic and gravitational zero point fields produces a vector potential squared value for all infinite superpositions which can be expressed as:

$$Q^2 A^2 = \frac{\left[8 + 8\sqrt{\alpha_{EMP}} + im_0 \sqrt{G_p / (2s\hbar c)} \right]^2 (\hbar^2 k^4 r^2)}{3\pi(sN)(1+\varepsilon)} \left[\frac{(sN)(1+\varepsilon)dk}{k} \right]$$

(Where the length of the complex vector is squared here.) The significance of the cancelling top and bottom factors (sN) is explained in section 2.1.2. Also the cancelling $(1+\varepsilon)$ factors are due to gravity and explained in section 4.2. The primary_colour coupling amplitude is conjectured to be 1 to each of the eight preons, and $\sqrt{\alpha_{EMP}}$ the primary electromagnetic coupling. This equation applies regardless of the individual preon colour or electric charge signs, whether positive or negative (section 2.2.3). The primary gravitational coupling is to the particle mass m_0 . The primary gravitational constant is G_p divided by $\hbar c$ to put it in the same form as the other two coupling constants. The magnitude of the total angular momentum vector of the infinite superposition is $|\mathbf{L}_{Total}| = \sqrt{s(s+1)}$. This $Q^2 A^2$ without the gravity term generates superpositions with probability $(N \cdot s)dk / k$ where s is the superposition spin, $N=1$ for massive spin $\frac{1}{2}$ fermion & massive boson superpositions but $N=2$ for infinitesimal mass boson superpositions (Table 4.3. 1, section 6 and its subsections cover this more fully). Section 4.2 includes gravity raising the superposition probability to $(1+\varepsilon)(N \cdot s)dk / k$ where the infinitesimal ε (not to be confused with infinitesimal mass) is

$\varepsilon \approx 2m_0^2 / spin$ (in Planck units $\hbar = c = G = 1$) $\approx 7 \times 10^{-45}$ for electrons, and $\varepsilon \approx 10^{-34}$ for a Z^0 . The ψ_k superpositions require at least three integral n members. The following three member superpositions fit the Standard Model best (see Table 4.3. 1)

$$\text{Spin } \frac{1}{2} \text{ massive } N=1 \text{ fermion superpositions} \quad \psi_k = \sum_{n=5,6,7} c_n \psi_{nk} \cdot$$

$$\text{Spin 1 massive } N=1 \text{ boson superpositions} \quad \psi_k = \sum_{n=4,5,6} c_n \psi_{nk} \cdot$$

$$\text{Spins 1 \& 2 infinitesimal mass } N=2 \text{ boson superpositions} \quad \psi_k = \sum_{n=3,4,5} c_n \psi_{nk} \cdot$$

Below are infinite superpositions $\psi_{\infty,s,m}$ for only spins $\frac{1}{2}$ & 1. The symbol ∞ refers to the infinite sum, s the spin of the resulting real particle, m its angular momentum state, and ss a spherically symmetric state. Section 3.1.3 explains this format. Also square cutoffs in wavenumber k are used here for simplicity. Infinitesimal mass superpositions are introduced in section 6.2. (Complex number factors are not included here for clarity.)

$$\begin{aligned} \text{Massive } N=1 \text{ Spin } \frac{1}{2}, \psi_{\infty,1/2,m} &= \sum_{n=5,6,7} c_n \int_0^{k(\text{cutoff})} \left[\frac{(\psi_{nk,ss})}{\gamma_{nk}} + \beta_{nk} (\psi_{nk,4m}) \right] \sqrt{\frac{1+\varepsilon}{2k}} dk & (1.1. 1) \\ \text{Infinitesimal mass } N=2 \text{ Spin 1}, \psi_{\infty,1,m} &= \sum_{n=3,4,5} c_n \int_0^{k(\text{cutoff})} \left[\frac{(\psi_{nk,ss})}{\gamma_{nk}} + \beta_{nk} (\psi_{nk,2m}) \right] \sqrt{\frac{2(1+\varepsilon)}{k}} dk \end{aligned}$$

In these infinite superpositions the probability that the wavefunction is spherically symmetric is $1/\gamma_{nk}^2 = 1 - \beta_{nk}^2$ and the probability that it is an m state is β_{nk}^2 where β_{nk} is the magnitude of the velocity of the centre of momentum of the primary interactions that generate each ψ_{nk} . This is similar to the superposition of time and spatially polarized virtual photons in QED. For example spin $\frac{1}{2}$ has probabilities of $1/\gamma_{nk}^2 = 1 - \beta_{nk}^2$ spherically symmetric ψ_{nk} wavefunctions, and $\beta_{nk}^2 \times (\psi_{nk}, m = \pm 2)$ wavefunctions. Each ψ_k is normalized to 1 but the infinite superpositions $\psi_{\infty,s,m}$ are not normalized, diverging logarithmically with k ; the same logarithmic divergence that applies to virtual photon emission. (*Real wavefunctions have to be normalized to one as they refer to finding a real particle somewhere but this need not apply here.*) Each member of these spin $\frac{1}{2}$ superpositions has probability $dk(1+\varepsilon)/2k$, and if electrically charged emits virtual photons with probability $4\alpha/\pi$. Ignoring the factor of $(1+\varepsilon) \approx 1 + \approx 10^{-44}$, the overall virtual scalar photon emission probability is the usual $(2\alpha/\pi)dk/k$. (Possible implications of the infinitesimal ε are discussed in section 6.6) We also find in section 3.1 that $m = +2$ virtual wavefunctions have β_{nk}^2 probability of leaving

an $m = -2$ debt in the zero point fields. Integrating this over all k produces a total angular momentum for a spin $\frac{1}{2}$ state of $(\hbar/2)(1 - 1/\gamma^2_{Cutoff})(1 + \varepsilon)$, (section 3.2.2). When $1/k_{Cutoff}$ is near the Planck length $(1 - 1/\gamma^2_{Cutoff}) = 1/(1 + \varepsilon)$. A similar integration over all k for the rest energy of the infinite superposition also leads to $\pm m_0 c^2 (1 - 1/\gamma^2_{Cutoff})(1 + \varepsilon)$, (section 3.2.1). The infinitesimal quantity ε vanishes in a zero gravity, zero Planck length universe where $k_{Cutoff} \& \gamma_{Cutoff} \rightarrow \infty$. In this paper each preon borrows virtual rest mass from a Higgs type scalar field. The superposition mass/energy is obtained by summing squared momenta over all k . The equations are based on probabilities of these in a similar manner to those for angular momentum. This suggests the superposition or equivalent particle mass is both energy borrowed from zero point *vector*, and mass borrowed from Higgs type *scalar* fields.

The final sections of this paper (5 & 6) argue that the limited zero point energies (required to generate virtual gravitons) available at causally connected cosmos wavelengths require it to expand exponentially in an accelerating manner (Figure 5.3. 3). Sections 5.2 & 5.2.2 argue that the warping of spacetime around mass concentrations is consistent with local observers measuring a constant background density of virtual gravitons. This can only happen if at any radius r around a mass m , space expands proportionally to m/r in accordance with the Schwarzschild solution. We argue that this implies General Relativity (in an infinitesimally modified form effective only at cosmic scale) and the warping of spacetime is a consequence of Quantum Mechanics. The first two thirds of this paper is about the *primary interactions between spin zero preons and spin one quanta* that build the fundamental particles. The Standard Model is about the secondary interactions between them. (*The weak force is only between spin $\frac{1}{2}$ particles and thus a secondary interaction. It can not be involved in primary interactions.*) Apart from infinitesimal effects, such as infinitesimal masses, the properties of fundamental particles covered in this paper seem consistent with their Standard Model counterparts. All $N = 1 \& N = 2$ superpositions as in Table 4.3. 1 are conjectured to cutoff at the Planck energy E_p . If this is so both colour and electromagnetic *interaction energies* must cutoff at $E_p / \langle n \rangle \approx 2.03 \times 10^{18} GeV.$, or $\approx 1/6$ of the Planck energy. (The expectation value $\langle n \rangle$ is ≈ 6.0135 for spin $\frac{1}{2}$ leptons and quarks Eq. (3.5. 16)). The electromagnetic and colour coupling constants predicted at this cutoff are consistent with Standard Model predictions assuming three families of fermions and one Higgs field. (See Figure 4.1. 1 & Figure 4.1. 2). Only after attempting to show that infinite superpositions can be equivalent to the Standard Model fundamental particles do we try to connect them with General Relativity.

2 Building Infinite Virtual Superpositions

2.1 The possibility of Infinite Superpositions

2.1.1 Early ideas

After World War II there was still much confusion about QED. In 1947 at the Long Island Conference the results of the Lamb shift experiment were announced [8]. Some of the first early explanations that gave approximately correct answers used simple semi classical thinking to get a better understanding of what seemed to be going on. These early ideas helped to eventually lead to the QED of today, perhaps in a similar manner to the way Bohr's original simple semi classical explanation of quantized atomic energy levels played such a large part in the eventual development of full three dimensional wavefunction solutions of atoms, and quantum mechanics. We start this paper with an example of a semi classical Lamb shift explanation that seems to lead into the possibility of fundamental particles and infinite virtual superpositions being one and the same.

The density of transverse modes of waves at frequency ω is $\omega^2 d\omega / \pi^2 c^3$ and the zero point energy for each of these modes is $\hbar\omega / 2$. The electrostatic and magnetic energy densities in electromagnetic waves are equal, thus for electromagnetic zero point fields:

$$\frac{\overline{\varepsilon_0 E^2}}{2} + \frac{\overline{\varepsilon_0 c^2 B^2}}{2} = \frac{\hbar\omega}{2} \left[\frac{\omega^2 d\omega}{\pi^2 c^3} \right] \quad \text{and} \quad \overline{\varepsilon_0 E^2} = \overline{\varepsilon_0 c^2 B^2} = \frac{\hbar\omega^4}{2\pi^2 c^3} \frac{d\omega}{\omega}.$$

For a fundamental charge e using $\alpha = e^2 / 4\pi\varepsilon_0\hbar c$, and provided $\beta \ll 1$, this gives an

$$\text{Average force squared of } \overline{F^2} = e^2 \overline{E^2} = \frac{2\alpha \hbar^2 \omega^4}{\pi c^2} \frac{d\omega}{\omega} \quad (2.1.1)$$

Thinking semi classically, for an electron of rest mass m this can generate simple harmonic motion of amplitude r , where $F^2 = m^2 \omega^4 r^2$ (if $\beta \ll 1$). Solving for r^2 (where r^2 is superimposed on the normal quantum mechanical electron orbit, $\tilde{\lambda}_c = \hbar / mc$ is the Compton

wavelength, and $k = \omega / c$):

$$r^2 = \frac{\hbar^2}{m^2 c^2} \frac{2\alpha}{\pi} \frac{d\omega}{\omega} = \left[\tilde{\lambda}_c^2 \right] \cdot \left[\frac{2\alpha}{\pi} \frac{dk}{k} \right]$$

Integrating r^2 (directions are random) : $r_{Total}^2 = \tilde{\lambda}_c^2 \frac{2\alpha}{\pi} \int_{k_{min}}^{k_{max}} \frac{dk}{k} = \tilde{\lambda}_c^2 \frac{2\alpha}{\pi} \log(k_{max} / k_{min})$.

The minimum and maximum values for k are chosen to fit atomic orbits, and a root mean square value for r can be found. Combining this with the small probability that the electron will be found in the nucleus, this small root mean square deviation shifts the average potential by approximately the Lamb shift. This can also be thought of as simple harmonic motion of amplitude $\approx \tilde{\lambda}_c$, occurring with probability $(2\alpha / \pi)dk / k$. It can also be interpreted as the electron recoiling by $\approx \tilde{\lambda}_c$ (provided $\beta_{recoil} \ll 1$) in random directions due to virtual photon emission with a probability of $(2\alpha / \pi)dk / k$.

2.1.2 Dividing probabilities into the product of two component parts

This probability $(2\alpha / \pi)dk / k$ can be thought of as the product of two terms A & B , where A includes the electromagnetic coupling constant α , B includes dk / k , and $AB = (2\alpha / \pi)dk / k$. This suggests that this same behaviour is possible if we have an appropriate superposition of virtual wavefunctions occurring with probability B , which emits virtual photons with probability A (by changing Eigenvalues $|\mathbf{p}_{nk}| = n\hbar k$ by $n = \pm 1$). For example, if a virtual superposition occurs with probability $B = (\mathbf{N} \cdot \mathbf{s})dk / k$, and has a virtual photon emission probability for each member of these superpositions of $A = (\mathbf{N} \cdot \mathbf{s})^{-1}(2\alpha / \pi)$, then the overall virtual photon emission probability remains as above at $AB = (2\alpha / \pi)dk / k$. This applies equally whether it is virtual gluon/photon/W&Z/graviton etc. emission. Provided A includes the appropriate coupling constant this same logic applies regardless of the type of boson emitted. As is usual to get integral or half integral total angular momentum $2s$ has to be integral and section 6.2 argues that N must also be integral. (This paragraph is simplified to illustrate the principle and will later be modified in section 3.3.)

In section 1.1.4 we said that these wavefunctions are built with squared vector potentials. If superpositions of them are to represent real particles they must be able to exist anywhere. This is possible only if they are generated by uniform fields. The only fields uniform in space-time are the zero point fields, and looking at the electromagnetic field first we can use section 2.1.1 above. Consider a vector \mathbf{r} from some central origin O and a magnetic field vector \mathbf{B} through origin O , then the vector potential at point \mathbf{r} is $\mathbf{A} = (\mathbf{B} \times \mathbf{r}) / 2$ and the vector potential squared is $A^2 = (B^2 r^2 \sin^2 \theta) / 4$ where the angle between vectors \mathbf{B} & \mathbf{r} is θ .

$$\text{As } \sin^2 \theta \text{ averages } 2/3 \text{ over a sphere: } \overline{A^2} = B^2 r^2 / 6 \quad (2.1. 2)$$

Here B^2 is the magnetic field squared at any point due to the cubic intensity of zero point EM also as in section 2.1.1. Putting Eq's. (2.1. 1) & (2.1. 2) together the vector potential squared is

$$\overline{e^2 A^2} = \frac{e^2 B^2 r^2}{6} = \frac{\alpha}{3\pi} \frac{\hbar^2 \omega^4 r^2}{c^4} \frac{d\omega}{\omega} = \frac{\alpha}{3\pi} \hbar^2 k^4 r^2 \frac{dk}{k} \quad (2.1. 3)$$

As in section 2.1.2 we can divide this into two parts, noting the inclusion of spin s and integer N in the numerator and denominator:

$$\overline{e^2 A^2} = \left[\frac{\alpha}{3\pi s N} \hbar^2 k^4 r^2 \right] \cdot \left[\frac{s N \cdot dk}{k} \right] \quad (2.1. 4)$$

But here a vector potential squared term $\left[\frac{\alpha}{3\pi s N} \hbar^2 k^4 r^2 \right]$ occurs with probability $\left[\frac{s N \cdot dk}{k} \right]$.

Another way of looking at this is that a wavefunction ψ_k that is generated by a vector potential squared term $\left[\frac{\alpha}{3\pi s N} \hbar^2 k^4 r^2 \right]$ can occur with $\left[\frac{s N \cdot dk}{k} \right]$ probability.

This is similar reasoning to that used in the semi classical Lamb shift explanation of section 2.1.1. In the first bracketed term of Eq. (2.1. 4), α is the electromagnetic coupling constant, but the same logic applies for the eight gluon and gravitational zero point vector fields where we will sum appropriate amplitudes of these and square this total as our effective coupling constant in Eq. (2.1. 4). But first we need to look at groups of spin zero preons that could build these wavefunctions. What mixtures of colours and electrical charges end up with the appropriate final colour and electrical charge for each of the fundamental particles or at least the ones we know of?

2.2 Spin Zero Virtual Preons from a Higgs type Scalar Field

2.2.1 Groups of eight preons that form superpositions

In this paper preons have zero spin and can have no weak charge. The only fields they can interact with (via Primary Interactions that build superpositions as in section 1.1.2) are colour, electromagnetic and gravity. In the simplest world there would be just one type of preon that comes in three colours, always positively charged say, with their three anti colours all negatively charged. We will assume that this is true unless it does not work. Looking at Table 2.2. 1 we see that a minimum of 6 preons is required to get the correct charge ratios of 3:2:1 between electrons, and up and down quarks. To get vector potential squared values that make all our equations work however, we need to couple to all 8 gluon fields requiring a total of 8 preons. Table 2.2. 1 has all the basic properties required to build infinite superpositions for the fundamental particles. We need to remember when looking at this table that from section 1.1.2 the effective secondary charge is much less than the primary charge and we have no idea yet of just what effective value the primary preon electric charge is.

Particles only are addressed in the groups of preons in Table 2.2. 1. To get anti particles it would seem that we can just change the signs of each preon in the groups of 8, excepting those that are already their own antiparticle. The first point to notice however is that both the electron and the W^- are predominantly anti preons, yet they are both defined as particles. Have we got something wrong? When we look at relativistic masses in section 3.2.1 we get the usual plus and minus solutions and Feynman showed us how to interpret the negative solutions as antiparticles. If this also applies in anti preons then because they are zero spin, and the weak force discriminates between particles and antiparticles by their helicity, this discrimination can apply only in secondary interactions. The preon anti preon content of the groups in Table 2.2. 1 does not necessarily tell us whether they produce particles or antiparticles. We will discuss this further in section 3.2.1, also as of now there is still no good understanding of the predominance of matter over antimatter in our universe. In Table 2.2. 1 only one example of colour is given for quarks and gluons. Different colours can be obtained by simply changing appropriate preon colours. Various combinations of 8 preons in this table are borrowed from a scalar field for time $\Delta T \approx \hbar / \Delta E$, this process continually repeating in time. Conservation of charge normally allows only opposite sign pairs of electric charges to appear out of the vacuum. Let us imagine that these virtual preons are building an electron for example whose electric charge exists continually unless it meets a positron and is annihilated.

Fundamental Particles	Preon colour	Preon electric charge.	Group colour	Group electric charge.
Spin ½ Neutrino family. Spin 1 Photons, Z ₀ & Neutral gluons. Spin 2 Gravitons.	Any colour + its Anticolour Red Antired Green Antigreen Blue Antiblue	1 -1 1 -1 1 -1 1 -1	Colourless	0
Spin ½ Electron family. Spin 1 W ⁻ .	Any colour + its Anticolour Antired Antired Antigreen Antigreen Antiblue Antiblue	1 -1 -1 -1 -1 -1 -1 -1	Colourless	-6
Spin ½ Blue up quark family.	Red Antired Green Antigreen Green Blue Blue Red	1 -1 1 -1 1 1 1 1	Blue	+4
Spin ½ Red down quark family.	Green Antigreen Red Antired Green Antigreen Antiblue Antigreen	1 -1 1 -1 1 -1 -1 -1	Red	-2
Spin 1 Red to green Gluons.	Red Antigreen Red Antired Green Antigreen Blue Antiblue	1 -1 1 -1 1 -1 1 -1	Red plus antigreen	0

Table 2.2. 1 Groups of 8 virtual preons forming the fundamental particles. The Higgs boson is discussed in section 6.5. If it is a superposition it would be in the neutral group at top.

This charged electron is thus due to a continuous appearance out of and back into the vacuum of virtual charged preons in a steady state process existing for the life of the superposition, and not conflicting with conservation of charge. If the electron itself does not conflict then neither do the borrowed preons that build it.

2.2.2 Primary coupling constants behave differently and actually are constant

Q.E.D. tells us that the bare (electric) charge of an electron for example increases logarithmically inversely with radius from its centre. Polarizations of the vacuum (of virtual charged pairs) progressively shield the bare charge from a radius of approximately one Compton radius $\hat{\lambda}_c$ inwards towards the centre. When an electron (for example) is created in some interaction the full bare charge is exposed for an infinitesimal time. Instantaneously after its creation, shielding due to polarization of the vacuum builds progressively outward from the centre of its creation at the velocity of light. For radii $\geq \hat{\lambda}_c$ we measure the usual fundamental charge e . There are similar but more complicated processes that occur to the colour charge. Camouflage is the dominant one where the colour charge grows with radius as the emitted gluons themselves have color charge. At the instant of their birth the preons are bare and at this time $t = 0$ say, all the zero point vector fields can act on these bare colour and electric charges as there is simply no time for shielding and other effects to build. The primary coupling constants that we use must consequently be the same for all values of k in complete contrast to those for secondary interactions. We don't know what this *primary electromagnetic coupling constant* is so we will just call it α_{EMP} . Also we will find that to get any sense out of our equations the *primary colour coupling* has to be very close to 1. A coupling of 1 is a natural number and simply reflects certainty of coupling. Provided the *secondary colour coupling can be in line with the Standard Model* and there does not seem to be any other good reason to pick a number less than 1, we will make the (apparently arbitrary) *assumption that the bare primary colour coupling is exactly 1*. (In section 4.1.1 we will find that this seems to be consistent with the Standard Model.)

2.2.3 Primary interactions also behave differently

Let us define a frame in which the central origin of the wavefunctions ψ_k of our infinite superposition is at rest: The laboratory or rest frame we will refer to as the LF. The preons that build each ψ_k are born from a Higg's type scalar field with zero momentum in this frame. This has very relevant consequences as their *wavelength is infinite* in this rest frame at time $t = 0$, and after they become wavefunction ψ_k their wavelength is of the order $1/k$ for

times $0 < t < \hbar/2E$. This implies that there could possibly be significant differences in the way amplitudes are handled between primary and secondary interactions.

Let us consider secondary interactions first with an electron and positron for example located approximately distance r apart. For photon wavelengths $\ll r$ both the electron and the positron each emit virtual photons with probabilities proportional to α , but for wavelengths $\gg r$ their amplitudes cancel. Returning to primary interactions, zero momentum preons must always have an infinite wavelength which is greater than the wavelengths (or $1/k$ values) of the zero point quanta they interact with, for all $k \neq 0$. This implies that we cannot simply add or subtract amplitudes algebraically as the charged preons can be always further apart than the wavelength of the interacting quanta (except when $k = 0$, but we will see there is always a minimum k value, ie $k_{\min} > 0$ in sections 5 & 0). In fact if algebraic addition of amplitudes did apply in primary interactions, infinite superpositions for colourless and electrically neutral neutrinos would be impossible. So how can infinitely far apart preons of differing charge generate wavefunctions of all dimensions down to Planck scale? This can happen only if the amplitudes of all 8 preons are somehow linked over infinite space, all at the same time $t = 0$ contributing to generating the wavefunction ψ_k . This non-local behaviour is not new. Recent experiments have confirmed that what Einstein struggled to come to terms with is in fact true; he called it “spooky action at a distance”. While these experiments are so far limited in the distance over which they demonstrate entanglement, there is now wide acceptance that it can reach across the Universe. In the same manner wavefunctions covering all space can instantly collapse. We want to suggest that this same non-locality applies in primary interactions: our 8 virtual preons all unite instantaneously at time $t = 0$ across infinite space in generating each ψ_k . Also the vector potential squared equations that they generate must always be the same for all the preon combinations in Table 2.2. 1. This can happen only if the amplitudes of all 8 are added regardless of charge sign for primary interactions. This applies to both colour and electric charge.

The opposite is true for the secondary interactions. At time $t = 0$ all 8 preons instantaneously collapse into some sort of virtual composite particle that for times $0 < t < \hbar/2E$ obeys wavefunction ψ_k . The dimensions of ψ_k are of the same order as the wavelength of the interacting quanta, and the usual algebraic total electric charge and nett colour charge must now apply as in the group charges in Table 2.2. 1. All of this may seem contrary to current thinking which has gradually been built up over several centuries of secondary interaction experiments; however it may not be so out of place when viewed in the context of the counter

intuitive results of entanglement experiments. *The key point to bear in mind is that the predictions of this paper must agree or at least be able to fit the Standard Model, or secondary interaction experiments; as we may never be able to look into virtual primary interactions, but only observe their effects.*

Amplitudes to interact are complex numbers which we can draw as a vector. This applies to both colour and electric coupling, where these two vectors can be at the same complex angle or at different angles. The simplest case is if they are in line and we will assume this is true for both colour and electromagnetic primary interactions which are both spin 1. This seems to work and when we later include gravity, a spin 2 interaction, we find that the spin 2 vector only works if it is at right angles to the two in line spin 1 vectors. *Let us start in a zero gravity world* by simply adding the 8 preon colour vectors of amplitude 1 and the eight primary electromagnetic vectors of amplitude $\sqrt{\alpha_{EMP}}$ together, as all this only works if they are all in line.

$$\text{The total colour plus electromagnetic primary amplitude is } 8 + 8\sqrt{\alpha_{EMP}} \quad (2.2. 1)$$

This equation is always true regardless of signs as in section 2.2.3

$$\text{The colour plus electromagnetic primary coupling constant is } (8 + 8\sqrt{\alpha_{EMP}})^2 \quad (2.2. 2)$$

Inserting this into Eq. (2.1. 4) we get

$$Q^2 A^2 = \left[\frac{[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2 \right] \cdot \left[\frac{sN \cdot dk}{k} \right] \quad (2.2. 3)$$

Again we interpret this just as we did in section 2.1.2 and Eq. (2.1. 4) as a vector potential squared term

$$Q^2 A^2 = \frac{[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2 \text{ occurring with probability } = \frac{sN \cdot dk}{k} \quad (2.2. 4)$$

Where Q is a symbol representing the entire 8 colour and 8 electric amplitudes combined, with s the spin and $N=1$ for massive superpositions, but $N=2$ for infinitesimal mass superpositions. (Table 4.3. 1, section 0 and its subsections cover this more fully.)

2.3 Virtual Wavefunctions that form Infinite Superpositions

2.3.1 Infinite families of similar virtual wavefunctions

Consider the family of wave functions where ignoring time:

$$\begin{aligned}\psi_{nk} &= U(nrk)Y(\theta\phi) \\ U(nrk) &= C_{nk}r^l \exp(-n^2k^2r^2/18)\end{aligned}\quad (2.3.1)$$

$U(nrk)$ is the radial part of ψ_{nk} , $Y(\theta\phi)$ the angular part, C_{nk} a normalizing constant, and we will find that l is the usual angular momentum quantum number. There is an infinite family of ψ_{nk} , one for each value k where $0 < k < \infty$ in a zero gravity world.

$$\text{Now put } R(nrk) = rU(nrk) = C_{nk}r^{l+1} \exp(-n^2k^2r^2/18) \quad (2.3.2)$$

As we are dealing with zero spin preons we use Klein-Gordon equations [9]. The Klein-Gordon equation is based on the relativistic equation $\mathbf{p}^2 = E^2/c^2 - m_0^2c^2$ and in a squared vector potential the Time Independent Klein Gordon Equation is

$$\hat{P}^2\psi = -\hbar^2\nabla^2\psi + Q^2A^2\psi = \left[\frac{E^2}{c^2} - m_0^2c^2 \right] \psi \quad (2.3.3)$$

Using $\frac{\nabla^2\psi}{\psi} = \frac{1}{R} \frac{\partial^2 R}{\partial r^2} - \frac{l(l+1)}{r^2}$ we get the Time Independent

$$\text{Radial Klein Gordon Equation } \frac{\hbar^2}{R} \frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + Q^2A^2 - \left[\frac{E^2}{c^2} - m_0^2c^2 \right] \quad (2.3.4)$$

For each ψ_{nk} the energy is E_{nk} a function of n & k , and we will label the rest mass as m_{0snk} a function of spin s , n & k , but also a function of the particle rest mass m_0 and this becomes

$$\frac{\hbar^2}{R} \frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + Q^2A^2 - \left[\frac{E_{nk}^2}{c^2} - m_{0snk}^2c^2 \right] \quad (2.3.5)$$

Differentiating $R(nrk) = rU(nrk) = C_{nk} r^{l+1} \exp\left(\frac{-n^2 k^2 r^2}{18}\right)$ twice with respect to r , multiplying

by \hbar^2 and dividing by R

$$\frac{\hbar^2}{R} \frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + \frac{n^4 \hbar^2 k^4 r^2}{81} - \frac{(2l+3)n^2 \hbar^2 k^2}{9} \quad (2.3. 6)$$

Comparing Eq's. (2.3. 5) & (2.3. 6) we see that l is the usual angular momentum quantum number and the vector potential squared required to generate these wavefunctions is

$$Q^2 A^2 = \frac{n^4 \hbar^2 k^4 r^2}{81} = \left[\frac{n}{3}\right]^4 \hbar^2 k^4 r^2 \quad (2.3. 7)$$

$$\text{The momentum squared is } \mathbf{p}_{nk}^2 = \frac{E_{nk}^2}{c^2} - m_{0snk}^2 c^2 = \frac{(2l+3)n^2 \hbar^2 k^2}{9} \quad (2.3. 8)$$

$$\text{For } l = 3 \text{ wavefunctions this becomes } \mathbf{p}_{nk}^2 = n^2 \hbar^2 k^2 \text{ \& } |\mathbf{p}_{nk}| = n\hbar k \quad (2.3. 9)$$

2.3.2 Eigenvalues of these virtual wavefunctions and parallel momentum vectors

From Eq.'s (2.3. 8) & (2.3. 9) as $k \rightarrow \infty$, the energy squared $E_{nk}^2 \rightarrow \mathbf{p}_{nk}^2 c^2 = n^2 \hbar^2 \omega^2$ and thus

$$\text{If } l = 3 \text{ when } k \rightarrow \infty \text{ energy } E_{nk} \rightarrow n\hbar\omega \text{ (considering only the positive solution)}. \quad (2.3. 10)$$

This suggests that n must be integral. If it is integral when $k \rightarrow \infty$ we will conjecture that it must be integral for all values of k . (This is a virtual process where the energy exchanged does not need to obey $E_x^2 = m_{0x}^2 c^4 + \mathbf{p}_x^2 c^2$ and $E_x^2 < \mathbf{p}_{nk}^2 c^2$ or $E_x^2 < n^2 \hbar^2 \omega^2$ when $k \ll \infty$.)

We can also perhaps think of Eq.(2.3. 9) as integral n parallel momentum vector $|\mathbf{p}| = \hbar k$ quanta, transferring total momentum $|\mathbf{p}_{nk}| = n\hbar k$ and energy $E_x \leq n\hbar\omega$ from the zero point fields to generate the virtual wavefunction ψ_{nk} . Thus provided $Q^2 A^2 = (n/3)^4 \hbar^2 k^4 r^2$ as in Eq. (2.3. 7) the operator $\hat{P}^2 = (-\hbar^2 \nabla^2 + Q^2 A^2)$ applied to the virtual wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta\phi)$ produces $\hat{P}^2 \psi_{nk} = -\hbar^2 \nabla^2 \psi_{nk} + Q^2 A^2 \psi_{nk} = n^2 \hbar^2 k^2 \psi_{nk}$, where n is integral but k is continuous as for free particles. Thus we conjecture that:

$$\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta\phi) \text{ are Eigenfunctions with} \quad (2.3. 11)$$

Eigenvalues $\mathbf{p}_{nk}^2 = n^2 \hbar^2 k^2$ with continuous k but integral n .

Also there are no scalar potentials involved, only squared vector potentials, so this is a magnetic or vector type interaction. Particles in classical magnetic fields have a constant magnitude of linear momentum which is consistent with the squared momentum Eigenvalues of Eq. (2.3. 11). This also implies that each ψ_{nk} is formed from quanta of wave number k only and that secondary interactions with ψ_{nk} emit or absorb $|\hbar k|$ virtual quanta if n changes by ± 1 . The wavefunction ψ_{nk} is virtual and in this sense both the energy E_{nk} and rest mass $m_{0_{snk}}$ in Eq. (2.3. 8) are also virtual quantities borrowed from zero point vector fields and a scalar Higgs type field. We use these virtual quantities to calculate the amplitude that ψ_{nk} is in an m state of angular momentum in section 3.1, and in section 3.2 to calculate the total angular momentum and rest mass. As in section 2.3.2 above, we can think of $|\mathbf{p}_{nk}| = n\hbar k$ as n parallel momentum vectors $|\mathbf{p}| = \hbar k$. As spin 3 (or $l=3$) needs at least 3 spin 1 quanta to build it n must be at least 3. When $n=3$ we can think of this as 3 of the 8 preons each absorbing quanta $|\hbar k|$ at time $t=0$. We will find that a spin $1/2$ state has a dominant $n=6$ Eigenfunction where 6 of the 8 preons each absorb quanta $|\hbar k|$. It needs at least two smaller side Eigenfunctions $n=5$ & $n=7$ with either 5 or 7 of the 8 preons each absorbing quanta $|\hbar k|$ respectively at $t=0$. (Figure 3.1. 4 illustrates the three n modes of a positron superposition.)

From Eq. (2.3. 7) $Q^2 A^2 = \left[\frac{n}{3}\right]^4 \hbar^2 k^4 r^2 = 16\hbar^2 k^4 r^2$ for this dominant $n=6$ mode.

Thus using Eq. (2.2. 4) $Q^2 A^2 = \frac{[8+8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2 = 16\hbar^2 k^4 r^2$ for an $n=6$ mode.

Now $s=1/2$ & $N=1$ for spin $1/2$ fermions and $\frac{2[8+8\sqrt{\alpha_{EMP}}]^2}{3\pi} = 16$ if we have only an $n=6$ mode.

Thus $8+8\sqrt{\alpha_{EMP}} = \sqrt{24\pi}$ and $\alpha_{EMP}^{-1} \approx 137.1$, but this is true for an $n=6$ Eigenfunction only, and we have a superposition where the amplitudes of the smaller side Eigenfunctions $n=5$ & $n=7$ determine the ratio between the primary to secondary (colour and electromagnetic) coupling amplitudes or the value of α_3^{-1} @ k_{cutoff} (Section 3.3). The $Q^2 A^2$ required to produce this superposition with amplitudes c_n is, using Eq. (2.3. 7)

$$Q^2 A^2 = \sum_{n=5,6,7} c_n^* c_n \frac{n^4 \hbar^2 k^4 r^2}{81} \quad (2.3. 12)$$

Repeating the same procedure as above for three member superpositions using Eq. (2.3. 12) we find the strength of α_{EMP} required increases considerably (see section 4.1 & Table 4.1. 1.) As the secondary electromagnetic coupling $\alpha_{EMS}^{-1} @ k_{cutoff}$ must be constant for all spin $\frac{1}{2}$ leptons and quarks the amplitudes of the smaller side Eigenfunctions $n=5$ & $n=7$ that determine this must also be constant for all the fermions, implying that Eq. (2.3. 12) must be the same for all fermions. The same arguments apply to the other groups of fundamental particles but we return to this in sections 3.3 where we see that the same also applies with graviton emission.

3 Properties of Infinite Superpositions

3.1 What is the Amplitude that ψ_{nk} is in an m state?

3.1.1 Four vector transformations

The rules of quantum mechanics tell us that if we carry out any measurement on a real spherically symmetric $l=3$ wavefunction it will immediately fall into one of the seven possible states $l=3, m=0, \pm 1, \pm 2, \pm 3$ [10]. But ψ_{nk} is a virtual $l=3$ wave function so we cannot measure its angular momentum. During its brief existence it must always remain in some virtual superposition of the above seven possible states and we can describe only the amplitudes of these. So is there any way to calculate these amplitudes as they must relate to the amplitudes of the angular momentum states of the spin 1 quanta it absorbs from the zero point vector fields? First consider the 4 vector wavefunction of a spin 1 particle and start with a time polarized state which has equal probability of polarization directions. It is thus spherically symmetric, which we will label as ss . Using 4 vector (t, x, y, z) notation:

In frame A, a time polarized or ss spin 1 state is (1,0,0,0).

Let frame B move along the z axis at velocity $\beta = v/c$ in the z direction.

In frame B the polarization state transforms to $(\gamma, 0, 0, \gamma\beta)$.

But this is γ^2 time polarized (ss states) minus $\gamma^2\beta^2 \times z$ polarized ($m=0$ states).

In frame B there are $\gamma^2 \times ss$ states $-\gamma^2\beta^2 \times m=0$ states.

Now $\gamma^2 - \gamma^2\beta^2 = \gamma^2(1 - \beta^2) = 1$ is an invariant probability in all frames and in removing $\gamma^2\beta^2 \times (m=0)$ states from $\gamma^2 ss$ states, the new ratio of spherical symmetry is

$(\gamma^2 - \gamma^2 \beta^2) / \gamma^2 = 1 - \beta^2$. Thus a spherically symmetric state is transformed from probability 1 in frame A, to $1 - \beta^2$ in frame B. Also removing $m = 0$ states from spherically symmetric states leaves a surplus of $m = \pm 1$ states, as spherically symmetric states are equal superpositions of $m = -1$, $m = 0$, & $m = +1$ states.

Thus in Frame B the probabilities are $(1 - \beta^2) \times ss$ states + $\beta^2 \times m = \pm 1$ states. (3.1. 1)

We can describe this as a virtual superposition of $(\frac{1}{\gamma} \times ss, +\beta \times m = \pm 1)$ states. (3.1. 2)

As $\beta^2 \rightarrow 1$ we have transverse polarized states, the same as real photons. Now transverse polarized spin 1 states can be either left ($m = -1$), or right ($m = +1$) circular polarization, or equal superpositions of $(1/\sqrt{2})L + (1/\sqrt{2})R$ as in x & y polarization. If we think of individual spin zero preons absorbing these spin 1 quanta at $t = 0$ they must also have this same β^2 probability of transversely polarized spin 1 states. If they then merge into some composite $l = 3$ particle (as in Figure 3.1. 4) for time $0 < t < \hbar / 2E$, the probability of it being in some particular state ($l = 3, m = 0$), ($l = 3, m = \pm 1$), ($l = 3, m = \pm 2$) or ($l = 3, m = \pm 3$), must be the same β^2 . If we look at Eq.'s (1.1. 1) we can see what is behind them. We initially write the amplitudes in these three equations in terms of β_{nk} & γ_{nk} as this is the most convenient way to express them. Velocity operators are momentum operators over relativistic masses. Our Eigenvalues are $\mathbf{p}_{nk}^2 = n^2 \hbar^2 k^2$ for each n & k , and this allows the velocity operators to give constant β_{nk}^2 . Later in Eq's. (3.1. 11) & (3.1. 12) we write β_{nk} & γ_{nk} in momentum terms. Even though the mass in these operators is virtual, we can still use it to calculate $|\beta_{nk}|$. For each k and integral n there will be a constant $|\beta_{nk}|$ and $\gamma_{nk} = (1 - \beta_{nk}^2)^{-1/2}$. As we will see, β_{nk} can be thought of as the magnitude of the velocity of an imaginary centre of momentum frame in which these interactions take place. We will also draw our Feynman diagrams of these interactions in terms of β_{nk} & γ_{nk} for convenience, even though this is unconventional. To proceed from here we define two frames as follows:

1) The Laboratory Frame (LF) or Fixed Frame as in section 2.2.3

The infinite superposition has rest mass m_0 and zero nett momentum in this frame. Each ψ_{nk} is centered here with magnitude of momentum $|\mathbf{p}_{nk}| = n\hbar k$. Even though we have no idea of the direction of this momentum vector we will define it as the z direction. The eight preons are born in this frame with zero momentum, and can thus be considered here as being at rest or

with zero velocity and *infinite wavelength at their birth*. The Feynman diagram of the interaction in this frame that builds ψ_{nk} is illustrated in Figure 3.1. 3.

2) The Center of Momentum Frame (CMF)

This (imaginary) frame is the center of momentum of the interaction that builds ψ_{nk} . The CMF moves at velocity β_{nk} relative to the laboratory frame in the z direction or parallel to the unknown momentum vector direction \mathbf{p}_{nk} . In this CMF the momenta and velocities of the preons at birth and after the interaction are equal and opposite. This is illustrated in Figure 3.1. 2 again in terms of $m_0, \beta_{nk}, \& \gamma_{nk}$. In the LF the velocity of the preons at birth is zero, in the CMF this is $-\beta_{nk}$ and after the interaction $+\beta_{nk}$, where both $-\beta_{nk}$ and $+\beta_{nk}$ are in the unknown z direction. In the LF the particle velocity $\beta_{nk}(\text{particle}) = \beta_{nkP}$ is the simple relativistic addition of the two equal velocities β_{nk} as in Figure 3.1. 1.

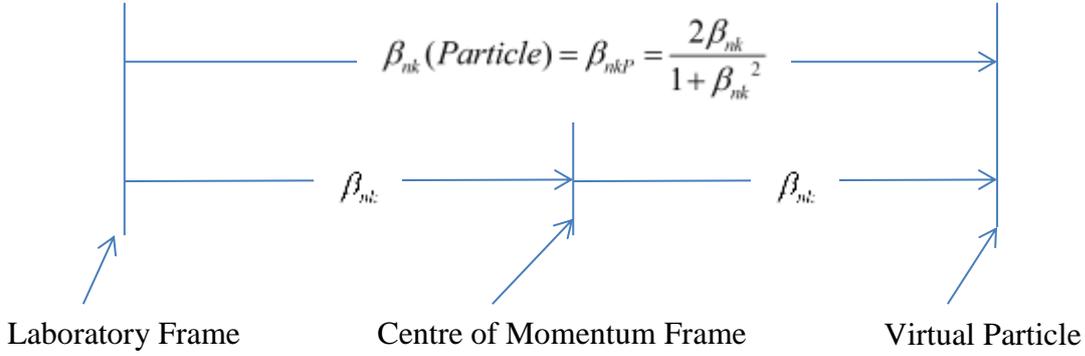


Figure 3.1. 1

3.1.2 Feynman diagrams of primary interactions

Let us start with

$$\beta_{nk}(\text{Particle}) = \beta_{nkP} = \frac{2\beta_{nk}}{1 + \beta_{nk}^2} \text{ and } \gamma_{nkP} = (1 - \beta_{nkP}^2)^{-1/2} = \gamma_{nk}^2(1 + \beta_{nk}^2) \quad (3.1. 3)$$

If the particle rest mass is m_0 let each preon have a virtual rest mass $m_0 / (8\gamma_{nk} \sqrt{2s})$.

$$\text{The eight preons are effectively a virtual particle of rest mass } m_{0snk} = \frac{m_0}{\gamma_{nk} \sqrt{2s}} \quad (3.1. 4)$$

The particle momentum in the LF is zero at birth. After the interaction using these equations

$$|\mathbf{p}_{nk}| = n\hbar k = m_{0snk} \beta_{nkP} \gamma_{nkP} c = \left[\frac{m_0}{\gamma_{nk} \sqrt{2s}} \right] \left[\frac{2\beta_{nk}}{1 + \beta_{nk}^2} \right] \left[\gamma_{nk}^2 (1 + \beta_{nk}^2) \right] c$$

(3.1. 5)

The particle momentum after the interaction in the LF $|\mathbf{p}_{nk}| = n\hbar k = \frac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}}$

Using Eq. (3.1. 4), in the LF the particle energy at birth is

$$m_{0snk}c^2 = \frac{m_0c^2}{\gamma_{nk}\sqrt{2s}} \quad (3.1. 6)$$

In the LF the particle energy after the interaction is using Eq's. (3.1. 3)

$$m_{0snk}\gamma_{pnk}c^2 = \frac{m_0}{\gamma_{nk}\sqrt{2s}}\gamma_{nk}^2(1+\beta_{nk}^2)c^2 = \frac{m_0\gamma_{nk}}{\sqrt{2s}}(1+\beta_{nk}^2)c^2 \quad (3.1. 7)$$

In the CMF the momentum at birth is using Eq. (3.1. 4)

$$-m_{0snk}\gamma_{nk}\beta_{nk} = \frac{-m_0\beta_{nk}}{\sqrt{2s}} \quad (3.1. 8)$$

In the CMF the momentum after the interaction is equal but in the opposite direction

$$= \frac{+m_0\beta_{nk}}{\sqrt{2s}} \quad (3.1. 9)$$

In the CMF the energy at birth, and after the interaction is

$$m_{0snk}\gamma_{nk}c^2 = \frac{m_0c^2}{\sqrt{2s}} \quad (3.1. 10)$$

These values are all summarized in Figure 3.1. 2 and Figure 3.1. 3 but with $c = 1$.

$$\text{From Eq. (3.1. 5)} \quad |\mathbf{p}_{nk}| = n\hbar k = \frac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}} \quad \text{and} \quad \beta_{nk}\gamma_{nk} = \frac{n\hbar k\sqrt{2s}}{2m_0c} = \frac{\tilde{\lambda}_c nk\sqrt{2s}}{2}$$

(where $\tilde{\lambda}_c$ is the Compton wavelength). We can now express β_{nk} & γ_{nk} in momentum terms:

$$\text{Let } K_{nk} = \beta_{nk}\gamma_{nk} = \frac{n\hbar k\sqrt{2s}}{2m_0c} = \frac{\tilde{\lambda}_c nk\sqrt{2s}}{2} \quad (3.1. 11)$$

$$\text{In terms of } K_{nk}: \quad \beta_{nk}^2 = \frac{K_{nk}^2}{1+K_{nk}^2} \quad \text{and} \quad \gamma_{nk}^2 = 1+K_{nk}^2 \quad (3.1. 12)$$

Each infinite superposition has fixed $\tilde{\lambda}_c$. Each wavefunction ψ_{nk} of this infinite superposition has fixed n & s , thus $K_{nk} \propto k$.

$$\text{For example we can put} \quad \frac{dK_{nk}}{K_{nk}} = \frac{dk}{k} \quad (3.1. 13)$$

These simple expressions and what follows are not possible if $m_{0snk} \neq m_0 / \gamma_{nk} \sqrt{2s}$, and when we include gravity we find $m_{0snk} = m_0 / (\gamma_{nk} \sqrt{2s})$ is essential (section 4.2).

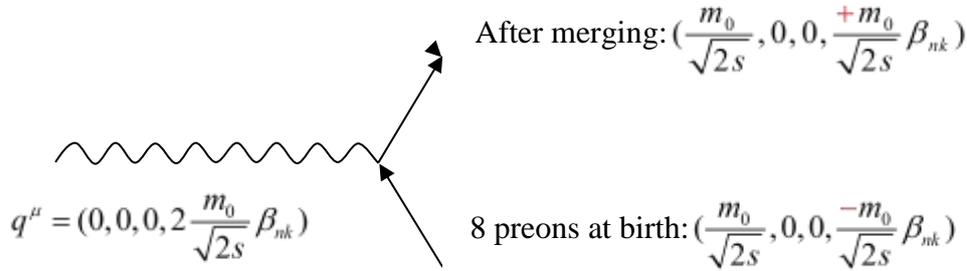


Figure 3.1. 2 Feynman diagram in an imaginary centre of momentum frame.

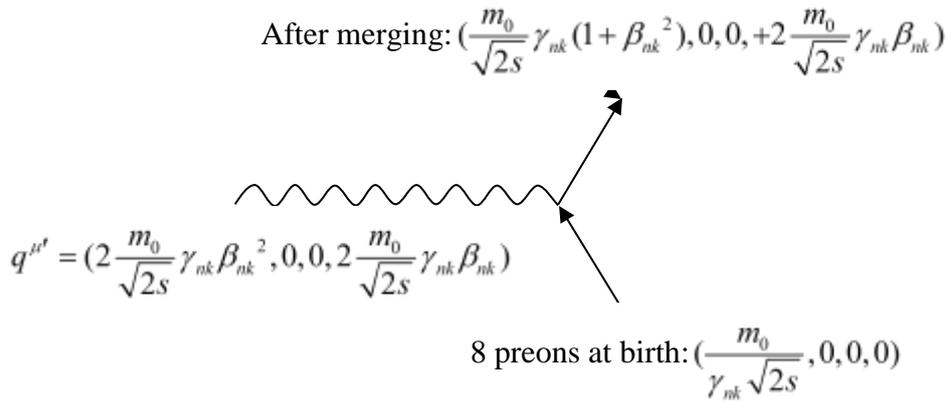


Figure 3.1. 3 Feynman diagram in the laboratory frame.

The interaction in the Feynman diagrams above is with spin 1 quanta. The Feynman transition amplitude of this interaction tells us that the polarization states of these exchanged quanta is determined by the sum of the components of the initial, plus final 4 momentum $(p_i + p_f)^\mu$. Ignoring all other common factors this tells us that the space polarized component is the sum of the momentum terms $(\mathbf{p}_i + \mathbf{p}_f)$ and the time polarized component is the sum of the energy terms $(p_i + p_f)^0$. We have defined our momentum as in an unknown z direction:

The ratio of z polarization to time polarization amplitudes is
$$\frac{(p_i + p_f)^z}{(p_i + p_f)^0} \tag{3.1. 14}$$

In the CMF $(p_i + p_f)^z = 0$, thus an interaction in the CMF exchanges only time polarized, or spherically symmetric $l = 1$ states. In the LF the ratio of z (or $m = 0$) polarization, to time polarization in the LF is β_{nk}^2 ,

$$\text{where } \frac{(p_i + p_f)^z}{(p_i + p_f)^0} = \frac{2m_0\gamma_{nk}\beta_{nk}}{2m_0\gamma_{nk}} = \beta_{nk} \quad (3.1.15)$$

From section 3.1.1 these are probabilities of $\gamma_{nk}^2 ss - \gamma_{nk}^2 \beta_{nk}^2 \times (m = 0)$ states, or $(1 - \beta_{nk}^2) ss + \beta_{nk}^2 \times (l = 1, m = \pm 1)$ states.

In the LF this is a virtual superposition of $(\frac{1}{\gamma_{nk}} \times ss, +\beta_{nk} \times m = \pm 1)$ states. (3.1.16)

From section 3.1.1 as these quanta from the scalar and vector zero point fields build each ψ_{nk} this implies that:

In the LF ψ_{nk} has virtual superposition amplitudes $(\frac{1}{\gamma_{nk}} \times ss, +\beta_{nk} \times m \text{ states})$ (3.1.17)

From section 3.1.1 appropriate $l = 1, m = \pm 1$ superpositions can build any $l = 3, m$ state. Figure 3.1. 4 is an example of such a ψ_{nk} for $n = 5, 6, \& 7$ ($l = 3, m = +2$) states.

3.1.3 Different ways to express superpositions

We have expressed all superpositions here in terms of spherically symmetric and m states for convenience and simplicity. We could have expressed them in the form:

$$\frac{1}{\gamma_{nk}\sqrt{7}} [(m = -3), (m = -2), (m = -1), (m = 0), (m = +1), (m = +2), (m = +3)] + \beta_{nk} (m = +2)$$

This is equivalent to (ignoring complex number amplitude factors for clarity)

$$\psi_{nk} = \frac{1}{\gamma_{nk}} \times ss, +\beta_{nk} \times (m = +2) \text{ where we have put } m = +2 \text{ for example.}$$

Because all these wavefunctions are virtual they cannot be measured in the normal way that collapses them into any of these Eigenstates, it is more convenient to use the method adopted here which is similar to QED virtual photons superpositions.

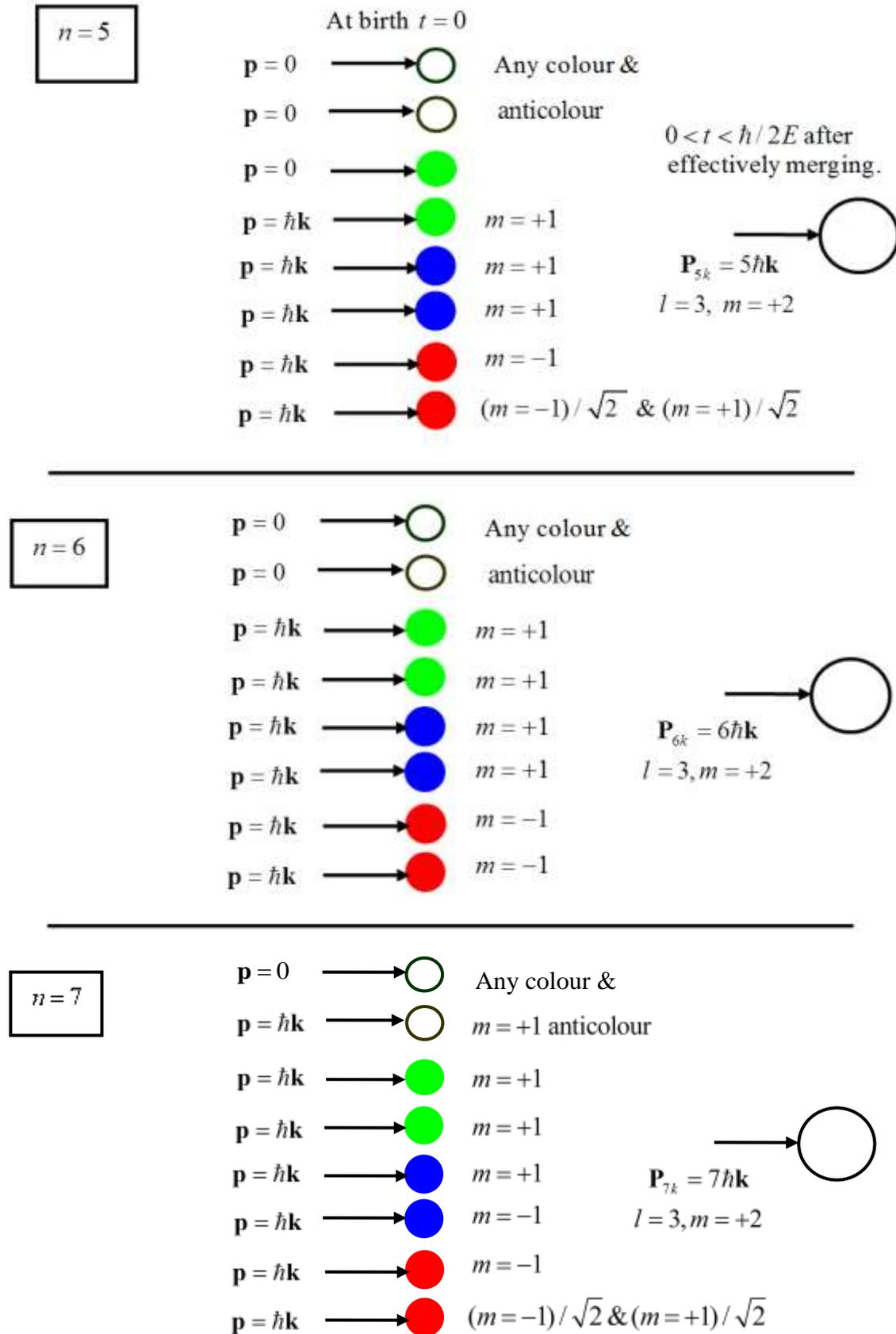


Figure 3.1. 4 Eight preons forming $m = +2$ states as part of a positron superposition.

There is no significance in which preons absorb quanta in the above.

3.2 Mass and Total Angular Momentum of Infinite Superpositions

3.2.1 Total mass of massive infinite superpositions

We will consider first the total mass of an infinite superposition, and to help illustrate, consider only one integral n Eigenfunction ψ_{nk} at a time; temporarily assuming that the amplitude c_n of each ψ_{nk} has magnitude $|c_n| = 1$. Each time ψ_{nk} is born it borrows virtual mass from a scalar Higgs field and virtual energy from vector zero point fields. Each time ψ_{nk} is born the virtual mass that it borrows is exactly cancelled by an equal debt in the Higgs scalar field so this should sum to zero for all k . But what about the momenta borrowed from the zero point fields, do these momenta also leave momentum debts in the vacuum? From section 2.3.2 as $k \rightarrow \infty$, $E_{nk}^2 \rightarrow \mathbf{p}_{nk}^2 c^2 = n^2 \hbar^2 \omega^2$ or $E_{nk} \rightarrow n \hbar \omega$ and n quanta of energy $\hbar \omega$ and momentum $|\hbar k|$ are absorbed. We know that in some unknown direction $\mathbf{p}_{nk} = n \hbar \mathbf{k}$, which implies these n absorbed quanta must leave a cancelling debt in the opposite direction of $\mathbf{p}_{nk} (debt) = -n \hbar \mathbf{k}$ in the vacuum. But this is true only as $k \rightarrow \infty$ & $\beta_{nk}^2 \rightarrow 1$ and the virtual quanta energy transferred $E_X \rightarrow \hbar \omega$. So what happens when $\beta_{nk}^2 \ll 1$? Our wavefunctions ψ_{nk} are generated from a vector potential squared term A^2 derived in section 2.1.2 which in turn came from a B^2 type term as in section 2.1.1. As discussed in section 2.3.2 the Eigenvalues $\mathbf{p}_{nk}^2 = n^2 \hbar^2 k^2$ confirm the constant momentum squared feature of magnetic type interactions. Also in section 2.1.1 the scalar virtual photon emission probability is directly related to the force squared term $F^2 = \varepsilon^2 E^2$. Magnetic type coupling probabilities are related to a magnetic type force squared term $F^2 = \beta^2 \varepsilon^2 B^2 / c^2 = \beta^2 \varepsilon^2 E^2$, where from section 3.1.2 and Eq's. (3.1. 14) & (3.1. 15) the ratio of this scalar to magnetic coupling is β_{nk}^2 . Thus when $k < \infty$ and the exchanged energy $E_X \neq \hbar \omega$, $\beta_{nk}^2 n$ quanta $|\hbar k|$ are absorbed from the vacuum and:

$$\text{We can expect a momentum debt of } \mathbf{p}_{nk} (debt) = -\beta_{nk}^2 n \hbar \mathbf{k} \quad (3.2. 1)$$

We could sum $\sum \mathbf{p}_{nk}^2$ & $\sum \mathbf{p}_{nk}^2 (debt)$ but both vectors \mathbf{p}_{nk} and $\mathbf{p}_{nk} (debt)$ are antiparallel in the same unknown direction. We can pair them together giving a nett momentum per pair of:

$$\mathbf{p}_{nk} (nett) = \mathbf{p}_{nk} + \mathbf{p}_{nk} (debt) = (1 - \beta_{nk}^2) n \hbar \mathbf{k} = \frac{n \hbar \mathbf{k}}{\gamma_{nk}^2} = \frac{\mathbf{p}_{nk}}{\gamma_{nk}^2} \text{ at wavenumber } k. \quad (3.2. 2)$$

We have said above that the mass of each virtual particle is cancelled by an equal and opposite debt in the Higgs scalar field so we can now use the relativistic energy expression

$$E_n^2 = \sum_{k=0}^{k=\infty} \mathbf{p}_{nk} (nett)^2 c^2 \text{ times the probability of each pair at each wavenumber } k.$$

We will initially look at only $N = 1$ massive infinite superpositions in Eq. (2.2. 4).

Thus using probability $sN \cdot dk / k = s \cdot dk / k$, also Eq's. (3.1. 11), (3.1. 12),(3.1. 13),&(3.2. 2).

$$E_n^2 = c^2 \int_{k=0}^{k=\infty} \mathbf{p}_{nk} (nett)^2 \frac{s \cdot dk}{k} = c^2 \int_0^{\infty} \frac{n^2 \hbar^2 k^2}{\gamma_{nk}^4} \frac{s \cdot dk}{k} = 4m_0^2 c^4 \int_0^{\infty} \frac{K_{nk}^2}{(1 + K_{nk}^2)^2} \frac{dK_{nk}}{2K_{nk}}$$

$$E_n^2 = m_0^2 c^4 \left[\frac{-1}{1 + K_{nk}^2} \right]_0^{\infty} = m_0^2 c^4 \text{ or } E_n = \pm m_0 c^2 \quad (3.2. 3)$$

This energy is due to summing momenta squared and it must be real, with a mass $\pm m_0$ for infinite superpositions of Eigenfunctions ψ_{nk} . These superpositions can form all the non infinitesimal mass fundamental particles. The equations do not work if the mass m_0 is zero. (We will look at infinitesimal masses in section 6.2.) Negative mass solutions in Eq. (3.2. 3) must be handled in the usual Feynman manner, and treated as antiparticles with positive energy going backwards in time. If they are spin $1/2$ this also determines how they interact with the weak force.

3.2.2 Angular momentum of massive infinite superpositions

We will use the same procedure for the total angular momentum of $N = 1$ type infinite superpositions with non infinitesimal mass in Eq. (2.2. 4).

Wavefunctions $\psi_{nk} = C_{nk} r^3 \exp(-n^2 k^2 r^2 / 18) Y(\theta, \varphi)$ have angular momentum squared Eigenvalues $\mathbf{L}^2 = 12\hbar^2$ and the various m states have angular momentum Eigenvalues $\mathbf{L}_z = m\hbar$. We will treat both angular momentum and angular momentum debts as real just as we did for linear momentum. Even though m state wavefunctions are part of superpositions they still have probabilities just as the linear momenta squared above and it seemed to work. Using exactly the same arguments as in section 3.2.1, if ψ_{nk} is in a state of angular momentum $\mathbf{L}_{zk} = m\hbar$, then it must leave an angular momentum debt in the vacuum of $\mathbf{L}_{zk}(\text{debt}) = -\beta_{nk}^2 m\hbar$ (or as in section 3.2.1) $\mathbf{L}_{zk}(\text{nett}) = \mathbf{L}_{zk} - \mathbf{L}_{zk}(\text{debt})$.

$$\mathbf{L}_{zk}(\text{nett}) = (1 - \beta_{nk}^2) m\hbar = (1 - \beta_{nk}^2) \mathbf{L}_{zk} = \frac{\mathbf{L}_{zk}}{\gamma_{nk}^2} \quad (\text{if } \mathbf{L}_{zk} \text{ is in state } m\hbar) \quad (3.2. 4)$$

But from Eq. (3.1. 17) the probability that \mathbf{L}_{zk} is in an m state is also β_{nk}^2 so that

$$\text{Including this extra } \beta_{nk}^2 \text{ probability term: } \mathbf{L}_{zk}(\text{nett}) = m\hbar \frac{\beta_{nk}^2}{\gamma_{nk}^2} \text{ at wavenumber } k. \quad (3.2. 5)$$

For an $N = 1$ type infinite superposition $\mathbf{L}_z(Total) = \int_{k=0}^{k=\infty} \mathbf{L}_{zk}(nett) \frac{s \cdot dk}{k} = sm\hbar \int_0^{\infty} \frac{\beta_{nk}^2}{\gamma_{nk}^2} \frac{dk}{2k}$

Using Eq's. (3.1. 11) to (3.1. 13) $\mathbf{L}_z(Total) = sm\hbar \int_0^{\infty} \frac{K_{nk}^2}{(1 + K_{nk}^2)^2} \frac{dK_{nk}}{K_{nk}} = \frac{sm\hbar}{2} \left[\frac{-1}{1 + K_{nk}^2} \right]_0^{\infty}$

$$\mathbf{L}_z(Total) = m'\hbar = \frac{sm\hbar}{2} \quad \text{or} \quad m' = \frac{s}{2}m \quad (3.2. 6)$$

Where m' is the angular momentum state of the infinite superposition and m the state of ψ_{nk} . Thus for spin $\frac{1}{2}$ particles with $s = \frac{1}{2}$ in Eq.(3.2. 6) $m' = m/4$ but m' can be only $\pm \frac{1}{2}$, implying the m state of ψ_{nk} that generates spin $\frac{1}{2}$ must be $m = \pm 2$. An $N = 1$ massive spin 1 particle has $s = 1$ with $m' = m/2$. ($N = 2$ is covered in section 6.2.) This is summarized in the following three member infinite superpositions *ignoring complex number factors*.

$$\text{Massive } (N = 1) \text{ Spin } \frac{1}{2}, \quad \psi_{\infty, 1/2, \pm 1/2} = \sum_{n=5,6,7} c_n \int_{k=0}^{k=\infty} \left[\frac{1}{\gamma_{nk}} (\psi_{nk,ss}) + \beta_{nk} (\psi_{nk, \pm 2}) \right] \sqrt{\frac{1}{2k}} dk \quad (3.2. 7)$$

$$\text{Massive } (N = 1) \text{ Spin } 1, \quad \psi_{\infty, 1, m} = \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{1}{\gamma_{nk}} (\psi_{nk,ss}) + \beta_{nk} (\psi_{nk, 2m}) \right] \sqrt{\frac{1}{k}} dk \quad (3.2. 8)$$

The spin vectors of each ψ_{nk} with $|\mathbf{L}| = 2\sqrt{3}\hbar$, and their spin vector debts in the zero point vector fields, have to be aligned such that the sum in each case is the correct value: $|\mathbf{L}| = \sqrt{3}\hbar/2$, $|\mathbf{L}| = \sqrt{2}\hbar$ or $|\mathbf{L}| = \sqrt{6}\hbar$ for spins $\frac{1}{2}$, 1 & 2 respectively. Gravity (the ε term) is included in Eq. (1.1. 1) in our summary also spin 1 in Eq. (3.2. 8) is for $N = 1$.

Spherically symmetric massive $N = 1$ spin 1 states are superpositions of the three states

$$\frac{1}{\sqrt{3}}(m' = -1), \frac{1}{\sqrt{3}}(m' = 0), \frac{1}{\sqrt{3}}(m' = +1), \text{ and using Eq. (3.2. 8) can be formed as follows}$$

$$\text{Massive spin 1} \left[\begin{array}{l} \frac{1}{\sqrt{3}}\psi_{\infty, 1, m'=-1} = \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{1}{\gamma_{nk}} (\psi_{nk,ss}) + \beta_{nk} (\psi_{nk, m=-2}) \right] \sqrt{\frac{1}{k}} dk \\ + \frac{1}{\sqrt{3}}\psi_{\infty, 1, m'=0} = \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{1}{\gamma_{nk}} (\psi_{nk,ss}) + \beta_{nk} (\psi_{nk, m=0}) \right] \sqrt{\frac{1}{k}} dk \\ + \frac{1}{\sqrt{3}}\psi_{\infty, 1, m'=+1} = \frac{1}{\sqrt{3}} \sum_{n=4,5,6} c_n \int_{k=0}^{k=\infty} \left[\frac{1}{\gamma_{nk}} (\psi_{nk,ss}) + \beta_{nk} (\psi_{nk, m=+2}) \right] \sqrt{\frac{1}{k}} dk \end{array} \right] \quad (3.2. 9)$$

3.2.3 Mass and angular momentum of multiple integer n superpositions

In sections 3.2.1 & 3.2.2 for simplicity we looked at single integer n superpositions ψ_{nk} . For superpositions $\psi_k = \sum_n c_n \psi_{nk}$, we replace K_{nk}^2 with $\langle K_k \rangle^2$. Equation (2.3. 9) appears to suggest $|\mathbf{p}_k|^2 = \sum_n c_n^* c_n n^2 \hbar^2 k^2 = \langle n^2 \rangle \hbar^2 k^2$ and $\langle |\mathbf{p}_k| \rangle = \hbar k \sqrt{\langle n^2 \rangle}$. In section (3.5.1) we discuss why $\langle |\mathbf{p}_k| \rangle \neq \hbar k \sqrt{\langle n^2 \rangle}$ but $\langle |\mathbf{p}_k| \rangle = \hbar k \sum_n c_n^* c_n \cdot n = \hbar k \langle n \rangle$. Thus using Eq. (3.1. 11)

$$\langle K_k \rangle = \frac{\tilde{\lambda}_c k \sqrt{2s}}{2} \langle n \rangle \quad \& \quad \langle K_k \rangle^2 = \frac{\tilde{\lambda}_c^2 k^2 s}{2} \langle n \rangle^2 \quad \text{but} \quad \langle K_k \rangle^2 \neq \frac{\tilde{\lambda}_c^2 k^2 s}{2} \langle n^2 \rangle \quad (3.2. 10)$$

Replacing K_{nk}^2 with $\langle K_k \rangle^2 = \tilde{\lambda}_c^2 k^2 s \langle n \rangle^2 / 2$ in the key equations (3.2. 3) & (3.2. 6) does not change the final results. The laws of quantum mechanics tell us the total angular momentum is precisely integral \hbar or half integral $\hbar/2$. Looking at the above integrals used to derive total angular momentum we see that N must be 1 (we discuss $N=2$ in section 6.2) also s must be exactly $1/2$ or 1 for spin $1/2$ & spin 1 massive particles respectively, in Eq. (2.2. 4) our probability formula. Also these integrals are infinite sums of positive and negative integral \hbar that are virtual and cannot be observed. If an infinite superposition for an electron is in a spin up state and flips to spin down in a magnetic field, a real $m = \pm 1$ photon is emitted carrying away the change in angular momentum. This is the only real effect observed from this infinity of $(l=3, m=+2)$ virtual wavefunctions all flipping to $(l=3, m=-2)$ states, plus an infinite flipping of the virtual zero point vector debts. Also Eq's. (3.2. 3) & (3.2. 6) are true *only if our high energy cutoff is at infinity and the low frequency cutoff is at zero*. We look at high frequency Planck scale cutoffs in section 4.2 and in section 6.1 low frequency cutoffs near the radius of the causally connected horizon.

3.3 Ratios between Primary and Secondary Coupling

3.3.1 Initial simplifying assumptions

Section 3.3 is long and can be skipped if preferred. The key equations especially Eq's. (3.3. 19), (3.3. 21) & (3.3. 22) and the paragraph following can be referred back to later. It is based on a special case thought experiment to try and illustrate as simply as possible how superpositions interact with one another, in the same way as virtual photons interact with electrons for example. *It is also important to remember here that because primary coupling*

constants are to bare charges (section 2.2.2), and thus fixed for all k , while secondary coupling constants run with k , that the coupling ratios can be defined only at the cutoff value of k applying to the bare charge (sections 4.1.1 & 4.2.2). From Table 2.2. 1 there are 6 fundamental primary charges for electrons and positrons. But electrons and positrons are defined as fundamental charges. In other words what we define as a fundamental electric charge is in reality 6 primary charges. Of course we can never in reality measure 6 as their effect is reduced by the ratio between primary and secondary coupling. Because electromagnetic and colour coupling are both via spin one bosons their *coupling ratios* are fundamentally the same but because of the above they are related simply as $6^2 = 36 : 1$.

$$\frac{1}{\chi_{Colour}} = \frac{36}{\chi_{EM}} \quad (3.3. 1)$$

We define the colour and electromagnetic ratios as follows (leaving gravity till section 6.2.3)

$$\frac{1}{\chi_{Colour}} = \frac{\alpha_{Colour(Secondary)}}{\alpha_{Colour(Primary)}} = \frac{\alpha_{3S}}{\alpha_{3P}} \quad \text{and} \quad \frac{1}{\chi_{EM}} = \frac{\alpha_{EM(Secondary)}}{\alpha_{EM(Primary)}} = \frac{\alpha_{EMS}}{\alpha_{EMP}} \quad (3.3. 2)$$

The secondary coupling constants α_{EMS} & α_{3S} are the bare charge values, both at the fermion interaction cutoff near the Planck length Eq. (4.2. 11). Also we assumed in section 2.2.2 that $\alpha_{3P} = 1$; thus from Eq.(3.3. 2)

$$\chi_C = \alpha_{3S}^{-1} = \alpha_3^{-1} @ k_{cutoff} \approx 2.029 \times 10^{18} GeV \quad (3.3. 3)$$

In other words provided $\alpha_{3P} = 1$, the ratio χ_C (or χ_{Colour}) is also the inverse of the colour coupling constant α_3 at the high energy interaction cutoff near the Planck length. In this respect χ_C or χ_{Colour} is the fundamental ratio we will use mainly from here on.

From the above paragraphs to find the coupling ratios we need secondary interactions that are between bare charges. But this implies extremely close spacing where the effects of spin dominate. If the spacing is sufficiently large the effects of spin can be ignored but then we are not looking at bare charges. However we can ignore the effects of shielding due to virtual charged pairs by imagining as a simple thought experiment, an interaction between bare charges even at such large spacing. We can also simplify things further by considering only

scalar or coulomb type interactions at this large spacing. We are also going to temporarily ignore Eq. (3.3. 2) and imagine that we have only one primary electric and or one colour charge. Consider two infinite superpositions and (*due to the above simplifying assumptions*) imagine them as spin zero charges. QED considers the interaction between them as a single covariant combination of two separate and opposite direction non-covariant interactions (a) plus (b) as in the Feynman diagram of Figure 3.3. 1 below. The Feynman transition amplitude is invariant in all frames [9]. So let us consider a special simple case in a CM frame where we have identical particles on a head on collision path with spatial momenta:

$$\mathbf{p}_a = -\mathbf{p}'_a = -\mathbf{p}_b = +\mathbf{p}'_b \quad (3.3. 4)$$

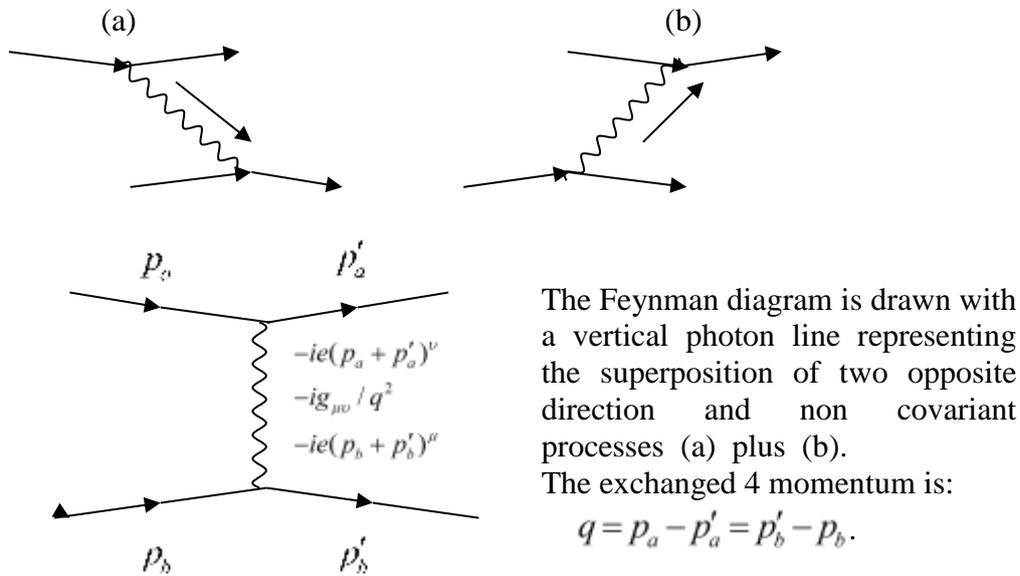


Figure 3.3. 1 Feynman diagram of virtual photon exchange between two spin zero particles of charge e .

From Eq. (3.3. 4) the initial and final spatial momenta are reversed with mirror images of each other at each vertex. Also in this simple special scalar case the transferred four momentum squared is simply the transferred three momentum squared.

$$q^2 = (p_a - p'_a)^2 = (p_b - p'_b)^2 = 4\mathbf{p}_a^2 = 4\mathbf{p}_b^2. \quad (3.3. 5)$$

If we look at Figure 3.3. 2 we see that at any fixed value of k , all modes ψ_{nk} in the groups of three overlapping superpositions for the various spins $\frac{1}{2}$, 1 & 2 occupy very similar regions of space (provided they are all on the same centre.) The directions of their linear momenta are unknown but let us imagine some particular vector $\hbar\mathbf{k}$ that is parallel to the above vectors $\mathbf{p}_a = \mathbf{p}_b$. As we are considering only scalar interactions, all these modes must be spherically symmetric (as in section 3.2.2 for spins 1 & 2, and for spin $\frac{1}{2}$ provided k or in turn β_{nk} is small enough the probability that it is not spherically symmetric can be extremely low) and at a fixed value of k they have momenta $\pm n\hbar\mathbf{k}$. Also as they overlap each other we can imagine units of $\pm\hbar\mathbf{k}$ quanta somehow transferring between these superpositions so that the values of n in each mode can change temporarily by ± 1 for times $\Delta T \approx \hbar / \Delta E$. The directions of these momentum transfers causing either repulsion or attraction depending on the charge signs of the superpositions at each vertex, whether the same or opposite.

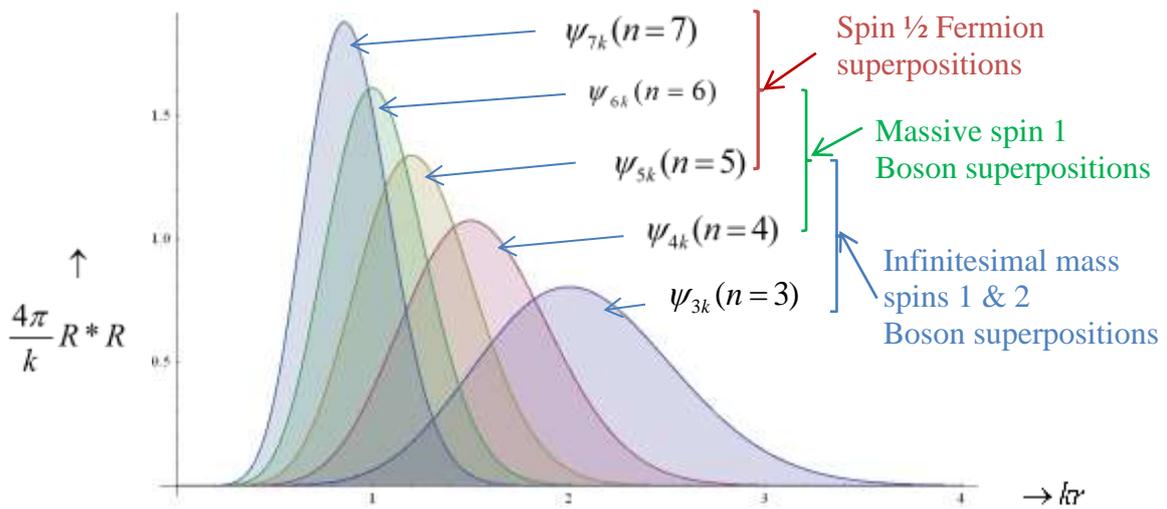


Figure 3.3. 2 All Eigenfunctions ψ_{nk} in the groups of three overlap at a fixed wavenumber k .

3.3.2 Restrictions on possible Eigenvalue changes

Before we look at changing these Eigenvalues by $n = \pm 1$ we need to consider what restrictions there are on these changes.

From Eq. (2.3. 12) superposition ψ_k requires $Q^2 A^2 = \sum_n c_n * c_n \frac{n^4 \hbar^2 k^4 r^2}{81}$ and Eq. (2.2. 4) tells

us the available $Q^2 A^2 = \frac{[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2$ occurs with probability $= \frac{sN \cdot dk}{k}$.

For very brief periods the required value of $Q^2 A^2$ can fluctuate, such as during these changes

of momentum, but if its average value changes over the entire process then Eq. (2.2. 4) tells us that probability $sN \cdot dk / k$ changes also, and we have shown in section 3.2.1 that this is not allowed. For example in a spin $\frac{1}{2}$ superposition $\psi_{5k}, \psi_{6k}, \psi_{7k}$, the average values of $|c_5|, |c_6|$ & $|c_7|$ must each remain constant. This can happen only if n remains within its pre-existing boundaries of $(5 \leq n \leq 7)$. For example if ψ_7 adds $+\hbar\mathbf{k}$, it can create ψ_8 , but $|c_8|$ must average zero, which it can do only if it fluctuates either side of zero, and $|c_n|$ cannot be negative. Similarly $|c_4|$ must average zero, thus ψ_4 & ψ_8 are forbidden states. Keeping the average values of $|c_n|$ constant is also equivalent to a constant internal average particle energy (we have shown in section 3.2.1 that rest mass is a function of $\sum c_n^* c_n \mathbf{p}_{nk}^2$). By changing these Eigenvalues by $n = \pm 1$ there are only four possibilities; ψ_6 & ψ_7 can both reduce by $-\hbar\mathbf{k}$ quanta, ψ_6 & ψ_5 can both increase by $+\hbar\mathbf{k}$ quanta. If ψ_6 becomes ψ_7 , $|c_7|$ also increases and $|c_6|$ decreases, but then ψ_7 has to drop back becoming ψ_6 , with $|c_7|$ decreasing back down and $|c_6|$ increasing back up in exact balance. If we view this as one overall process the average values of both $|c_6|$ and $|c_7|$ remain constant but fluctuate continuously. We can use exactly the same argument if ψ_5 increases which has to be followed by ψ_6 dropping, where if we view this as one process again, the average values of both $|c_5|$ and $|c_6|$ remain constant. This is similar to a particle not being able to absorb a photon in a covariant manner, it has to reemit within time $\Delta T \approx \hbar / E$. With *spherical symmetry* the momentum $\mathbf{p} = \pm n\hbar\mathbf{k}$. If we change n by ± 1 the sign of $\mathbf{p} = \pm n\hbar\mathbf{k}$ determines the *direction of the momentum transfer* $\Delta\mathbf{p}$. In the above if $\psi_{5k} \rightarrow \psi_{6k}$ then returns $\psi_{6k} \rightarrow \psi_{5k}$, and $\mathbf{p} = \pm n\hbar\mathbf{k}$ keeps the same sign during this process, there is *no nett momentum transfer* and there is a probability of this, but it is not the probability we need. However if this process is as in Figure 3.3. 3.

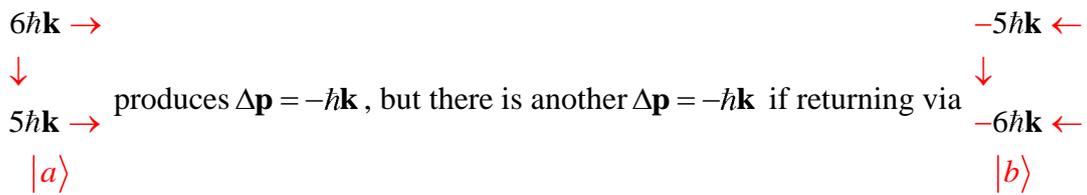


Figure 3.3. 3

To get a net momentum transfer the momenta have to be in opposite directions for each half of this process. (Conservation of momentum allows this only if there is an equal and opposite transfer of momentum at the other vertex of the interaction.) The problem with this is that a total transfer of $\Delta\mathbf{p} = -2\hbar\mathbf{k}$ implies superpositions ψ_k interact with virtual $2k$ photons. Section 3.5 shows that interactions only with virtual k photons give the correct Dirac spin $\frac{1}{2}$ magnetic energy. However just as transversely polarized photons are equal left and right polarization

superpositions $|L\rangle/\sqrt{2}+|R\rangle/\sqrt{2}$, we can perhaps regard the Figure 3.3. 3 process as a similar equal superposition $|a\rangle/\sqrt{2}+|b\rangle/\sqrt{2}$.

The figure 3.3.3 process becomes the superposition $\frac{|a\rangle}{\sqrt{2}} + \frac{|b\rangle}{\sqrt{2}} = \frac{|-\hbar\mathbf{k}\rangle}{\sqrt{2}} + \frac{|-\hbar\mathbf{k}\rangle}{\sqrt{2}}$ (3.3. 6)

We have two equal 50% probabilities of states $|a\rangle$ & $|b\rangle$ producing the required total $\Delta\mathbf{p} = -\hbar\mathbf{k}$. Also as from the above paragraphs the average values of $|c_5|$ and $|c_6|$ remain constant:

The probability of transitions $\psi_5 \rightleftharpoons \psi_6$ must be the same in either direction. (3.3. 7)

As spherically symmetric states have momentum $\mathbf{p} = \pm n\hbar\mathbf{k}$:

We can also think of $\mathbf{p} = \pm n\hbar\mathbf{k}$ as a superposition $\mathbf{p} = |+\hbar\mathbf{k}\rangle/\sqrt{2} + |-\hbar\mathbf{k}\rangle/\sqrt{2}$. (3.3. 8)

3.3.3 Looking at just one vertex of the interaction first

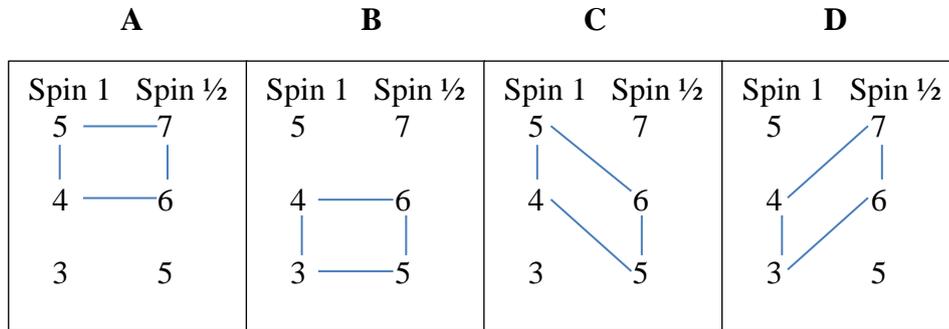
In Table 4.3. 1 and section 6.2 we introduce infinitesimal rest mass photons and gluons as superpositions of $\psi_{3k}, \psi_{4k}, \psi_{5k}$ where $N=2$ in Eq. (2.2. 4). Consider just one vertex of an infinitesimal rest mass spin 1 photon superposition $\psi_{3k}, \psi_{4k}, \psi_{5k}$ interacting with a spin $1/2$ superposition $\psi_{5k}, \psi_{6k}, \psi_{7k}$ at the same k . Looking at one possibility first, ψ_{4k} & ψ_{5k} for spin 1 and ψ_{6k} & ψ_{7k} for spin $1/2$, we can apply the Figure 3.3. 3 process to get a nett momentum transfer. For this combination of Eigenfunctions there are four possible ways of getting the momentum transfer as in Figure 3.3. 4. In each of these 4 cases the amplitude for this to happen includes the factors $c_4 \cdot c_5 \cdot c_6 \cdot c_7$. Let us temporarily imagine $|c_4 \cdot c_5 \cdot c_6 \cdot c_7| = 1$. Then $\mathbf{p} = +n\hbar\mathbf{k}$ as in $|a\rangle$ of Figure 3.3. 3 with an amplitude of $1/\sqrt{2}$ from Eq. (3.3. 8) transfers $\Delta\mathbf{p} = -\hbar\mathbf{k}$ also with an amplitude of $1/\sqrt{2}$, which is the required first half of our superposition Eq.(3.3. 6) $|a\rangle/\sqrt{2} + |b\rangle/\sqrt{2}$. Similarly $\mathbf{p} = -n\hbar\mathbf{k}$ as in $|b\rangle$ of Figure 3.3. 3 gives the second half. It would thus seem that our amplitude is simply $c_5 \cdot c_6 \cdot c_6 \cdot c_7$. However from Eq. (3.3. 7) there is a 50% probability of the transitions $\psi_5 \rightleftharpoons \psi_6$ in either direction, or an extra $1/\sqrt{2}$ amplitude factor for $\psi_5 \rightleftharpoons \psi_6$ in either direction, similarly an extra $1/\sqrt{2}$ amplitude factor for $\psi_6 \rightleftharpoons \psi_7$. These two extra $1/\sqrt{2}$ factors reduce the amplitude $c_4 \cdot c_5 \cdot c_6 \cdot c_7$ to $c_4 \cdot c_5 \cdot c_6 \cdot c_7 / (\sqrt{2} \times \sqrt{2}) = c_4 \cdot c_5 \cdot c_6 \cdot c_7 / 2$. Thus adding the four cases in Figure 3.3. 4 together and *treating all other factors as 1*:

Figure 3.3. 4 process amplitude factor is $4 \times (c_4 \cdot c_5 \cdot c_6 \cdot c_7) / 2 = 2c_4 \cdot c_5 \cdot c_6 \cdot c_7$ (3.3. 9)

Spin 1 4 goes to 5 with $\mathbf{p} \leftarrow$ 5 returns to 4 with $\mathbf{p} \rightarrow$	Spin 1/2 7 goes to 6 with $\mathbf{p} \leftarrow$ 6 returns to 7 with $\mathbf{p} \rightarrow$	Spin 1 4 goes to 5 with $\mathbf{p} \leftarrow$ 5 returns to 4 with $\mathbf{p} \rightarrow$	Spin 1/2 6 goes to 7 with $\mathbf{p} \rightarrow$ 7 returns to 6 with $\mathbf{p} \leftarrow$
Spin 1 5 goes to 4 with $\mathbf{p} \rightarrow$ 4 returns to 5 with $\mathbf{p} \leftarrow$	Spin 1/2 7 goes to 6 with $\mathbf{p} \leftarrow$ 6 returns to 7 with $\mathbf{p} \rightarrow$	Spin 1 5 goes to 4 with $\mathbf{p} \rightarrow$ 4 returns to 5 with $\mathbf{p} \leftarrow$	Spin 1/2 6 goes to 7 with $\mathbf{p} \rightarrow$ 7 returns to 6 with $\mathbf{p} \leftarrow$

Figure 3.3. 4

The four possibilities in Figure 3.3. 4 are all between the same sets of Eigenfunctions ψ_{4k} & ψ_{5k} for spin 1, ψ_{6k} & ψ_{7k} for spin 1/2. But there are also four different sets of these A, B, C & D, between groups of four Eigenfunctions as in Figure 3.3. 5; with their amplitudes from Eq. (3.3. 9) below each relevant box, which we also label as A, B, C & D. (Subscripts a refer to spin 1/2 and b to spin 1.)



Amplitudes: $A = 2c_{4b}c_{5b}c_{6a}c_{7a}$, $B = 2c_{3b}c_{4b}c_{6a}c_{5a}$, $C = 2c_{4b}c_{5b}c_{6a}c_{5a}$, $D = 2c_{3b}c_{4b}c_{6a}c_{7a}$.

Figure 3.3. 5

3.3.4 Assumptions when looking at both vertexes of the interaction

Because we are looking at an interaction between identical spin 1/2 fermions each vertex has the same groups of Eigenfunctions A,B,C&D as in Figure 3.3. 5. From section 2.2.2 and Figure 3.1. 4 the three Eigenfunctions forming each of the interacting particles are born simultaneously. It would thus seem reasonable to assume that *the amplitudes of each group of three Eigenfunctions have the same complex phase angle*. The two fermions and one boson can be at different complex phase angles to each other but each one individually is a superposition of three Eigenfunctions at the same complex phase angle. Thus the four

amplitudes $A, B, C & D$ from Figure 3.3. 5 ($A, B, C & D$ each comprising two fermion amplitudes and two boson amplitudes) must all have the same complex phase angle. Similarly the four amplitudes $A', B', C' & D'$ of vertex 2 in Figure 3.3. 6 also have a common phase angle.

Eigenfunction Groups	A	B	C	D
Vertex 1	Amplitude A	Amplitude B	Amplitude C	Amplitude D
Vertex 2	Amplitude A'	Amplitude B'	Amplitude C'	Amplitude D'

Figure 3.3. 6

We are also going to assume that Eigenfunctions A of vertex 1 interact only with Eigenfunctions A of vertex 2 and Eigenfunctions B of vertex 1 interact only with Eigenfunctions B of vertex 2 etc. Eigenfunctions A of vertex 1 do not interact with Eigenfunctions B of vertex 2 etc. Thus *if all other amplitude factors are 1*:

$$\text{The total interaction amplitude} = AA' + BB' + CC' + DD' \quad (3.3. 10)$$

Apart from a different complex phase angle this is equivalent to: ($A & A', B & B'$ etc. all differ by *the same complex phase angle*.)

$$\text{Total interaction amplitude} = A^2 + B^2 + C^2 + D^2 \quad (3.3. 11)$$

$$\text{Interaction probability} = (A^2 + B^2 + C^2 + D^2) * (A^2 + B^2 + C^2 + D^2) \quad (3.3. 12)$$

Using $(A^2 * A^2) = (A * A)(A * A)$ etc. this is equivalent to

$$\text{Interaction probability} = (A * A + B * B + C * C + D * D)^2 \quad (3.3. 13)$$

From Figure 3.3. 5 $A = 2c_{4b}c_{5b}c_{6a}c_{7a}$, $B = 2c_{3b}c_{4b}c_{6a}c_{5a}$, $C = 2c_{4b}c_{5b}c_{6a}c_{5a}$, $D = 2c_{3b}c_{4b}c_{6a}c_{7a}$.

Putting $P_{5a} = c_{5a} * c_{5a}$, $P_{4b} = c_{4b} * c_{4b}$ etc. & $A * A = 4P_{4b}P_{5b}P_{6a}P_{7a}$ etc. this is equivalent to

$$\begin{aligned} (A * A + B * B + C * C + D * D)^2 &= 16[P_{4b}P_{5b}P_{6a}P_{7a} + P_{3b}P_{4b}P_{6a}P_{5a} + P_{4b}P_{5b}P_{6a}P_{5a} + P_{3b}P_{4b}P_{6a}P_{7a}]^2 \\ &= 16[P_{4b}(P_{3b} + P_{5b})]^2 [P_{6a}(P_{5a} + P_{7a})]^2 \end{aligned}$$

Then using $c_{3b}^*c_{3b} + c_{4b}^*c_{4b} + c_{5b}^*c_{5b} = c_{5a}^*c_{5a} + c_{6a}^*c_{6a} + c_{6a}^*c_{6a} = 1$ the interaction probability is

$$(A^*A + B^*B + C^*C + D^*D)^2 = 2^4 \left[c_{4b}^*c_{4b}(1 - c_{4b}^*c_{4b}) \right]^2 \left[c_{6a}^*c_{6a}(1 - c_{6a}^*c_{6a}) \right]^2 \quad (3.3. 14)$$

We have assumed to here that *all other amplitude factors are 1*. However at each vertex there are both fermion and boson superposition probabilities from Eq. (2.2. 4). Writing the superposition probability at each vertex $sN \cdot dk / k$ as $s_{1/2}N_1dk / k$, s_1N_2dk / k for clarity where spin $1 = s_1$, $N = 1$ is N_1 etc. Including these factors (*if all other factors are one*) in Eq. (3.3. 14) our overall probability at wavenumber k is

$$\begin{aligned} & \left[\frac{2s_{1/2}N_1c_{6a}^*c_{6a}(1 - c_{6a}^*c_{6a})}{k} \right]^2 \left[\frac{2s_1N_2c_{4b}^*c_{4b}(1 - c_{4b}^*c_{4b})}{k} \right]^2 \\ &= \frac{\left[2s_{1/2}N_1c_{6a}^*c_{6a}(1 - c_{6a}^*c_{6a}) \right]^2 \left[2s_1N_2c_{4b}^*c_{4b}(1 - c_{4b}^*c_{4b}) \right]^2}{(k)^4}. \end{aligned}$$

The momentum per transfer is a total of $\pm\hbar\mathbf{k}$ and using Eq's. (3.3. 5), (3.3. 6) & Figure 3.3. 3 we have $(\pm\hbar\mathbf{k})^4 = q^4$ (then putting $\hbar = 1$) the interaction probability:

$$= \frac{\left[2s_{1/2}N_1c_{6a}^*c_{6a}(1 - c_{6a}^*c_{6a}) \right]^2 \left[2s_1N_2c_{4b}^*c_{4b}(1 - c_{4b}^*c_{4b}) \right]^2}{q^4} \quad (3.3. 15)$$

This is the *scalar interaction probability between two spin 1/2 fermions exchanging infinitesimal rest mass spin 1 bosons at very large spacings, where the fermions are effectively spin zero, imagining them as bare charges and all other factors being one*. Going through exactly the same procedure but similarly exchanging spin 2 infinitesimal rest mass scalar gravitons (with $N = 2 = N_2$ for clarity) the *gravitational interaction probability* between fermions becomes (using *subscript c* for spin 2) *if all other amplitude factors are 1*:

$$= \frac{\left[2s_{1/2}N_1c_{6a}^*c_{6a}(1 - c_{6a}^*c_{6a}) \right]^2 \left[2s_2N_2c_{4c}^*c_{4c}(1 - c_{4c}^*c_{4c}) \right]^2}{q^4} \text{ for fermions.} \quad (3.3. 16)$$

And if for example two spin 1 photons exchange spin 2 gravitons (all infinitesimal rest mass with $N = 2 = N_2$) the *interaction probability* becomes *if all other amplitude factors are 1*:

$$= \frac{[2s_1 N_2 c_{4b} * c_{4b} (1 - c_{4b} * c_{4b})]^2 [2s_2 N_2 c_{4c} * c_{4c} (1 - c_{4c} * c_{4c})]^2}{q^4} \text{ for } N = 2 \text{ photons.} \quad (3.3. 17)$$

If two massive $N = 1$ photons (as in Figure 3.3. 2) exchange spin 2 gravitons the *interaction probability* becomes if all other factors are 1:

$$= \frac{[2s_1 N_1 c_{5b} * c_{5b} (1 - c_{5b} * c_{5b})]^2 [2s_2 N_2 c_{4c} * c_{4c} (1 - c_{4c} * c_{4c})]^2}{q^4} \text{ for } N = 1 \text{ photons.} \quad (3.3. 18)$$

General Relativity (section 1.1.1) tells us the emission of gravitons is identical for both mass and energy. Keeping all other factors (such as mass/energy) in Eq's. (3.3. 16), (3.3. 17) & (3.3. 18) constant, the exchange probabilities must be the same in each. We can thus put them equal to each other and cancel out the red terms:

$$2s_1 N_2 c_{4b} * c_{4b} (1 - c_{4b} * c_{4b}) = 2s_1 N_1 c_{5b} * c_{5b} (1 - c_{5b} * c_{5b}) = 2s_{1/2} N_1 c_{6a} * c_{6a} (1 - c_{6a} * c_{6a})$$

or

$$\frac{4c_{4b} * c_{4b} (1 - c_{4b} * c_{4b})}{N = 2 \text{ Spin } 1} = \frac{2c_{5b} * c_{5b} (1 - c_{5b} * c_{5b})}{N = 1 \text{ Spin } 1} = \frac{c_{6a} * c_{6a} (1 - c_{6a} * c_{6a})}{N = 1 \text{ Spin } 1/2} \quad (3.3. 19)$$

Now assume that all other factors (other than coupling constants) are 1, and remember that we are simplifying with a thought experiment by looking at spin 1/2 superpositions sufficiently far apart so we can treat them as *approximately spherically symmetric or effectively spin zero even if they are supposed to be bare charges with spin*. Under these same scalar exchange conditions QED tells us that with electrons for example:

$$\text{The probability of scalar or coulomb exchange in Eq (3.3. 15).} = \frac{4\alpha^2}{q^4}. \quad (3.3. 20)$$

Let us temporarily ignore the fact that gluons have limited range, and imagine our thought experiment applying to colour charges exchanging gluons. The α of Eq. (3.3. 20) becomes the usual colour coupling α_3 . To get the *fundamental coupling ratio* labelled as $\chi_C = \alpha_3^{-1}$ @ k_{cutoff} we substitute the α of Eq. (3.3. 20) with $\alpha = \chi_C^{-1}$ as we have assumed $\alpha_{3Primary} = 1$. Also substitute $2s_{1/2} = 1$, $2s_1 = 2$, $N_1 = 1$ & $N_2 = 2$ and equate Eq's. (3.3. 15) & (3.3. 20)

$$\frac{[c_{6a} * c_{6a} (1 - c_{6a} * c_{6a})]^2 [4c_{4b} * c_{4b} (1 - c_{4b} * c_{4b})]^2}{q^4} = \frac{4(\chi_C^{-1})^2}{q^4} \quad (3.3. 21)$$

or $[c_{6a} * c_{6a} (1 - c_{6a} * c_{6a})][4c_{4b} * c_{4b} (1 - c_{4b} * c_{4b})] = 2\chi_C^{-1}$

But from Eq. (3.3. 19) the blue and green terms are equal (also the magenta terms) and we can solve for *the fundamental coupling ratio* by combining Eq's. (3.3. 19) & (3.3. 21).

$$\begin{array}{lll} N = 2 \text{ Spin } 1 & N = 1 \text{ Spin } 1 & N = 1 \text{ Spin } 1/2 \\ \text{Photons or Gluons} & \text{Massive Photons} & \text{Fermions} \end{array} \quad (3.3. 22)$$

$$4c_{4b} * c_{4b} (1 - c_{4b} * c_{4b}) = 2c_{5b} * c_{5b} (1 - c_{5b} * c_{5b}) = c_{6a} * c_{6a} (1 - c_{6a} * c_{6a}) = \sqrt{2}\chi_C^{-1}$$

The coupling ratio is fundamentally the same for colour and electromagnetism apart from the six primary electric charges of Eq. (3.3. 1) because of the way electric charge is defined. Equations (3.3. 19), (3.3. 21) & (3.3. 22) tell us that for any interactions between two superpositions, *the inverse coupling ratio always involves the product of the central superposition member probability by the probability of the other two members combined* $\times N \times \text{spin}$ of the first superposition, times *the equivalent product for the other superposition*. In section 4 we introduce gravity and solve these ratios. Despite all the simplifications the above equations are surprisingly consistent with the Standard Model provided there are only three families of fermions. Even though we used gravity to derive Eq.(3.3. 19) we leave discussing the gravity coupling ratio till section 6.2.3.

3.4 Electrostatic Energy between two Infinite Superpositions

3.4.1 Using a simple quantum mechanics early QED approach

In section 3.3 we have shown that fermion superpositions can exchange boson superpositions in the same way as electrons can exchange virtual photons for example. Providing the superposition amplitudes are appropriate, the coupling constants can be just as in QED, though we will look further at this in section 4.1.1. So it might seem that evaluating electrostatic energy between superpositions is unnecessary. However when we look at gravity we find that spacetime warping around mass concentrations is consistent with constant cosmic wavelength virtual graviton probability densities. Now QED looks at particle scattering crosssections due to virtual particle exchange probabilities, but as we later focus on virtual graviton probability densities we will use a simple, but only approximate, quantum

mechanical method based on virtual photon probability densities to find the scalar potentials between two charges (or infinite superpositions) that also allows a simple solution to magnetic energy between superpositions in Section 3.5 where we modify relevant equations in a simple manner. We also use some of these same equations when looking at why borrowing energy and mass from zero point fields requires the universe to expand after the Big Bang and distort spacetime around mass concentrations.

We assume spherically symmetric $l=3$ superpositions emit virtual scalar photons in this section and $l=3, m=\pm 2$ superpositions emit virtual $m=\pm 1$ photons in section 3.5. As section 3.3 has shown that we can achieve the same electromagnetic coupling constant α we can use the scalar photon emission probability $(2\alpha / \pi)(dk / k)$ covered in section 2.1.1. From section 3.3 we can also see that the effective average emission point has to be the center of superpositions. The probability of finding this interacting virtual photon (or spin 1 superposition) decays exponentially with radial distance travelled. The normalized wavefunction ψ for such a virtual scalar photon of wave number k emitted at $r = 0$ is:

$$\psi = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr} e^{+i(kr-\omega t)}}{r} = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr} e^{+ikr}}{r} \text{ @ time } t = 0.$$

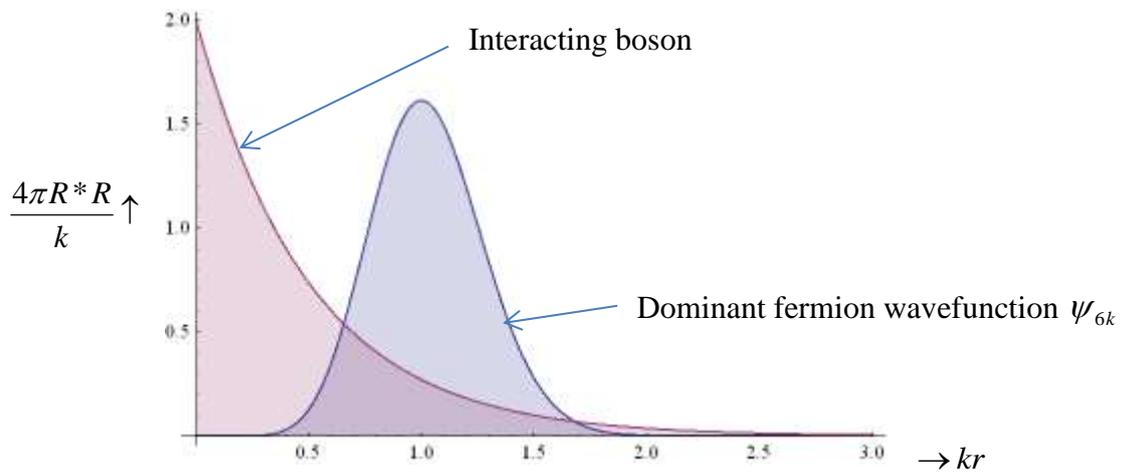


Figure 3.4. 1 Radial probability of ψ_{6k} and the exponential decay with radius of its interacting virtual boson $R^* R \propto 2ke^{-2kr}$. These curves are the same for all k , applying equally to virtual photons, gravitons and to large k value gluons etc.

Wavefunction ψ is spherically symmetric as scalar photons are time polarized. Figure 3.4. 1 plots the radial probabilities of the exponential range of the virtual photon and the dominant

$n = 6$ mode of its relating superposition ψ_k . The effective range of the interacting photon is of a similar order to the radial probability dimensions of ψ_{6k} . For simplicity in what follows we locate two superpositions (which we refer to as sources) in cavities that are small in relation to the distance between them. The accuracy of our results depends on how far apart they are in relation to the cavity size. Consider two spherically symmetric sources distance $2C$ apart emitting virtual scalar photons as in Figure 3.4. 2 where point P is r_1 from source 1, & r_2 from source 2. Let ψ_1 be the amplitude from source 1, and ψ_2 be the amplitude from source 2 and for simplicity and clarity let $t = 0$.

$$\text{Thus } \psi_1 = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_1+ikr_1}}{r_1} \quad \& \quad \psi_2 = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_2+ikr_2}}{r_2} \quad (3.4. 1)$$

$$\text{Consider } (\psi_1 + \psi_2)^*(\psi_1 + \psi_2) = \psi_1^*\psi_1 + \psi_1^*\psi_2 + \psi_2^*\psi_1 + \psi_2^*\psi_2$$

Now $\psi_1^*\psi_1$ & $\psi_2^*\psi_2$ are just the normal probability densities around sources 1 & 2 as though they are infinitely far apart but the work done per pair of superpositions k on bringing 2 sources closer together is in the interaction term: $\psi_1^*\psi_2 + \psi_2^*\psi_1$.

$$\psi_1^*\psi_2 = \frac{2k}{4\pi r_1 r_2} e^{-kr_1} e^{-kr_2} e^{-ikr_1} e^{+ikr_2} = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{-ik(r_1-r_2)}$$

$$\psi_2^*\psi_1 = \frac{2k}{4\pi r_1 r_2} e^{-kr_2} e^{-kr_1} e^{-ikr_2} e^{+ikr_1} = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{+ik(r_1-r_2)}$$

$$\psi_1^*\psi_2 + \psi_2^*\psi_1 = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \left[e^{+ik(r_1-r_2)} + e^{-ik(r_1-r_2)} \right]$$

$$= \frac{4k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \cos k(r_1 - r_2)$$

$$\text{Now put } (A = r_1 + r_2, B = r_1 - r_2) \quad \& \quad \psi_1^*\psi_2 + \psi_2^*\psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos(kB) \quad (3.4. 2)$$

Real work is done when bringing superpositions together and we can treat these interacting virtual photons as having real energy $\hbar\omega = \hbar kc$. Using virtual photon emission probability $(2\alpha / \pi)(dk / k)$ from section 2.1.1

$$\text{Energy per virtual photon} \times \text{Probability} = \hbar kc \times \left[\text{Probability} \frac{2\alpha}{\pi} \frac{dk}{k} \right] = \frac{2\alpha\hbar c}{\pi} dk \quad (3.4. 3)$$

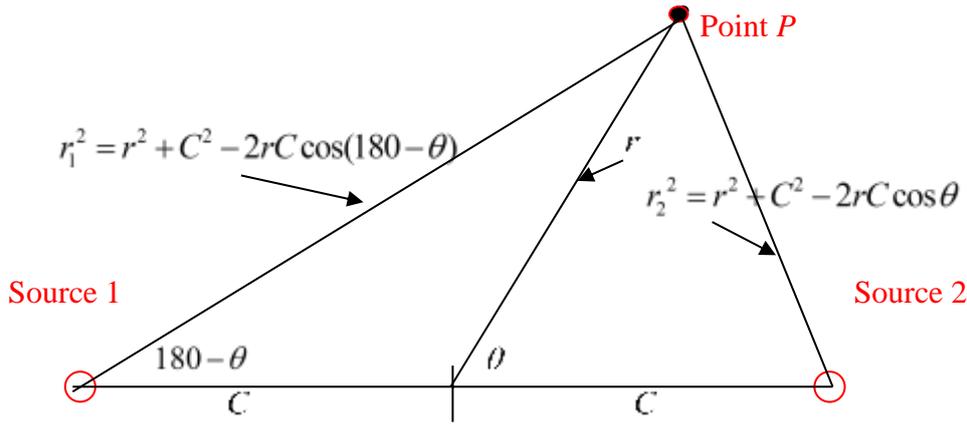


Figure 3.4. 2

Including Eq.(3.4. 3) the interaction energy @ k is thus $(\psi_1^* \psi_2 + \psi_2^* \psi_1) \left[\frac{2\alpha\hbar c}{\pi} dk \right]$ and

using Eq. (3.4. 2) the interaction energy @ k is $\left[\frac{2\alpha\hbar c}{\pi} dk \right] \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos(kB)$.

The total interaction energy density due to $\psi_1^* \psi_2 + \psi_2^* \psi_1$ for all k is

$$\frac{2\alpha\hbar c}{\pi} \frac{4}{4\pi r_1 r_2} \int_0^{\infty} k e^{-Ak} \cos(Bk) dk \quad (3.4. 4)$$

$$\int_0^{\infty} k e^{-Ak} \cos(Bk) dk = \frac{A^2 - B^2}{(A^2 + B^2)^2} \quad (3.4. 5)$$

Where $A^2 = (r_1 + r_2)^2 = r_1^2 + 2r_1 r_2 + r_2^2$ & $B^2 = (r_1 - r_2)^2 = r_1^2 - 2r_1 r_2 + r_2^2$

$$\text{Thus } A^2 = (r_1 + r_2)^2 = r_1^2 + 2r_1 r_2 + r_2^2 \text{ \& } A^2 + B^2 = 2(r_1^2 + r_2^2) \quad (3.4. 6)$$

$$= 2(r^2 + C^2) \text{ as } \cos(180 - \theta) = -\cos \theta$$

$$\text{and } A^2 + B^2 = 4(r^2 + C^2) \quad (3.4. 7)$$

Putting Eq's. (3.4. 4), (3.4. 5), (3.4. 6) & (3.4. 7) together $\frac{A^2 - B^2}{(A^2 + B^2)^2} = \frac{4r_1 r_2}{16(r^2 + C^2)^2}$

$$\int_0^{\infty} k e^{-Ak} \cos(Bk) dk = \frac{r_1 r_2}{4(r^2 + C^2)^2}$$

$$\frac{2\alpha\hbar c}{\pi} \frac{4}{4\pi r_1 r_2} \int_0^{\infty} k e^{-Ak} \cos(Bk) dk = \frac{2\alpha\hbar c}{\pi} \frac{4}{4\pi r_1 r_2} \frac{r_1 r_2}{4(r^2 + C^2)^2}$$

$$\frac{2\alpha\hbar c}{\pi} \frac{4}{4\pi r_1 r_2} \int_0^\infty k e^{-Ak} \cos(Bk) dk = \frac{2\alpha\hbar c}{\pi} \frac{1}{4\pi} \frac{1}{(r^2 + C^2)^2} \quad (3.4. 8)$$

This is the total interaction energy density of time polarized virtual photons at point P due to $\psi_1^* \psi_2 + \psi_2^* \psi_1$ for all k and there are no directional vectors to take into account. We will use similar equations for the vector potential ($m = \pm 1$) photons for magnetic energies but will then need directional vectors. Equation (3.4. 8) is the energy due to the interaction of amplitudes at any radius r from the centre of the pair. It is independent of θ , and to get the total energy of interaction we multiply by $4\pi r^2 dr$ for layer dr and integrate from $r = 0 \rightarrow \infty$.

The total interaction energy is
$$\frac{2\alpha\hbar c}{\pi} \int_0^\infty \int_0^\infty (\psi_1^* \psi_2 + \psi_2^* \psi_1) dk \ 4\pi r^2 dr$$

Using Eq. (3.4. 8)
$$= \frac{2\alpha\hbar c}{\pi} \frac{1}{4\pi} \int_0^\infty \frac{4\pi r^2 dr}{(r^2 + C^2)^2}$$

Thus
$$\frac{2\alpha\hbar c}{\pi} \int_0^\infty \int_0^\infty (\psi_1^* \psi_2 + \psi_2^* \psi_1) dk dv = \frac{2\alpha\hbar c}{\pi} \int_0^\infty \frac{r^2 dr}{(r^2 + C^2)^2}$$

$$\int_0^\infty \frac{r^2 dr}{(r^2 + C^2)^2} = \frac{1}{2C} \frac{\pi}{2}$$

The interaction or potential energy is
$$\frac{\alpha\hbar c}{2C} = \frac{\alpha\hbar c}{R} \quad (3.4. 9)$$

If $R = 2C$ is the distance between the centres of our assemblies, this is the classical potential. The procedure used here with small changes, simplifies the derivation of the magnetic moment; we reuse some equations, but in a slightly modified form taking polarization vectors into account. We also reuse some simple derivations when looking at gravity in Section 5.

3.5 Magnetic Energy between two spin aligned Infinite Superpositions

In this section we are going to consider two infinite superpositions that form Dirac spin $\frac{1}{2}$ states. We will look at the magnetic energy between them when they are both in a spin up state say along some z axis as in Figure 3.5. 1. *We are not looking at the magnetic energy*

here when they are both coupled in a spin 0 or spin 1 state. That is, both Dirac spin $\frac{1}{2}$ states have their $\sqrt{3}\hbar/2$ spin vectors randomly oriented around the z axis with $\hbar/2$ components aligned along this z axis. Also in this section we will be dealing with transversely polarized virtual photons and must take account of polarization vectors. In section 3.2.2 and Eq. (3.2. 7) spin $\frac{1}{2}$ states are generated only from $l=3, m=2$ states and as transversely polarized photons are superpositions of $m=\pm 1$ photons they can only be emitted from these $l=3, m=2$ states, the remaining states are spherically symmetric and cannot emit transversely polarized photons. We don't yet know the value of amplitudes $|c_n|$ so we will derive the magnetic energy in terms of these. We will then equate this energy to the Dirac values assuming a g value of 2 before QED corrections; this allows us to evaluate in section 4.3 the amplitudes $|c_n|$ in terms of the ratio χ_{EM} between primary and secondary electromagnetic coupling. We can then evaluate in section 4.1 the primary electromagnetic coupling constant α_{EMP} in terms of the ratio χ_{EM} . (Section 3.5 uses the same format as Chapter 18, "The Feynman Lectures on Physics" Volume 3, Quantum Mechanics [11].)

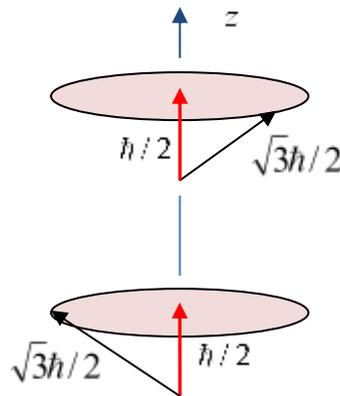


Figure 3.5. 1

An $l=3, m=2$ state can emit a right hand circularly (R.H.C.) polarized ($m=+1$) photon in the $+z$ direction. Let the amplitude for this be temporarily $|R\rangle$.

An $l=3, m=-2$ state can emit a left hand circularly (L.H.C.) polarized ($m=-1$) photon in the $+z$ direction. Let the amplitude for this also be temporarily $|L\rangle$.

First rotate the z axis about the y axis by angle θ (call this operation $S|R\rangle$) then use

$$\langle x'| = (1/\sqrt{2})[\langle R'| + \langle L'|]$$
 and multiply on the right by operation $S|R\rangle$.

The amplitude to emit a transversely polarized photon in the x' direction is thus

$$\langle x'|S|R\rangle = \frac{1}{\sqrt{2}}[\langle R'|S|R\rangle + \langle L'|S|R\rangle]$$

Where $\langle R'|S|R\rangle = \langle 3,+2'|S|3,+2\rangle = (1/4)\left[2+2\cos\theta-4\sin^2\theta+3\sin^2\theta\cos\theta\right]$ is the amplitude an $l=3,m=2$ state remains in an $l=3,m=2$ state after rotation by angle θ .

Also $\langle L'|S|R\rangle = -\langle 3,-2'|S|3,+2\rangle = (1/4)\left[2-2\cos\theta-4\sin^2\theta-3\sin^2\theta\cos\theta\right]$ is minus the amplitude that an $l=3,m=2$ state is in an $l=3,m=-2$ state after rotation by θ .

Putting this together
$$\langle x'|S|R\rangle = \frac{1-2\sin^2\theta}{\sqrt{2}} = \frac{\cos 2\theta}{\sqrt{2}} \quad (3.5. 1)$$

An $l=3,m=2$ state can also emit an ($m=+1$) photon in the $-z$ direction but it will now be left hand circularly polarized. Let this amplitude be temporarily: $|L\rangle$.

Similarly an $l=3,m=-2$ state can emit an ($m=-1$) photon in the $-z$ direction which is right hand circularly polarized. Let this amplitude be temporarily: $|R\rangle$.

We can go through the same procedure as above to get
$$\langle x'|S|L\rangle = \frac{\cos 2\theta}{\sqrt{2}} \quad (3.5. 2)$$

This amplitude Eq. (3.5. 2) is for a photon emitted in the opposite direction to amplitude Eq. (3.5. 1) but $\cos 2\theta = \cos 2(180+\theta)$ and we can simply add these two amplitudes. Let us assume however that an $l=3,m=2$ state has equal amplitudes to emit in the $+z$ & $-z$ directions of $|R\rangle/\sqrt{2}$ and $|L\rangle/\sqrt{2}$.

With these amplitudes;
$$\frac{1}{\sqrt{2}}\left[\langle x'|S|R\rangle + \langle x'|S|L\rangle\right] = \frac{\cos 2\theta}{2} + \frac{\cos 2\theta}{2} = \cos 2\theta \quad (3.5. 3)$$

Equation (3.5. 3) is the angular component of the amplitude for a transverse x' polarization in the new z' direction where $x \rightarrow x'$ & $z \rightarrow z' = \theta$. When $\theta=0$ or 180 the on axis amplitude for transverse polarization is one as expected ignoring other factors. Using the same normalization factors (we check the validity of this in section 3.5.2 we can still use the amplitudes and phasing of our original time mode photons Eq's. (3.4. 1) but instead of including polarization vectors we will for simplicity just use the cosine of the angle ($\gamma-\delta$) between them (as in Figure 3.5. 2) as a multiplying factor. Including the angular factor Eq. (3.5. 3) in our earlier scalar amplitudes Eq's. (3.4. 1) we have for our new wavefunctions:

$$\psi_1 = \cos 2\delta \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_1+ikr_1}}{r_1} \quad \& \quad \psi_2 = \cos 2\gamma \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_2+ikr_2}}{r_2} \quad (3.5. 4)$$

The transverse polarized photons from sources (1) & (2) have polarization vectors $|x_1\rangle$ and $|x_2\rangle$ at angle to each other $(\gamma - \delta)$, (Figure 3.5. 2) and the complex product becomes:

$$(\psi_1 + \psi_2)^*(\psi_1 + \psi_2) = \psi_1^*\psi_1 + (\psi_1^*\psi_2 + \psi_2^*\psi_1)\cos(\gamma - \delta) + \psi_2^*\psi_2$$

Where the interaction term is now: $(\psi_1^*\psi_2 + \psi_2^*\psi_1)\cos(\gamma - \delta)$ and as in the scalar case (section 3.4.1) but now using Eq's. (3.5. 4)

$$\psi_1^*\psi_2 \cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{-ik(r_1-r_2)} \cos(\gamma - \delta)$$

$$\psi_2^*\psi_1 \cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{+ik(r_1-r_2)} \cos(\gamma - \delta)$$

$$(\psi_1^*\psi_2 + \psi_2^*\psi_1)\cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos(kB) \cos(\gamma - \delta) \quad (3.5. 5)$$

(Where as in section 3.4.1, Eq. (3.4. 2) $A = r_1 + r_2$ & $B = r_1 - r_2$.)

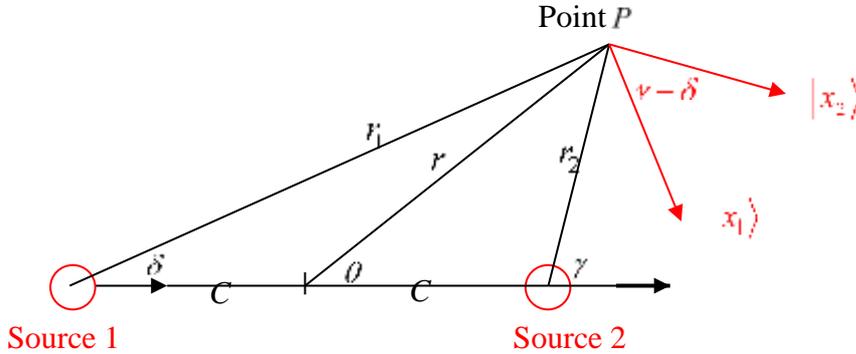


Figure 3.5. 2 Two sources $2C$ apart, both with $\beta_{nk}^2 \times (m = +2)$ states along the joining line, δ & γ are the respective angles to P , r_1 & r_2 are the respective distances to point P .

3.5.1 Amplitudes of transversely polarized virtual emitted photons

In the laboratory frame ψ_{nk} has amplitude β_{nk} to be in an $m = +2$ state (section 3.1). For a multiple integer n superposition $\psi_k = \sum_n c_n \psi_{nk}$. At each fixed wavenumber k we cannot distinguish which integer n a virtual photon comes from, so we must add amplitudes from each individual integer n superposition. To keep integrals simple we will assume that $\beta_{nk} \ll \ll \ll 1$ or that spacing $2C$ is very large, and our interacting k values are very small.

(We can make a comparison with the Dirac values at any large spacing so accuracy need not be affected.) Thus if $\beta_{nk} \lll 1$ & $\gamma_{nk} \approx 1$, we can approximate Eq. (3.1. 11) as

$$K_{nk} = \beta_{nk} \gamma_{nk} \approx \beta_{nk} \approx \frac{n\hbar k \sqrt{2s}}{2m_0 c} \approx \frac{|\mathbf{p}_{nk}| \sqrt{2s}}{2m_0 c} \approx \frac{\tilde{\lambda}_c n k \sqrt{2s}}{2} \approx \frac{\tilde{\lambda}_c n k}{2} \text{ for spin } 1/2 \text{ fermions.}$$

$$\text{Adding amplitudes for multiple integer } n \text{ superpositions } \langle \beta_k \rangle \approx \frac{\tilde{\lambda}_c \langle n \rangle k}{2} \quad (3.5. 6)$$

(When deriving Eq. (3.2. 10) we said $\langle |\mathbf{p}_k| \rangle = \hbar k \langle n \rangle$ and not $\langle |\mathbf{p}_k| \rangle = \hbar k \sqrt{\langle n^2 \rangle}$. How do we justify this? When $\beta_{nk} \ll 1$ as above $\beta_{nk} \propto n\hbar k = |\mathbf{p}_{nk}|$ So adding amplitudes β_{nk} to get $\langle \beta_k \rangle$ is equivalent to adding \mathbf{p}_{nk} to get $\langle \mathbf{p}_k \rangle$ and not adding $\mathbf{p}_{nk}^2 = n^2 \hbar^2 k^2$ to get $\langle |\mathbf{p}_k| \rangle = \hbar k \sqrt{\langle n^2 \rangle}$. If this is true when $\beta_{nk} \ll 1$ it must be true for $0 \leq \beta_{nk} \leq 1$.)

3.5.2 Checking our normalization factors

Let us pause and check the reasonableness of all this and our normalization factors. From

Eq's. (3.4. 1) for scalar photons $\left[\psi^* \psi = \frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \right] \times (\text{emission probability } \frac{2\alpha}{\pi} \frac{dk}{k})$ gives a

$$\text{Scalar } \psi_k \text{ emission probability density } \psi^* \psi \left[\frac{2\alpha}{\pi} \frac{dk}{k} \right] = \frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \left[\frac{2\alpha}{\pi} \frac{dk}{k} \right].$$

The transversely polarized probability density, using Eq's. (3.5. 4) & (3.5. 7) plus $\langle \beta_k \rangle^2$ is

$$\text{Transverse emission probability density } \langle \beta_{nk} \rangle^2 \psi'^* \psi' \frac{2\alpha}{\pi} \frac{dk}{k} = \langle \beta_{nk} \rangle^2 \left[\cos^2 2\delta \frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \right] \frac{2\alpha}{\pi} \frac{dk}{k}$$

(Where $2\delta = 2\gamma$ & $r_1 = r_2$.) If we now consider the on axis $\delta = 0$ case the transverse polarized on axis emission probability density at k is:

$$\langle \beta_k \rangle^2 \left[\frac{2k}{4\pi} \frac{e^{-2kr}}{r^2} \right] \frac{2\alpha}{\pi} \frac{dk}{k} = \langle \beta_k \rangle^2 \psi^* \psi \frac{2\alpha}{\pi} \frac{dk}{k}$$

Just as in QED the factor $\langle \beta_k \rangle^2$ is the factor we need for this on axis emission probability density *ratio between transverse and scalar polarization*. This justifies using the same normalization constant $\sqrt{2k/4\pi}$ for both the scalar and magnetic wavefunctions. We seem to be on the right track and using the same virtual photon emission probability and energy $\hbar kc$ as in Eq. (3.4. 3) for both the scalar and transverse polarization cases ie

$$\text{Energy per transverse photon} \times \text{Probability} = \hbar kc \times \left[\text{Probability} \frac{2\alpha}{\pi} \frac{dk}{k} \right] = \frac{2\alpha\hbar c}{\pi} dk \quad (3.5. 7)$$

Multiplying Eq. (3.5. 5) by Eq. (3.5. 6) squared, and Eq. (3.5. 7) we get the transverse interaction energy @ wavenumber k :

$$\begin{aligned} & \langle \beta_k \rangle^2 (\psi_1^* \psi_2 + \psi_2^* \psi_1) \cos(\gamma - \delta) \left[\frac{2\alpha\hbar c}{\pi} dk \right] \\ &= \left[\frac{\tilde{\lambda}_c^2 \langle n \rangle^2 k^2}{4} \right] \cos 2\delta \cos 2\gamma \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos(kB) \cos(\gamma - \delta) \left[\frac{2\alpha\hbar c}{\pi} dk \right] \end{aligned}$$

$$\begin{aligned} \text{Rearranging this:} \quad & \langle \beta_k \rangle^2 (\psi_1^* \psi_2 + \psi_2^* \psi_1) \cos(\gamma - \delta) \left[\frac{2\alpha\hbar c}{\pi} dk \right] \\ &= \frac{2\langle n \rangle^2 \tilde{\lambda}_c^2 \alpha\hbar c}{\pi} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{4\pi r_1 r_2} \left[k^3 e^{-Ak} \cos(kB) dk \right] \end{aligned} \quad (3.5. 8)$$

As in the scalar case we integrate over k first but now with a k^3 term due to the inclusion of the $\langle \beta_k \rangle^2$ factor which is approximately proportional to k^2 from Eq. (3.5. 6).

Using $A = r_1 + r_2$ & $B = r_1 - r_2$ and Eq's. (3.4. 6) & (3.5. 6)

$$\int_0^{\infty} \left[k^3 e^{-Ak} \cos(kB) dk \right] = \frac{3}{8} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right]$$

$$\begin{aligned} \text{And thus:} \quad & \int_0^{\infty} \langle \beta_k \rangle^2 (\psi_1^* \psi_2 + \psi_2^* \psi_1) \cos(\gamma - \delta) \frac{2\alpha\hbar c}{\pi} dk \\ &= \frac{2\langle n \rangle^2 \tilde{\lambda}_c^2 \alpha\hbar c}{\pi} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{4\pi r_1 r_2} \times \frac{3}{8} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] \end{aligned} \quad (3.5. 9)$$

Equation (3.5. 9) is the magnetic interaction energy density at point P for all wave numbers k .

Figure 3.5. 2 is a plane of symmetry that can be rotated through angle 2π around the axis of symmetry (the joining line along the axis of the 2 spin aligned sources). To evaluate the total magnetic energy density over all space we just multiply by $4\pi r^2 \sin\theta d\theta dr$.

We thus integrate Eq. (3.5. 9) $\times 4\pi r^2 \sin\theta d\theta dr$. =

$$\frac{3\langle n \rangle^2 \tilde{\lambda}_c^2 \alpha\hbar c}{4\pi} \int_0^{\infty} \int_0^{\pi/2} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{r_1 r_2} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] r^2 \sin\theta d\theta dr \quad (3.5. 10)$$

Now $\int_0^\infty \int_0^{\frac{\pi}{2}} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{r_1 r_2} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] r^2 \sin \theta d\theta dr$ can be reduced to the

single integral: $\frac{1}{8C^3} \int_0^1 \sqrt{1-x^2} \left[\frac{(7-5x^2)}{x^3} \ln \frac{1+x}{1-x} - \frac{14}{x^2} + \frac{16}{3} \right] dx$ which can be also expressed

as an infinite series in p (to not confuse with superposition value n):

$$\frac{1}{8C^3} \sum_{p=1}^{p=\infty} \left[\frac{14}{2p+3} - \frac{10}{2p+1} \right] \frac{(2p-1)!}{(p-1)!(p+1)!4^p} \cdot \frac{\pi}{2} = \frac{1}{8C^3} \frac{(160-51\pi)}{6} \cdot \frac{\pi}{2}$$

$$\text{(Putting } R = 2C \text{)} = \frac{1}{R^3} \frac{(160-51\pi)}{6} \cdot \frac{\pi}{2} \quad (3.5. 11)$$

$$\text{This infinite series is approximately } \approx -\frac{1}{R^3} \frac{\pi}{54(1.0045062\dots)} \quad (3.5. 12)$$

Putting Eq.(3.5. 12) into Eq.(3.5. 9) the total magnetic interaction energy over all frequencies and all space for 2 spin aligned infinite superpositions is:

$$U' \approx \frac{3\langle n \rangle^2 \tilde{\lambda}_c^2 \alpha \hbar c}{4\pi} \left[-\frac{1}{R^3} \frac{\pi}{54(1.0045062\dots)} \right]$$

$$\text{We will call this } U(\text{superpositions}) \approx -\left[\frac{\langle n \rangle^2 \tilde{\lambda}_c^2 \alpha \hbar c}{72R^3(1.0045062\dots)} \right] \quad (3.5. 13)$$

We can equate this magnetic energy to the classical value assuming the Dirac value of $g = 2$ for spin $\frac{1}{2}$ (No QED corrections have been applied so it must be $g = 2$). For the arrangement of spins as in Figure 3.5. 1 the Dirac magnetic energy between two spin $\frac{1}{2}$ states is

$$U(\text{Dirac}) = -\left[\frac{2\mu^2}{4\pi\epsilon_0 c^2 R^3} \right] \quad (3.5. 14)$$

Using the Dirac magnetic moment $\mu = \frac{e\hbar}{2m_0} = \frac{e\hbar c}{2m_0 c} = \frac{ec\tilde{\lambda}_c}{2}$ the Dirac magnetic energy is

$$U(\text{Dirac}) = -\left[\frac{\tilde{\lambda}_c^2 \alpha \hbar c}{2R^3} \right]$$

The approximation used in deriving Eq. (3.5. 6) $\gamma^2 \beta^2 \approx \beta^2$ for $\beta^2 \lll 1$ is true only when $R \gg \hat{\lambda}_c$. This error in β^2 is of the order of $\hat{\lambda}_c^2 / R^2$ and rapidly tends to zero with increasing R . There is no upper limit on the value of distance R we can choose. Thus comparing our estimate of the magnetic energy with Dirac's value when $R \gg \hat{\lambda}_c$.

$$U(\text{Dirac}) = U(\text{Superpositions}) \text{ or } - \left[\frac{\hat{\lambda}_c^2 \alpha \hbar c}{2R^3} \right] \approx - \left[\frac{\langle n \rangle^2 \hat{\lambda}_c^2 \alpha \hbar c}{72R^3 (1.0045062\dots)} \right] \quad (3.5. 15)$$

All symbols cancel except $\langle n \rangle$ leaving: $\langle n \rangle^2 \approx 36(1.0045062\dots)$

The expectation value $\langle n \rangle$ in our superposition is slightly more than $n = 6$ our dominant mode. This is why we have used a three member superposition centred on this dominant $n = 6$ mode. The two side modes $n = 5$ & $n = 7$ are smaller so that:

$$\langle n \rangle = \sum_{n=5,6,7} (c_n^* c_n) n \approx \sqrt{36(1.0045062\dots)} \approx 6.01350345 \quad (3.5. 16)$$

This is for Dirac spin $\frac{1}{2}$ particles. This mean value of n creates a $g = 2$ fermion which *QED corrections (which are secondary interactions) increase slightly to the experimental value.* In section 4.1 we solve the primary electromagnetic coupling constant in terms of ratio χ_{EM} using Eq. (3.5. 16). It is important to remember this magnetic energy derivation applies to two infinite assemblies (or particles) localized in small cavities in relation to their distance R apart. They must be both on the z axis with spins aligned (or anti aligned) along this z axis as in Figure 3.5. 1 & Figure 3.5. 2. Also the agreement with Dirac and in what follows is possible *if superposition ψ_k interacts only with virtual photons of the same wavenumber k .*

4 High Energy Superposition Cutoffs

4.1 Electromagnetic Coupling to Spin $\frac{1}{2}$ Infinite Superpositions

Equation (3.5. 16) is the key requirement for spin $\frac{1}{2}$ superpositions to behave as Dirac fermions, allowing us to solve α_{EMP}^{-1} as a function of coupling ratio χ using Eq. (3.5. 16).

$$\langle n \rangle = \sum_{n=5,6,7} (c_n^* c_n) n \approx \sqrt{36(1.0045062\dots)} \approx 6.01350345$$

Thus $5c_5^* c_5 + 6c_6^* c_6 + 7c_7^* c_7 = 6.01350345$ but $6c_5^* c_5 + 6c_6^* c_6 + 6c_7^* c_7 = 6$

and $c_7^* c_7 - c_5^* c_5 = 0.01350345$

As $c_7^* c_7 + c_5^* c_5 = 1 - c_6^* c_6$ we can now solve for $c_7^* c_7$ & $c_5^* c_5$ in terms of $c_6^* c_6$

$$c_7 * c_7 \approx 0.50675172 - \frac{c_6 * c_6}{2} \quad \& \quad c_5 * c_5 \approx 0.49324827 - \frac{c_6 * c_6}{2} \quad (4.1. 1)$$

From Eq. (2.3. 12) the $Q^2 A^2$ required to produce this superposition with amplitudes c_n is

$$Q^2 A^2 = \sum_{n=5,6,7} c_n * c_n \frac{n^4 \hbar^2 k^4 r^2}{81} \text{ and using Eq. (4.1. 1)}$$

$$\sum_{n=5,6,7} c_n * c_n n^4 = 625c_5 * c_5 + 1296c_6 * c_6 + 2401c_7 * c_7 \approx 1524.991 - 217c_6 * c_6$$

Thus $Q^2 A^2 = \sum_{n=5,6,7} c_n * c_n \frac{n^4 \hbar^2 k^4 r^2}{81} \approx [18.82705 - 2.67901c_6 * c_6] \hbar^2 k^4 r^2$ is the *required* vector

potential squared to produce this spin $\frac{1}{2}$ superposition. From Eq. (2.2. 4) with $s = \frac{1}{2}$ &

$N = 1$ for massive fermions $Q^2 A^2 = \frac{2[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi} \hbar^2 k^4 r^2$ is the *available* $Q^2 A^2$.

Equating *required and available*: $2[8 + 8\sqrt{\alpha_{EMP}}]^2 \approx 3\pi[18.82705 - 2.67901c_6 * c_6]$

$$[1 + \sqrt{\alpha_{EMP}}]^2 \approx [1.386256 - 0.197258c_6 * c_6]$$

$$\alpha_{EMP} \approx \left[\sqrt{1.386256 - 0.197258c_6 * c_6} - 1 \right]^2 \quad (4.1. 2)$$

From Eq's. (3.3. 1) & (3.3. 22), $c_6 * c_6(1 - c_6 * c_6) = \sqrt{2/\chi_C} = 6\sqrt{2/\chi_{EM}}$ and we can solve for α_{EMP} as a function of either χ_{EM} or χ_C . We then use Eq. (3.3. 22) again to get α_{EMS}^{-1} @ k_{cutoff} . Now both χ_{EM} and χ_C are fundamentally the same ratio differing only by 36:1, because electron superpositions have six primary charges whereas we define them as one fundamental charge (section 3.3.1) and quarks have only one colour charge (Table 2.2. 1). Because $\chi_C = \alpha_3^{-1}$ at the cutoff near L_p it is more convenient to work with. From Eq. (3.3. 22)

$$c_6 * c_6 = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4 \frac{\sqrt{2}}{\chi_C}} \quad \text{and there are two solutions for each } \chi_C.$$

One has $c_6 * c_6$ *dominant* with two smaller $c_5 * c_5$ & $c_7 * c_7$ side modes, the other is *the reverse* with $c_6 * c_6$ *the minor player* and two larger $c_5 * c_5$ & $c_7 * c_7$ side modes. As the values for α_{EMP} with $c_6 * c_6$ dominant fit the Standard Model very closely, we include only these. (This only applies to spin $\frac{1}{2}$ fermions and in Table 4.3. 1 spins 1 & 2 boson superpositions have minor centre modes.) Table 4.1. 1 shows these dominant $c_6 * c_6$ mode results for $\chi_C = \alpha_3^{-1}$ at various possible cutoffs in the range $\chi_C = 50 \rightarrow 51$, as this range fits the Standard Model. Of course there can be *only one solution for this cutoff*.

Coupling Ratio χ_C	$c_6^*c_6$	α_{EM}^{-1} Primary	α_{EM}^{-1} Secondary @ k_{cutoff}
50.00	≈ 0.723607	≈ 75.4414	≈ 104.7798
50.20	≈ 0.724497	≈ 75.5447	≈ 105.3429
50.40	≈ 0.725378	≈ 75.6472	≈ 105.9060
50.4053	≈ 0.725401	≈ 75.6499	≈ 105.9210
50.60	≈ 0.726250	≈ 75.7488	≈ 106.4692
50.80	≈ 0.727115	≈ 75.8497	≈ 107.0324
51.00	≈ 0.727970	≈ 75.9499	≈ 107.5956

Table 4.1. 1 Possible coupling ratios χ_C versus α_{EM}^{-1} in the range $\chi_C = 50 \rightarrow 51$. The yellow row corresponds to the interaction cutoff energy in Figure 4.1. 2 & Eq. (4.2. 11).

4.1.1 Comparing this with the Standard Model

In the real world of Standard Model secondary interactions the electromagnetic force splits into two components α_1 & α_2 at energies greater than the mass/energy of the Z_0 boson or ≈ 91.1876 GeV. [12]. However we want to compare these Standard Model couplings with the values derived in Table 4.1. 1 at the $\approx 2.0288 \times 10^{18}$ GeV. cutoff of Eq. (4.2. 11). Assuming three families of fermions and one Higgs field the SM [13] predicts

$$\begin{aligned}\alpha_1^{-1} &\approx 58.98 \pm 0.08 - \frac{4.1}{2\pi} \log_e \frac{Q}{91.1876} \\ \alpha_2^{-1} &\approx 29.60 \pm 0.04 + \frac{19}{6 \times 2\pi} \log_e \frac{Q}{91.1876} \\ \alpha_3^{-1} &\approx 8.47 \pm 0.22 + \frac{7}{2\pi} \log_e \frac{Q}{91.1876}\end{aligned}\tag{4.1. 3}$$

The weak force split obeys
$$\alpha_{EM}^{-1} = \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1}\tag{4.1. 4}$$

Also $\alpha_1^{-1} = \frac{3}{5} \alpha_{EM}^{-1} \cos^2 \theta_W$ & $\alpha_2^{-1} = \alpha_{EM}^{-1} \sin^2 \theta_W$ where θ_W is the Weinberg angle.

Combining Eq's. (4.1. 3) & (4.1. 4)

$$\alpha_{EM}^{-1} = \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1} \approx 127.90 \pm 0.173 - \frac{11}{3 \times 2\pi} \log_e \frac{Q}{91.1876}\tag{4.1. 5}$$

Figure 4.1. 1 plots these four inverse coupling constants. Figure 4.1. 2 plots the intersection of α_{EM}^{-1} predicted in Table 4.1. 1 and the Standard Model prediction for α_{EM}^{-1} in Eq. (4.1. 5). It would initially seem in Figure 4.1. 2 that there is an unusually large error band in the predicted results. However $\Delta \alpha_{EM}^{-1} / \Delta \chi^{-1} \approx 2.8$ is approximately constant in this table and the error band in the Standard Model colour coupling α_3^{-1} of ± 0.22 in Eq's (4.1. 3) translates into the larger error band for α_{EM}^{-1} of $\pm 0.22 \times 2.8 \approx \pm 0.62$ in Figure 4.1. 2.

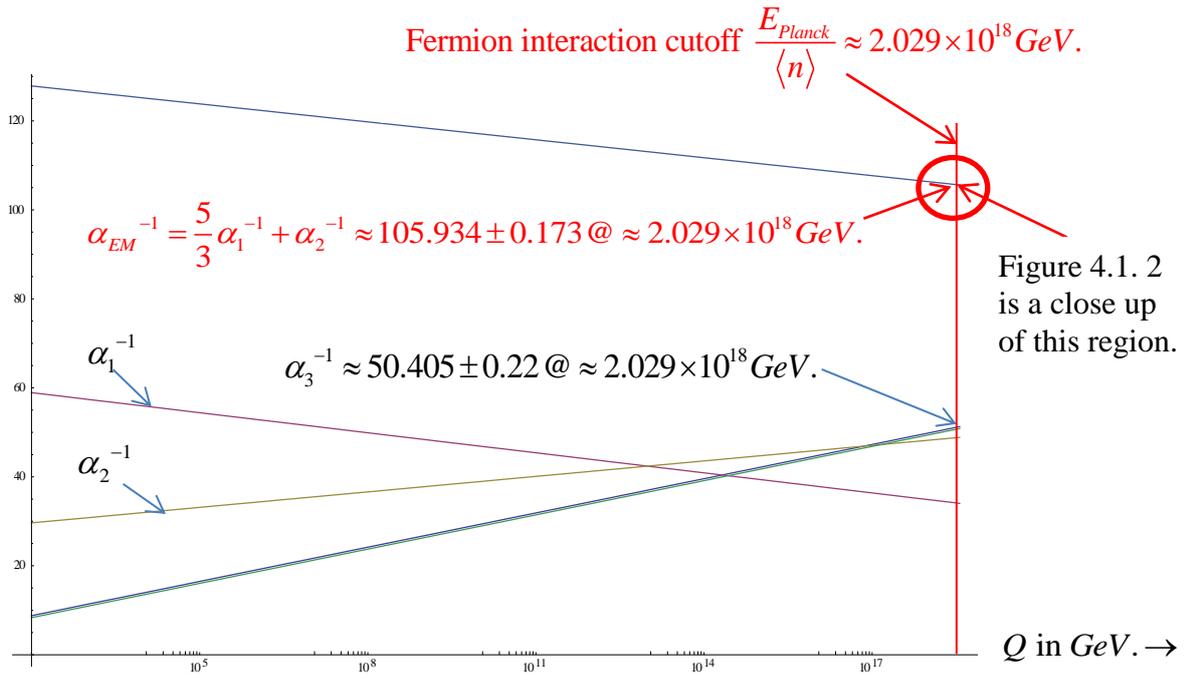


Figure 4.1. 1 Standard Model based on three families of fermions and one Higgs field.

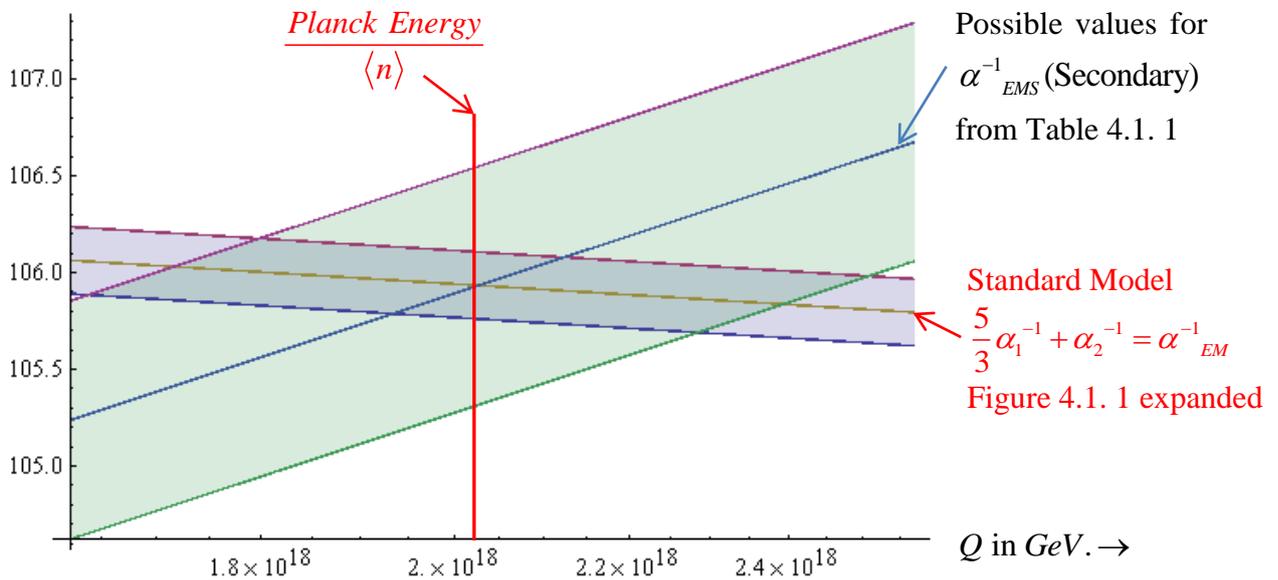


Figure 4.1. 2 A close up of the intersecting region of the Standard Model Eq. (4.1. 5) and Table 4.1. 1 predictions. The fermion interaction cutoff is consistent with the Standard Model.

4.2 Introducing Gravity into our Equations

4.2.1 Simple square superposition cutoffs

In section 3.2 we looked at single integer n superpositions of ψ_{nk} initially for clarity, and later found multiple integer n superpositions gave the same results; we will do the same here. We also found in Eq's. (3.2. 3) & (3.2. 6) that the integrals for both angular momentum and rest masses are of similar form. They both ended up including the term

$$\left[\frac{-1}{1+K_{nk}^2} \right]_0^\infty \text{ which if } K_{nk} \text{ cutoff} < \infty \text{ becomes } \left[\frac{-1}{1+K_{nk}^2} \right]_0^{K_{nk} \text{ cutoff}} \text{ and this is equal to}$$

$$1 - \frac{1}{1+K_{nk}^2 \text{ cutoff}} = \frac{K_{nk}^2 \text{ cutoff}}{1+K_{nk}^2 \text{ cutoff}} = \frac{1}{1+1/K_{nk}^2(\text{cutoff})} = \frac{1}{1+\varepsilon} \quad (4.2. 1)$$

where using Eq. (3.1. 11) the infinitesimal $\varepsilon = \frac{1}{K_{nk}^2 \text{ cutoff}} = \frac{2m_0^2 c^2}{n^2 \hbar^2 (k_{\text{cutoff}})^2 s}$ (4.2. 2)

For integral or half integral \hbar angular momentum precision is required but Eq. (3.2. 6) now

gives us $\mathbf{L}_z(\text{Total}) = \frac{sm\hbar}{2} \left[\frac{-1}{1+K_{nk}^2} \right]_0^{K_{nk} \text{ cutoff}} = \frac{sm\hbar}{2} \frac{1}{1+\varepsilon}$. So can the effect of gravity increase

our probabilities from $sN \cdot \frac{dk}{k}$ to $sN \cdot (1+\varepsilon) \frac{dk}{k}$? We will initially address only *massive infinite superpositions* where $N=1$ in Eq. (2.2. 4).

The first question we need to address is what is the effective preon mass to be used when coupling to gravity? In Eq. (3.1. 4) we said the preon rest mass is $m_0 / (8\gamma_{nk} \sqrt{2s})$ for each of the 8 preons that build a spin $\frac{1}{2}$ particle of rest mass m_0 . Now gravity couples to the total mass including the kinetic energy. It also couples to other terms in Einstein's energy-momentum tensor, but *we conjecture that in primary interactions such as this (section 1.1.2), gravitons only couple to the mass/energy*, and the equations are consistent only if this is so. (Sections 6.2.1 & 6.2.2 also discuss this further.)

At the start of the interaction each preon mass is $m_0 / (8\gamma_{nk} \sqrt{2s})$ and after the interaction (Figure 3.1. 3) it is $m_0 \gamma_{nk} (1+\beta_{nk}^2) / (8\sqrt{2s})$. Let us think semi classically again and see where it leads us. We have been using magnitudes of velocities as they are the most convenient way to express our equations even if not the conventional language of quantum mechanics. The

interaction with the zero point fields takes the momentum of each preon from zero to $2m_0\gamma_{nk}\beta_{nk}c/(8\sqrt{2s})$ (Figure 3.1. 3). While this happens as a quantum step change let us imagine it as a virtually infinite acceleration from zero velocity to $2\beta_{nk}/(1+\beta_{nk}^2)$, which is the relativistic velocity addition (see Figure 3.1. 1) of 2 equal steps of β_{nk} . At the half way point after one step the velocity is β_{nk} (the velocity of the CMF, the preon mass has increased to $m_0/(8\sqrt{2s})$). We can imagine this as being like the central point of a quantum interaction.

We will conjecture this midway point preon mass $m_0/(8\sqrt{2s})$ is the mass value that gravity acts on and we will see that it is indeed the only value that fits all equations. Also it does not make sense to choose either of the end point masses. We can also get reassurance from the properties of the Feynman transition amplitude which tells us in Eq. (3.1. 15)

$$\frac{(p_i + p_f)^z}{(p_i + p_f)^0} = \frac{2m_0\gamma_{nk}\beta_{nk}}{2m_0\gamma_{nk}} = \beta_{nk} \text{ and the ratio of space to time polarization in the LF is } \beta_{nk}^2.$$

This centre of momentum velocity tells us the key properties of the interaction. We will thus assume we have 8 preons in each ψ_{nk} of effective gravitational mass $m_0/(8\sqrt{2s})$ with effective *total* gravitational mass $m_0/\sqrt{2s}$. To put the gravitational constant in the same form as the other coupling constants we need to divide it by $\hbar c$. The gravitational coupling amplitude is thus $m_0\sqrt{G_p/(2s\hbar c)}$ to the gravitational zero point field, where $\sqrt{G_p}$ is the *primary amplitude for a Planck mass to emit or absorb a graviton*. Now this gravitational amplitude can be regarded as a complex vector just as colour and electromagnetism. We assumed for simplicity, as they are both spin 1 field particles, that colour and electromagnetism are parallel. Spin 2 gravity could be at a different complex angle to the other two. In fact the equations only have the correct properties if gravity is at right angles to colour and electromagnetism. Putting $G_{Primary} = \chi'_G \cdot G_{Secondary}$ we conjecture that:

$$\begin{aligned} \text{The gravitational coupling amplitude is } im_0\sqrt{G_p/(2s\hbar c)} &= im_0\sqrt{\chi'_G \cdot G_s/(2s\hbar c)} \\ &= im_0\sqrt{\chi'_G \cdot G/(2s\hbar c)} \end{aligned} \quad (4.2. 3)$$

Where we have put the *secondary gravitational coupling constant to a bare Planck mass* G_s in Eq. (4.2. 3) equal to the measured gravitational constant G and temporarily labelled the ratio between the primary the primary and secondary gravitational constants as χ'_G and return to this in section 6.2.3. So modifying Eq's. (2.2. 1) to (2.2. 3) by adding Eq. (4.2. 3)

$$Q^2A^2 = \left[\frac{\left[8 + 8\sqrt{\alpha_{EMP}} + im_0\sqrt{\chi'_G \cdot G/(2s\hbar c)} \right]^2}{3\pi sN} \hbar^2 k^4 r^2 \right] \cdot \left[\frac{sN \cdot dk}{k} \right]$$

$$Q^2 A^2 = \left[\frac{[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2 \right] \cdot \left[\frac{sN(1 + \varepsilon') dk}{k} \right] \text{ where } \varepsilon' = \frac{m_0^2 \chi'_G \cdot G}{2s\hbar c(8 + 8\sqrt{\alpha_{EMP}})^2}$$

Our previous wavefunctions ψ_k required $Q^2 A^2 = \frac{[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2$ from Eq. (2.2. 4).

Thus primary graviton interaction can increase the probability of our previous wavefunctions ψ_k by $1 + \varepsilon'$ as required to obtain precision in our integrals for $\hbar / 2 \& \hbar$ if $K_{nk} \text{ cutoff} < \infty$.

$$\text{Using Eq.(4.2. 2) now put } \varepsilon' = \frac{m_0^2 \chi'_G \cdot G}{2s\hbar c(8 + 8\sqrt{\alpha_{EMP}})^2} = \varepsilon = \frac{1}{K_{nk}^2 \text{ cutoff}} = \frac{2m_0^2 c^2}{sn^2 \hbar^2 (k_{\text{cutoff}})^2} \quad (4.2. 4)$$

Thus

$$\frac{\chi'_G \cdot G}{4\hbar c(8 + 8\sqrt{\alpha_{EMP}})^2} \approx \frac{c^2}{n^2 \hbar^2 (k_{\text{cutoff}})^2}$$

$$\frac{\chi'_G}{256(1 + \sqrt{\alpha_{EMP}})^2} \frac{G\hbar}{c^3} \approx \frac{1}{n^2 (k_{\text{cutoff}})^2}$$

$$\text{But } L_P^2 = \frac{G\hbar}{c^3} \quad \text{and} \quad \frac{\chi'_G \cdot L_P^2}{256(1 + \sqrt{\alpha_{EMP}})^2} \approx \frac{1}{n^2 (k_{\text{cutoff}})^2}$$

$$\text{For } N = 1 \text{ single integer } n \text{ superpositions } \chi'_G \approx \frac{256(1 + \sqrt{\alpha_{EMP}})^2}{n^2 (k_{\text{cutoff}} L_P)^2} \quad (4.2. 5)$$

For $N = 1$ superpositions $\psi_k = \sum_n c_n \psi_{nk}$, we can use the logic of section 3.5.1; replacing K_{nk}^2 with $\langle K_k \rangle^2$, and n^2 with $\langle n \rangle^2$ in Eq. (4.2. 4), so that Eq. (4.2. 5) becomes

$$\text{For } N = 1 \text{ multiple integer } n \text{ superpositions } \chi'_G \approx \frac{256(1 + \sqrt{\alpha_{EMP}})^2}{\langle n \rangle^2 (k_{\text{cutoff}} L_P)^2} \quad (4.2. 6)$$

If we now go back to Eq's. (2.3. 9) & (2.3. 10) as $k \rightarrow \infty$ the energy squared $E_{nk}^2 \rightarrow \mathbf{p}_{nk}^2 c^2 = n^2 \hbar^2 \omega^2$. Again using the logic of section 3.5.1) for multiple integer n superpositions the expectation value for energy squared as $k \rightarrow \infty$ is $\langle E_k \rangle^2 \rightarrow \langle |\mathbf{p}_k| \rangle^2 c^2 = \langle n \rangle^2 \hbar^2 k^2 c^2$ thus

$$\text{For multiple integer } n \text{ superpositions as } k \rightarrow \infty, \quad \langle E_k \rangle \rightarrow \langle |\mathbf{p}_k| \rangle c = \langle n \rangle \hbar kc \quad (4.2. 7)$$

4.2.2 All $N = 1$ superpositions cutoff at Planck Energy but interactions at less

It is reasonable to assume that the cutoff superposition energy cannot exceed the Planck energy E_{Planck} (at least for square cutoffs) and that this is true for all $N = 1$ superpositions. (Section 6.2.1 discusses $N = 2$ superposition E_{Planck} cutoffs.) So for *simple square cutoffs*:

$$N = 1 \text{ multiple integer } n \text{ superpositions cutoff energy } \langle E_{k(cutoff)} \rangle = \langle n \rangle \hbar k_{cutoff} c = E_{Planck} \quad (4.2. 8)$$

$$\text{This can be written as } \langle n \rangle k_{cutoff} \hbar c = E_{Planck} = \frac{\hbar c}{L_{Planck}}$$

$$\text{For } N = 1 \text{ multiple integer } n \text{ superpositions } \langle n \rangle k_{cutoff} = \frac{1}{L_{Planck}} \quad \& \quad \langle n \rangle k_{cutoff} L_P = 1 \quad (4.2. 9)$$

$$N = 1 \text{ multiple integer } n \text{ superposition interaction cutoff energy } \hbar c k_{cutoff} = \frac{E_{Planck}}{\langle n \rangle} \quad (4.2. 10)$$

Using Eq. (4.2. 10) with Planck energy $1.22 \times 10^{19} \text{ GeV}$. and $\langle n \rangle \approx 6.0135$ from Eq.(3.5. 16) for *simple square cutoffs* (also see Figure 4.1. 2).

$$\text{Interactions between } N = 1 \text{ fermions cutoff @ } \approx 2.0288 \times 10^{18} \text{ GeV}. \quad (4.2. 11)$$

From Table 4.3. 1 we see that all other particles such as photons, gluons and gravitons etc. have $\langle n \rangle < 6$ and thus higher interaction cutoff energies than fermions ie. $> 2.03 \times 10^{18} \text{ GeV}$., but $< E_p$. Putting $2.0288 \times 10^{18} \text{ GeV}$. in the Standard Model equations (4.1. 3) & (4.1. 4).

$$\begin{aligned} \alpha_1^{-1} @ 2.0288 \times 10^{18} \text{ GeV}. &\approx 34.4179 \pm 0.08 @ k(cutoff) \\ \alpha_2^{-1} &\dots\dots\dots \approx 48.5707 \pm 0.04 \dots\dots\dots \\ \alpha_3^{-1} &\dots\dots\dots \approx 50.4053 \pm 0.22 \dots\dots\dots \\ \alpha_{EM}^{-1} &= \frac{5}{3} \alpha_1^{-1} + \alpha_2^{-1} \dots\dots \approx 105.934 \pm 0.173 \dots\dots\dots \end{aligned} \quad (4.2. 12)$$

Real world high energy secondary interactions only involve α_1, α_2 & α_3 , but spin zero primary interactions do not involve the weak force. Table 4.1. 1 can thus only predict $\alpha_{EM}^{-1} \approx 105.921$ at the cutoff compared to the **Standard Model combination of $(5/3)\alpha_1^{-1} + \alpha_2^{-1} = \alpha_{EM}^{-1} \approx 105.934 \pm 0.173$** of Eq. (4.2. 12). (See Figure 4.1. 1 & Figure 4.1. 2). Also using Eq's. (3.3. 3) & (4.2. 12) we get the primary to secondary fundamental coupling ratio χ_C .

$$\text{Coupling Ratio } \chi_C = \alpha_3^{-1} @ k_{cutoff} \approx 50.405 \pm 0.22 \text{ (ie. @ } 2.0288 \times 10^{18} \text{ GeV).} \quad (4.2. 13)$$

If we now put Eq. (4.2. 9) into Eq. (4.2. 6) we get $\chi'_G \approx \frac{256(1+\sqrt{\alpha_{EMP}})^2}{\langle n \rangle^2 (k_{cutoff} L_P)^2} = 256(1+\sqrt{\alpha_{EMP}})^2$

From Eq's.(4.1. 2) and Table 4.3. 1 we find $(1+\sqrt{\alpha_{EMP}}) \approx 1.115$ and Eq.(4.2. 6) becomes

$$\chi'_G \approx 256(1.115)^2 \approx 318.3 \quad (4.2. 14)$$

Using Eq. (4.2. 3) $\chi'_G \approx 318.3$ is the ratio between the primary graviton coupling to bare preons, and the *normal measured gravitational constant (Big G)*. In other words the primary graviton coupling to preons is $\alpha_G(\text{Primary}) \approx (318.3)G$. (Section 5.1.1, Eq. (5.1. 8) defines the secondary graviton coupling between Planck masses α_G and section 5.3.2, Eq. (5.3. 14) finds contrary to expectations that $\alpha_G \approx 1/2,800$ so as in Eq.(6.2. 7) *the primary to secondary graviton coupling ratio is $\chi_G \approx 2,800$ and $\chi'_G \approx 2,800 \times 318.3 \approx 890,000$.*) When $\chi'_G \approx 318.3$ in Eq.(4.2. 4) the contribution from gravity (the ε' in Eq.(4.2. 4)) cancels any deficit in primary interactions (the ε in Eq.(4.2. 4)) if these superpositions cutoff at Planck energy, which we argue is true for all $N=1$ superpositions. (Sections 6.2 & 6.2.1 discuss $N=2$ superposition E_p cutoffs.) To enable high energy interactions $N=2$ bosons must also cutoff at Planck energy just as $N=1$ superpositions do, or as in Eq. (4.2. 10). Figure 4.2. 1 plots radial probabilities for all $n=3,4,5,6\&7$ Planck Energy cutoff modes. They are identical as the radial probability $P_R \propto r^8 \times \text{Exp}(n^2 k^2 r^2 / 9)$, but from Eq. (4.2. 7) $nk=1$ in each Planck energy mode, so they all have radial probability $P_R \approx 8.74 \times 10^{-6} \times r^8 \text{Exp}(r^2 / 9)$.

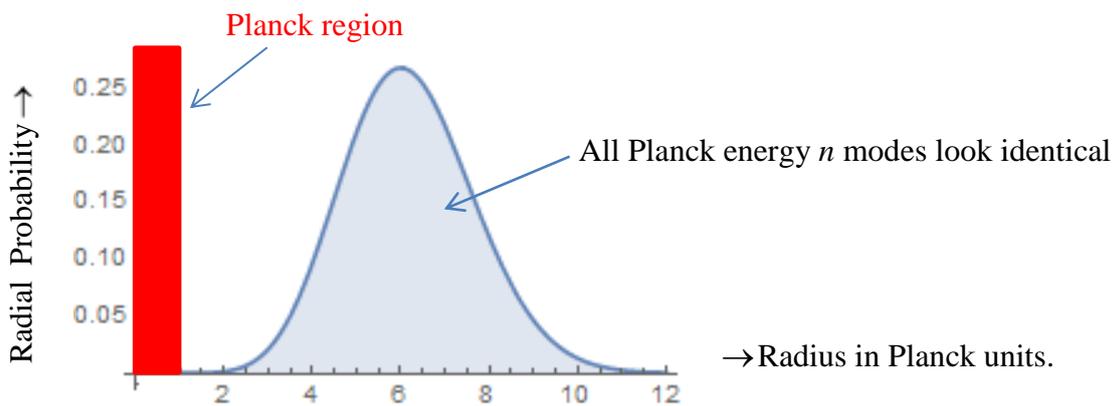


Figure 4.2. 1

Despite each $n=3,4,5,6\&7$ mode having Planck energy the probability in every case of being inside the Planck region is virtually zero at $\approx 8.9 \times 10^{-7}$.

4.3 Solving for spin 1/2, spin 1 and spin 2 superpositions

Superpositions with $N=2$ are covered in section 6.2 but Eq.(4.2. 13) and Eq. (3.3. 22) extended by keeping $N \cdot s$ constant as in Eq. (4.4. 1) allow us to solve various combinations of spins 1/2, 1 or 2 and $N=1$ or $N=2$.

$$\begin{aligned}
 & (N=2) \times (\text{Spin } 1) & (N=1) \times (\text{Spin } 1) & (N=1) \times (\text{Spin } 1/2) \\
 & \text{or } (N=1) \times (\text{Spin } 2) & \text{or } (N=2) \times (\text{Spin } 1/2) & \\
 4c_{4b} * c_{4b} (1 - c_{4b} * c_{4b}) & = 2c_{5b} * c_{5b} (1 - c_{5b} * c_{5b}) & = c_{6a} * c_{6a} (1 - c_{6a} * c_{6a}) & (4.4. 1) \\
 & = \sqrt{2 / \chi_C} \approx \sqrt{2 / 50.4053} \approx 0.199194 & &
 \end{aligned}$$

Starting with spin 1/2 we can solve this to get $c_6 * c_6 \approx 0.7254$ as the dominant value.

Putting $c_6 * c_6 \approx 0.7254$ into Eq.(4.1. 2) or alternatively using Table 4.1. 1

$$\alpha_{EMP} \approx \left[\sqrt{1.386256 - 0.197258 c_6 * c_6} - 1 \right]^2 \approx 75.6499^{-1} \quad (4.4. 2)$$

From Eq. (2.2. 4) the available $Q^2 A^2 = \frac{[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2$ with probability $\frac{sN \cdot dk}{k}$

where we ignore the infinitesimal factor of $(1 + \varepsilon)$ due to gravitons. And from Eq. (2.3. 12)

$$\begin{aligned}
 Q^2 A^2 & = \sum_n c_n * c_n \frac{n^4 \hbar^2 k^4 r^2}{81} = \frac{[8 + 8\sqrt{\alpha_{EMP}}]^2}{3\pi sN} \hbar^2 k^4 r^2 \\
 \sum c_n * c_n \cdot n^4 & \approx 1367.58 \text{ for (spin } 1/2 \times N = 1) \\
 & \approx 683.79 \text{ for (spin } 1 \times N = 1) \text{ or (spin } 1/2 \times N = 2) \\
 & \approx 341.9 \text{ for (spin } 1 \times N = 2) \text{ or (spin } 2 \times N = 1) \\
 & \approx 170.95 \text{ for (spin } 2, N = 2) \text{ by extension.} & (4.4. 3)
 \end{aligned}$$

The same primary electromagnetic coupling α_{EMP} builds all fundamental particles, allowing Eq.(4.4. 3) to be true. Using Eq's (4.4. 1),(4.4. 3) & $\sum_n c_n * c_n = 1$ we get Table 4.3. 1. We define the coupling ratio for gravitons $\chi_G \approx 827,000$ in Eq.(6.2. 7) section 6.2.3, where we also solve infinitesimal mass graviton superpositions. In Table 4.3. 1 three member superpositions fit the Standard Model best. In section 4.1 we solved spin 1/2 superpositions with a dominant centre mode $c_6 * c_6$ that fitted the Standard Model. However when solving for spins 1 & 2 we must initially comply with Eq. (4.4. 1) which defines interaction probabilities (see Eq. (3.3. 22) and final paragraph section 3.3.4). We must also comply with Eq.(4.4. 3) which determines centre or side mode dominance. In this table we have also included a massive $N=1$ spin 2 graviton type Dark Matter possibility interacting only with $N=2$ spin 2 gravitons. There are other possibilities which we have not included. To this point this paper has attempted to demonstrate that infinite superpositions can behave as the

Standard Model fundamental particles. The methods used may seem unconventional, but it is important to remember that primary interactions are very different to secondary interactions (see sections 7& 2.2.3). These methods are however based on simple quantum mechanics and relativity, and there is also surprising consistency with the Standard Model. If the principles behind the outcomes of these derivations are at least on the right track and fundamental particles can be built by borrowing energy and mass from zero point fields then, as we will see in what follows, this may possibly have some significant and profound consequences.

Mass Type	Spin	N	$c_3 * c_3$	$c_4 * c_4$	$c_5 * c_5$	$c_6 * c_6$	$c_7 * c_7$
Infinitesimal mass gravitons	2	2	0.8346	1.4×10^{-6}	0.1653		
Infinitesimal mass bosons	1	2	0.4847	0.0526	0.4627		
Massive (dark matter?) gravitons	2	1	0.4847	0.0526	0.4627		
Massive bosons	1	1		0.0134	0.8878	0.0988	
Massive fermions	$\frac{1}{2}$	1			0.1305	0.7254	0.1441

Table 4.3. 1 Approximate probabilities for various possible superpositions.

5 The Expanding Universe and General Relativity

5.1 Zero point energy densities are limited

If the fundamental particles can be built from energy borrowed from zero point fields and as this energy source is limited, (particularly at cosmic wavelengths) there must be implications for the maximum possible densities of these particles. In section 2.2.3 we discussed how the preons that build fundamental particles are born from a Higg's type scalar field with zero momentum in the laboratory rest frame. In this frame they have an infinite wavelength and can thus be borrowed from anywhere in the universe. This would suggest that there should be little effect on localized densities, but possibly on overall average densities in any or all of these universes. So which fundamental particle is there likely to be most of? *Working in Planck, or natural units with $G=1$ we will temporarily assume the graviton coupling constant between Planck masses is one.* (We will modify this later but it helps to illustrate the problem.) As an example there are approximately $M \approx 10^{61}$ Planck masses within our causally connected region of the universe. They have an average distance between them of approximately the radius R_{CCH} of this region. Thus there should be approximately $M^2 \approx 10^{122}$ virtual gravitons with wavelengths of the order of radius R_{CCH} within this same volume. No other fundamental particle is likely to approach these values, for example the number of virtual photons *of this extreme wavelength* is much smaller. (Virtual particles emerging from the vacuum are covered in section 6.2.2.) If this density of virtual gravitons needs to borrow more energy from the zero point fields than what is available at these extreme wavelengths does this somehow control the maximum possible density of a causally connected universe?

5.1.1 Virtual graviton density at wavenumber k in a causally connected Universe

From here on we will work in natural or Planck units where $\hbar = c = G = 1$.

General Relativity predicts nonlinear fields near black holes, but in the low average densities of typical universes we can assume approximate linearity. The majority of mass moves slowly relative to comoving coordinates so we can ignore momentum (i.e. $\beta \ll 1$), provided we limit this analyses to comoving coordinates. In these comoving coordinates the vast majority of virtual gravitons will thus be scalar. We should also be able to simply apply the equations in sections 3.4 & 3.5 to spin 2 virtual graviton emissions, as they should apply equally to both spins 1 & 2. We will assume spherically symmetric $l=3$ wavefunctions emit both spin 1 & 2 scalar virtual bosons, and $l=3, m=\pm 2$ states can emit both $m=\pm 1$ spin 1 bosons and $m=\pm 2$ spin 2 gravitons. Section 3.4 derived the electrostatic energy between infinite superpositions. In flat space we looked at the amplitude that each equivalent point charge emits a virtual photon, and then focused on the interaction terms between them. Thus we can use the same scalar wavefunctions Eq's. (3.4. 1) for virtual scalar gravitons as we did for virtual scalar photons. Using $(\psi_1 + \psi_2) * (\psi_1 + \psi_2) = \psi_1 * \psi_1 + \psi_1 * \psi_2 + \psi_2 * \psi_1 + \psi_2 * \psi_2$ we showed in section 3.4.1 that the interaction term for virtual photons is

$$\psi_1 * \psi_2 + \psi_2 * \psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \cos k(r_1 - r_2) \quad (5.1. 1)$$

This equation is strictly true only in flat space but it is still approximately true if the curvature is small or when $2m/r \ll \ll 1$, which we will assume applies almost everywhere throughout the universe except in the infinitesimal fraction of space close to black holes. In both sections 3.4 & 3.5, for simplicity and clarity, we delayed using coupling constants and emission probabilities in the wavefunctions until necessary. We do the same here. There will also be some minimum wavenumber k which we call k_{\min} where for all $k < k_{\min}$ there will be insufficient zero point energy available. We need Eq. (5.1. 1) to apply for all values of k down to this minimum value which we find is $k_{\min} \approx 1/R_{\text{CausallyConnectedHorizon}}$. In Section 6 we find gravitons have an infinitesimal rest mass m_0 of the same order as this minimum wavenumber k_{\min} . At these extreme k values this rest mass must be included in the wavefunction exponential term. It is normally irrelevant for infinitesimal masses. Section 6.2 looks at $N = 2$ infinitesimal rest masses finding $\langle K_{k_{\min}} \rangle^2 \approx 1$. Using Eq.(3.1. 11) with $\hbar = c = 1$

$$\langle K_{k_{\min}} \rangle^2 = \frac{s \langle n \rangle^2 k_{\min}^2}{2m_0^2} \approx 1 \text{ and for spin 2 gravitons } \frac{\langle n \rangle^2 k_{\min}^2}{m_0^2} \approx 1 \text{ or } m_0 \approx \langle n \rangle k_{\min} \quad (5.1. 2)$$

From Table 4.3. 1 we find

$$\text{For } N = 2 \text{ spin 2 gravitons } \langle n \rangle \approx 3.33 \text{ so that } m_0 \approx 3.33 k_{\min} \quad (5.1. 3)$$

This virtual mass m_0 introduces an extra exponential decay term $e^{-m_0 r}$ in the virtual graviton wavefunction Eq. (3.4. 1) $e^{-kr+ikr} \rightarrow e^{-kr+ikr} e^{-m_0 r} \rightarrow e^{-(k+m_0)r+ikr}$. Define k' using Eq. (5.1. 3)

$$k' = k + m_0 \approx k + 3.33k_{\min} \quad \text{and} \quad k'_{\min} \approx k_{\min} + 3.33k_{\min} \approx 4.33k_{\min} \quad (5.1. 4)$$

The normalized virtual graviton wavefunction in Eq. (3.4. 1)

$$\psi = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr+ikr}}{r} \quad \text{becomes} \quad \sqrt{\frac{2k'}{4\pi}} \frac{e^{-k'r+ikr}}{r} \quad (5.1. 5)$$

Thus the interaction term in Eq. (5.1. 1) becomes

$$\psi_1 * \psi_2 + \psi_2 * \psi_1 = \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1+r_2)} \cos k(r_1 - r_2) \quad (5.1. 6)$$

In Eq. (5.1. 6) constant $r_1 + r_2$ describe ellipses, and constant $r_1 - r_2$ describe hyperbolae. Integrating over all space at constant k and k' using elliptical coordinates in Figure 3.4. 2 and putting r as the distance between the two Planck masses or charges:

$$\text{At any wavenumber } k; \quad \iiint (\psi_1 * \psi_2 + \psi_2 * \psi_1) dv = \frac{2e^{-k'r} \sin(kr)}{kr} \quad (5.1. 7)$$

Now consider *one Planck mass* at any point P somewhere in the interior region of a typical universe, and let the average density be ρ_U (subscript U for homogeneous universe density) Planck masses per unit volume. Consider spherical shells around point P of radius r and thickness dr with $dm = \rho_U dv = 4\pi r^2 dr \rho_U$. Now we expect the graviton coupling constant α_G to be =1 between Planck masses but because we do not really know this let us define

$$\text{The Secondary graviton coupling constant between Planck masses} = \alpha_G \quad (5.1. 8)$$

Section 3.4.1 in Eq. (3.4. 3) used a scalar emission probability $(2\alpha / \pi)(dk / k)$ which becomes $(2\alpha_G / \pi)(dk / k)$ between Planck masses. (We return to this in section 5.3.2) Now quantum interactions are instantaneous over all space but distant galaxies recede at light like velocities. However at the *same cosmic time* T in all comoving coordinate systems, clocks tick at the same rate, and a wavenumber k (or frequency) in one comoving coordinate system measures the same in all comoving coordinate systems. Thus as we integrate from radius $r = 0 \rightarrow \infty$ we can still use the same equations as if the distant galaxies were not moving. (The vast majority of mass is moving relatively slowly in these comoving coordinate systems and we have already used this constant clock rate when integrating Eq. (5.1. 6) over all space to get

Eq.(5.1. 7) and we return to this important comoving coordinate property in section 5.3.1). Integrating over all radii the total number of virtual gravitons *interacting with this point Planck mass* using Eq's. (5.1. 7) & (5.1. 4) is

$$\begin{aligned}
\left(\frac{2\alpha_G}{\pi} \cdot \frac{dk}{k}\right) \int_{r=0}^{\infty} \frac{2e^{-kr} \sin(kr)}{kr} \cdot 4\pi r^2 dr \rho_U &= \frac{16\alpha_G \rho_U dk}{k^2} \int_{r=0}^{\infty} r e^{-kr} \sin(kr) dr \\
&= \frac{16\alpha_G \rho_U dk}{k^2} \left[\frac{2k'k}{(k'^2 + k^2)^2} \right] \\
&\approx \frac{16\alpha_G \rho_U dk}{k^2} \left[\frac{2(k + 3.33k_{\min})k}{[(k + 3.33k_{\min})^2 + k^2]^2} \right] \\
&\approx \frac{32\alpha_G \rho_U dk}{k} \left[\frac{k + 3.33k_{\min}}{[2k^2 + 6.66k_{\min}k + 11.09k_{\min}^2]^2} \right] \quad (5.1. 9)
\end{aligned}$$

Where we have reexpanded $k' \rightarrow k + m_0 \approx k + 3.33k_{\min}$ using Eq. (5.1. 4). This is the virtual graviton coupling at wavenumber k between one Planck mass and all other Planck masses. In a homogeneous universe we can carry out this same integral at all points (*at the same cosmic time T*). To get the total virtual graviton density we thus multiply Eq. (5.1. 9) by $\rho_U / 2$ (so as to not count all pairs of Planck masses twice). We need to also integrate over all wavenumbers $k < k_{\min}$ and for simplicity assume a square cutoff at k_{\min} . So we integrate from $k = k_{\min}$ to $k = 1$. As $k_{\min} \approx 10^{-61}$ in Planck units, to aid integration we use the variable u

Define the variable $u = kR_{\text{CausallyConnectedHorizon}}$ in radians. (5.1. 10)

Let $Y = u_{\min} = k_{\min} R_{\text{CausallyConnectedHorizon}}$

Multiplying Eq. (5.1. 9) by $\rho_U / 2$ and using Eq.(5.1. 10) then integrating over $u = Y \rightarrow \infty$

$$\begin{aligned}
\text{Total Graviton Density } \rho_G &\approx 16\alpha_G \rho_U^2 R_{\text{CCH}}^3 \int_Y^{\infty} \frac{du}{u} \left[\frac{u + 3.33Y}{[2u^2 + 6.66uY + 11.09Y^2]^2} \right] \\
&\approx 16\alpha_G \rho_U^2 R_{\text{CCH}}^3 A \quad (5.1. 11)
\end{aligned}$$

Where for brevity we have simply put $A = \int_Y^{\infty} \left[\frac{u + 3.33Y}{[2u^2 + 6.66uY + 11.09Y^2]^2} \right] \frac{du}{u}$

5.2 Relating this to General Relativity

The above assumes that at any cosmic time T , there is always some value k_{\min} where the borrowed energy density $E_{Gk_{\min}} = E_{ZP_{\min}}$ the available zero point energy density @ k_{\min} . We have also assumed so far that the mass in the universe is like a perfect fluid and homogeneous, also that space is essentially flat on average. Thus all observers fixed relative to comoving coordinates and at the same cosmic time T must measure the same virtual

graviton density ρ_G as in Eq. (5.1. 11). This equation must be true for all such comoving observers in a homogeneous universe. If this graviton density ρ_G is at a zero point energy borrowing limit we can perhaps expect it to always be uniform at any cosmic time T or at least some form of upper limit. So what happens if we now change the uniform fluid mass density ρ_U by putting an initially small mass concentration $+m_1$ at some point? Because near mass concentrations we would expect the local graviton density ρ_G to increase. However General Relativity tells us that near mass concentrations the metric changes, radial rulers shrink and local observers measure larger radial lengths. This expands volumes locally and should lower their measurement of the background ρ_G . Can these effects balance each other, so that the extra gravitons generated by a nearby mass concentration can bring this density back to the flat space or background ρ_G of Eq.(5.1. 11). Changes in the metric will also change the local measurement of $k_{\min} \rightarrow k''_{\min} = k_{\min} (1 - 2m/r)^{-1/2}$ but we will initially only consider the case where $m/r \ll \ll \ll 1$ and look again at this at the end of section 5.2.2.

5.2.1 Approximations that are necessary with possibly important consequences

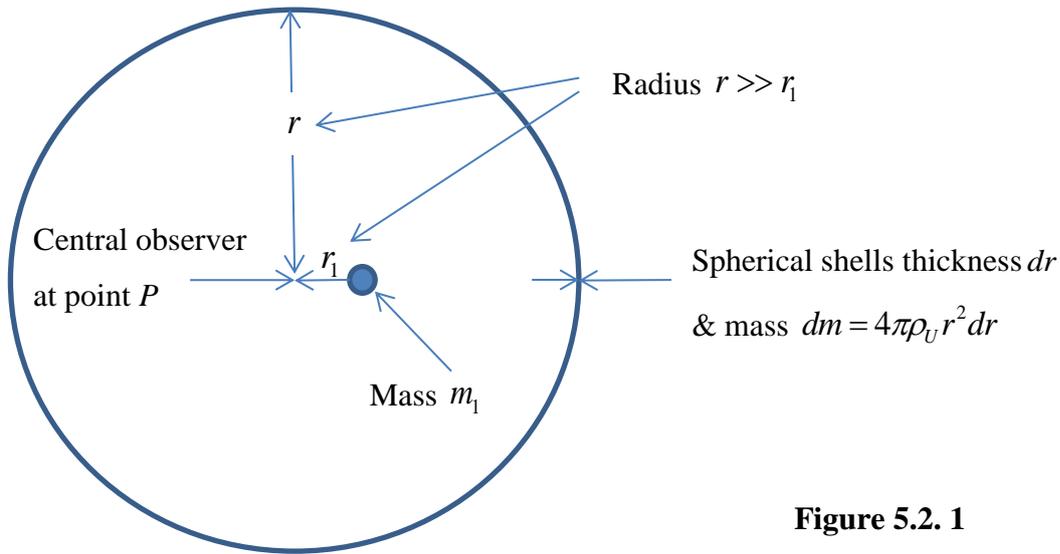


Figure 5.2. 1

Let us refer back to Eq. (3.4. 2) and the steps we took in section 3.4.1 to derive it; but now including $k' = k + m_0 \approx k + 3.33k_{\min}$ as in Eq. (5.1. 6)

$$\psi_1 * \psi_2 + \psi_2 * \psi_1 = \frac{4k'}{4\pi r_1 r_2} e^{-k'(r_1+r_2)} \cos[k(r_1 - r_2)] \quad (5.2. 1)$$

And assume that *space has to be approximately flat* with errors $\propto 1 - (1 - 2m/r)^{1/2} \approx m/r$. If we now focus on Figure 3.4. 2 , equation (5.2. 1) is the probability that a virtual graviton of wavenumber k is at the point P if all other factors are one. Let us now put a mass of m_1 Planck masses at the Source 1 point in Figure 3.4. 2 or as in Figure 5.2. 1. Also assume that the point P is reasonably close to mass m_1 (in relation to the horizon radius) at distance r_1 as in Figure 5.2. 1 and the vast majority of the rest of the mass inside the causally connected

horizon R_{CCH} is at various radii r , equal to the r_2 of Eq. (5.2. 1) where $r_2 = r \gg r_1$ and thus $\cos[k(r_1 - r)] \approx \cos(-kr)$. Only under these conditions can we approximate Eq. (5.2. 1) as

$$\psi_1^* \psi_2 + \psi_2^* \psi_1 \approx \frac{4k'}{4\pi r_1 r} e^{-kr} \cos(-kr) \quad (5.2. 2)$$

As we have assumed average particle velocities are low (relative to comoving coordinates) this is a *scalar interaction* (as in section 3.4.1) and as *there are no directional effects* we can consider simple spherical shells of thickness dr and radius r around a central observer at the point P which have mass $dm = \rho_U 4\pi r^2 dr$. At each radius r the coupling factor $(2\alpha/\pi)(dk/k)$ we used in Eq. (3.4. 3) using Eq. (5.1. 8) becomes $(2\alpha_G/\pi)(dk/k)$ between Planck masses *and again we use the fact that all instantaneously connected comoving clocks tick at the same rate.*

$$\text{Coupling factor} = \frac{2\alpha_G m_1}{\pi} dm \frac{dk}{k} = \frac{2\alpha_G m_1}{\pi} \frac{dk}{k} \rho_U 4\pi r^2 dr \quad (5.2. 3)$$

Including this coupling factor in Eq. (5.2. 2)

$$\begin{aligned} \left(\frac{2\alpha_G m_1}{\pi} \frac{dk}{k} \rho_U 4\pi r^2 dr \right) (\psi_1^* \psi_2 + \psi_2^* \psi_1) &\approx \left(\frac{2\alpha_G m_1}{\pi} \frac{dk}{k} \rho_U 4\pi r^2 dr \right) \left(\frac{4k'}{4\pi r_1 r} e^{-kr} \cos(-kr) \right) \\ &\approx \alpha_G \frac{m_1}{r_1} \frac{8\rho_U}{\pi} \frac{k' dk}{k} r e^{-kr} \cos(-kr) dr \end{aligned} \quad (5.2. 4)$$

This is virtual graviton density at point P due to each spherical shell. (Ignoring the relatively small number of gravitons emitted by mass m_1 itself, see addendum 8). Integrating over radius $r = 0 \rightarrow \infty$ the virtual graviton density at wavenumber k using Eq's.(5.1. 4) & (5.2. 4)

$$\begin{aligned} \Delta\rho_G &= \alpha_G \frac{m_1}{r_1} \frac{8\rho_U}{\pi} \frac{k' dk}{k} \int_0^\infty r e^{-kr} \cos(-kr) dr \\ &= \alpha_G \frac{m_1}{r_1} \frac{8\rho_U}{\pi} \frac{k' dk}{k} \left[\frac{(k'^2 - k^2)}{(k'^2 + k^2)^2} \right] \\ &= \alpha_G \frac{m_1}{r_1} \frac{8\rho_U R_{CCH}}{\pi} \left[\frac{(u + 3.33\Upsilon)(6.66u\Upsilon + 11.09\Upsilon^2)}{u(2u^2 + 6.66u\Upsilon + 11.09\Upsilon^2)} \right] du \end{aligned} \quad (5.2. 5)$$

Integrating over wavenumbers $k = k_{\min} \rightarrow 1$ or $u = \Upsilon \rightarrow \infty$, the extra virtual graviton density $\Delta\rho_G$ at point P distance r_1 from mass m_1 is

$$\Delta\rho_G = \alpha_G \frac{m_1}{r_1} \frac{8\rho_U R_{CCH}}{\pi} \cdot B \quad (5.2. 6)$$

Where again for brevity we have put $\int_{\Upsilon}^{\infty} \left[\frac{(u + 3.33\Upsilon)(6.66u\Upsilon + 11.09\Upsilon^2)}{(2u^2 + 6.66u\Upsilon + 11.09\Upsilon^2)} \right] \frac{du}{u} = B$

But we have conjectured that the expansion of space due to GR is such that extra gravitons near a local mass concentration do not change the background density ρ_G in Eq.). If the local expansion of space due to GR is $\Delta V / V$ and the *new background graviton density* ρ_G' is to remain unchanged then

$$\text{New } \rho_G' \approx \frac{\rho_G + \Delta\rho_G}{1 + \Delta V / V} \approx \text{original } \rho_G \text{ implying } \frac{\Delta\rho_G}{\rho_G} \approx \frac{\Delta V}{V} \quad (5.2. 7)$$

Using Eq's. (5.1. 11) &(5.2. 6) the graviton coupling constant α_G cancels out:

$$\frac{\Delta V}{V} \approx \frac{\Delta\rho_G}{\rho_G} \approx \frac{\alpha_G \left[\frac{m_1}{r_1} \right] \frac{8\rho_U R_{CCH} \cdot B}{\pi}}{\alpha_G 16\rho_U^2 R_{CCH}^3 \cdot A} \approx \left[\frac{m_1}{r_1} \right] \left[\frac{1}{2\pi\rho_U R_{CCH}^2} \frac{B}{A} \right] \quad (5.2. 8)$$

Both integrals B & A are functions of $\Upsilon = k_{\min} R_{CCH}$. In comoving coordinates, at any cosmic time T , the red part of this equation must be a fixed value at all points inside the horizon and thus the expansion of space around any mass is proportional to m_1 / r_1 . The Schwarzschild solution to Einstein's equations tells us that the radial metric around mass m_1 changes as

$$\frac{\Delta r_{local}}{\Delta r_\infty} = \left(1 - \frac{2m_1}{r_1}\right)^{-1/2} \approx 1 + \frac{m_1}{r_1} \text{ when } r_1 \gg m_1 \text{ and the local change in volume } \frac{\Delta V}{V} \approx \frac{m_1}{r_1}.$$

We have been approximating to the first order in m_1 / r_1 so to this first order we can say

$$\frac{\Delta V}{V} \approx \frac{m_1}{r_1} \approx \left[\frac{m_1}{r_1} \right] \left[\frac{1}{2\pi\rho_U R_{CCH}^2} \frac{B}{A} \right] \quad (5.2. 9)$$

Provided the background graviton density ρ_G remains constant; GR tells us the **red highlighted part is approximately one in Planck units**. Numerically integrating both B & A

$$\text{The average density of the universe } \rho_U \approx \frac{1}{2\pi R_{CCH}^2} \left[\frac{B}{A} \right] \approx \frac{1}{2\pi R_{CCH}^2} [25.26\Upsilon^2] \quad (5.2. 10)$$

Putting Eq. (5.2. 10) the average density ρ_U into Eq.(5.1. 11) gives ρ_G in terms of B^2 & A which we can again numerically integrate in terms of $\Upsilon = k_{\min} R_{CCH}$.

$$\rho_G = 16\alpha_G (\rho_U^2) R_{CCH}^3 \cdot A = 16\alpha_G \left[\frac{1}{4\pi^2 R_{CCH}^4} \frac{B^2}{A^2} \right] \cdot R_{CCH}^3 A \quad (5.2. 11)$$

Graviton density

$$\rho_G = \frac{4\alpha_G}{\pi^2 R_{CCH}} \left[\frac{B^2}{A} \right] \approx \frac{4\alpha_G}{\pi^2 R_{CCH}} [5.28\Upsilon]$$

If our conjecture is true, this is the average density of gravitons excluding possible effects of virtual particles emerging from the vacuum. In section 6.2.2 we argue these do not change graviton density ρ_G in Eq. (5.2. 11). However graviton density ρ_G in Eq.(5.2. 11) does depend on the graviton coupling constant α_G between Planck masses but it cancels out in Eq.(5.2. 8) and does not effect the allowed universe average density ρ_U in Eq.(5.2. 10).

5.2.2 The Schwarzschild metric near large masses

At a radius r_1 from a mass m_1 the Schwarzschild metric is $(1-2m_1/r_1)^{\pm 1/2}$ for the time and radial terms. The radial term can be written as

$$\frac{1}{\sqrt{1-2m_1/r_1}} = \frac{1}{\sqrt{1-\beta^2}} = \gamma \quad (5.2. 12)$$

Velocity β ($c = 1$) is that reached by a small mass falling from infinity and $\gamma^{\pm 1}$ is the metric change in clocks and rulers due to mass m_1 . Differentiating the metric at fixed radius r_1

$$d \left[1 - \frac{2m_1}{r_1} \right]^{-1/2} = \frac{dm_1}{r_1} \left[1 - \frac{2m_1}{r_1} \right]^{-3/2} = \gamma^3 \frac{dm_1}{r_1}$$

We can write this as the change in the radial metric $\Delta\gamma = \gamma^3 \frac{\Delta m_1}{r_1}$ (5.2. 13)

Where $\Delta\gamma$ is the change in the metric from adding mass Δm_1 to mass m_1 . With zero mass $m_1 = 0$ the unexpanded unit volume $V = 1$. When $m_1 > 0$ this volume becomes $V = \gamma$ so that

$$\frac{\Delta V}{V} = \frac{\Delta\gamma}{\gamma} = \gamma^2 \frac{\Delta m_1}{r_1} \quad (5.2. 14)$$

Now the mass m_1 in the Schwarzschild metric is that of *the mass dispersed at infinity before it comes together*. So let us start with a central mass m_1 (as measured at infinity) and then bring in from infinity a small extra mass Δm_1 (as measured at infinity), and repeat the derivation of Eq.(5.2. 5) in section 5.2.1. Because of the metric $\gamma > 1$ at point P in Figure 5.2. 1 due to m_1 (compared to the metric $\gamma = 1$ at infinity) an observer at point P measures the *total mass of all the spherical shells* inside his causally connected horizon as $a \equiv \gamma \int (dm)$. This same observer at P also measures the added mass Δm_1 as $b \equiv \gamma(\Delta m_1)$. These *two factors a & b* of γ together modify Eq.(5.2. 6) where $\Delta\rho_G$ is now the *extra graviton density* at point P due to adding mass Δm_1 :

$$\Delta\rho_G \approx \gamma^2 \alpha_G \left[\frac{\Delta m_1}{r_1} \right] \frac{8\rho_U R_{CCH}}{\pi} \cdot B \quad (5.2. 15)$$

Equations (5.2. 8) & (5.2. 9) are also modified, where ΔV is now the modified volume expansion at point P predicted by GR from adding mass Δm_1 where α_G cancels out again:

$$\frac{\Delta V}{V} = \frac{\Delta \rho_G}{\rho_G} \approx \gamma^2 \frac{\Delta m_1}{r_1} \approx \gamma^2 \left[\frac{\Delta m_1}{r_1} \right] \left[\frac{1}{2\pi\rho_U R_{CCH}^2} \frac{B}{A} \right]$$

Again the red parts are equal to approximately one and we have agreement with Eq. (5.2. 14).

$$\frac{\Delta V}{V} \approx \frac{\Delta \rho_G}{\rho_G} \approx \frac{\Delta \gamma}{\gamma} \approx \gamma^2 \frac{\Delta m_1}{r_1} \quad (5.2. 16)$$

Thus the Schwarzschild metric around any mass concentration is consistent with the conjecture that the *background virtual graviton density* ρ_G is uniform at any cosmic time T for comoving observers. But we have not yet discussed the effect of the metric on the local measured value of $k'' = \gamma k$. The vast majority of gravitons have wavelengths $\tilde{\lambda} = 1/k$ spanning towards the horizon. Almost 99% span $> R_{CCH} / 4$. We can thus let k tend towards k_{\min} in the product kr of the key factors in our integrals, in particular e^{-kr} & $\cos(-kr)$. Close to large black holes in the region where the metric $\gamma \gg 1$ the relevant radius $r \llll (R_{CCH} \approx 1/k_{\min})$ and the product $k''_{\min} r \rightarrow \gamma k_{\min} r \rightarrow 0$, also $e^{-k'r} \approx \cos(-kr) \approx 1$. There is an extremely localized perturbation of the exponentially decaying wavefunction. The metric γ rapidly $\rightarrow 1$ with radius r , and γk_{\min} also rapidly $\rightarrow k_{\min}$. The overall effect on the integrals is insignificant. This is only true if *outside observers* see infalling mass remaining on the event horizon, and *gravitons emitted from that horizon*. Also we assumed in our derivations that space is approximately flat on average. For the same reasons as above, as the region around even large black holes (where space is far from flat) is insignificant in relation to the volume over which we integrate, the overall effect is again very small.

5.3 The Expanding Universe

Equation (5.2. 11) tells us the density of gravitons is $\rho_G = \frac{4\alpha_G}{\pi^2 R_{CCH}} \frac{B^2}{A} \approx \frac{4\alpha_G}{\pi^2 R_{CCH}} (5.28Y)$ but using Eq.(5.1. 10) $Y = k_{\min} R_{CCH}$ we can write this as $\rho_G \approx \frac{(21.11)\alpha_G k_{\min}}{\pi^2}$. All these gravitons are superpositions of wavefunctions ψ_k occurring with probability $\frac{sN \cdot dk}{k}$ from Eq. (2.1. 4).

The density of any ψ_k wavefunction required is thus $\rho_{\psi k} = \rho_G \frac{sN \cdot dk}{k} \approx \left[\frac{(21.11)\alpha_G k_{\min}}{\pi^2} \right] \frac{sN \cdot dk}{k}$
or $\rho_{\psi k} \approx \frac{(21.11)\alpha_G sN}{\pi^2} \frac{k_{\min}}{k} \cdot dk$ and for **spin 2 & $N = 2$** gravitons

$$\text{Wavefunction density } \rho_{\psi k} \approx \frac{21.11 \times 2 \times 2 \times \alpha_G}{\pi^2} \frac{k_{\min}}{k} \cdot dk \approx \frac{84.44 \alpha_G}{\pi^2} \frac{k_{\min}}{k} \cdot dk \quad (5.3. 1)$$

From Eq.(3.2. 1) the vacuum debt for a superposition is $\langle \mathbf{p}_k(\text{debt}) \rangle = -\langle \beta_k \rangle^2 \langle n \rangle \hbar \mathbf{k}$. From Eq's.(3.1. 11), (3.1. 12) & (3.2. 10) $\langle \beta_k \rangle^2 = \frac{\langle K_k \rangle^2}{1 + \langle K_k \rangle^2}$ and for $N = 2$ spin 2 $\langle K_k \rangle = \frac{\langle n \rangle k}{m_0}$. From

Eq. (5.1. 3) $m_0 \approx 3.33k_{\min}$ from which we can show $\langle \beta_k \rangle^2 = \frac{k^2}{k^2 + k_{\min}^2}$. From Table 4.3. 1 we

find $\langle n \rangle \approx 3.33$ for gravitons, so *each wavefunction ψ_k borrows from the zero point fields*

$$\langle \beta_k \rangle^2 \langle n \rangle \approx \frac{(3.33)k^2}{k^2 + k_{\min}^2} \text{ quanta of wavenumber } k \text{ so the:}$$

$$\begin{aligned} \text{Density of quanta required @ } k \text{ by gravitons } \rho_{\text{Quanta}@k} &\approx \frac{(3.33)k^2}{k^2 + k_{\min}^2} \frac{84.44\alpha_G}{\pi^2} \frac{k_{\min}}{k} dk \\ &\approx \frac{281\alpha_G}{\pi^2} \frac{kk_{\min}}{k^2 + k_{\min}^2} dk \end{aligned} \quad (5.3. 2)$$

When $k \rightarrow k_{\min}$ Eq. (5.3. 2) becomes $\rho_{\text{Quanta}@k} \approx (14)\alpha_G dk$, but the density of zero point modes available @ k_{\min} is $k_{\min}^2 dk / \pi^2$ (ignoring some small factors). Even if $\alpha_G \ll 1$ this is ridiculously too small, by about $k_{\min}^2 \approx 1/R_{CCH}^2$. But the area of the causally connected horizon is $4\pi R_{CCH}^2$ suggesting a possible solution?

5.3.1 Holographic horizons and red shifted Planck scale zero point modes

Zero Point energies are invariant in all rest frames. The Lorenz transformations tell us this, but this is due to Special Relativity which applies locally. In section 2.2.3 we defined a rest frame in which zero momentum preons with infinite wavelength build infinite superpositions. If we have a spherical horizon with Planck scale modes, receding locally at the velocity of light, these Planck modes can be absorbed by infinite wavelength preons (from that receding horizon) and red shifted in a radially focussed manner inwards. We will argue in what follows, that at the centre where the infinite superpositions are built, approximately 1/6 of these Planck modes can be absorbed from that horizon with wavelengths of the order of the horizon radius. *This potential possibility only exists because zero momentum preons have an infinite wavelength.* If any source of radiation recedes at velocity $\beta = v/c$ the frequency/wavenumber reduces as $k_{\text{observer}} = k_{\text{source}} [\gamma(1-\beta)]$ where $\gamma = (1-\beta^2)^{-1/2}$. In the extreme relativistic limit $\beta \rightarrow 1$ & we can put $1-\beta = \Delta\beta = \varepsilon$.

Putting $1 - \beta = \Delta\beta = \varepsilon$ implies $\beta = 1 - \varepsilon$ and $\beta^2 \approx 1 - 2\varepsilon$

$$1 - \beta^2 = \gamma^{-2} \approx 2\varepsilon \text{ and } \gamma \approx 1/\sqrt{2\varepsilon} \quad (5.3. 3)$$

Thus $\frac{k_{Observer}}{k_{Source}} = [\gamma(1 - \beta)] \approx \frac{\varepsilon}{\sqrt{2\varepsilon}} \approx \sqrt{\frac{\varepsilon}{2}} \approx \frac{1}{2\gamma}$

There is always some rest frame travelling at nearly light velocity that can redshift Planck energy modes into a $k_{min} \approx 1/R_{CCH}$ mode and also many other frames travelling at various lower velocities that can redshift Planck energy modes into any $k > k_{min}$ mode . This is special relativity applying locally. But in sections 5.1.1 & 5.2.1 we used the fact that clocks in comoving coordinates tick at the same rate. So how does Eq.(5.3. 3) help? Space between comoving galaxies expands with cosmic or proper time t and is called the scale factor $a(t)$. It is normally expressed as $a(t) \propto t^p$.

Thus $\dot{a}(t) \propto pt^{p-1}$ and the Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{p}{t}$ (5.3. 4)

Writing the present scale factor normalized to one so that $a(T) = 1$ implies $a(t) = t^p / T^p$, we can get the causally connected horizon radius and the horizon velocity V . Using Eq.(5.3. 4)

The horizon radius $R_{CCH} = \int_0^T \frac{dt}{a(t)} = T^p \int_0^T \frac{dt}{t^p} = \frac{T}{1-p}$ only when p is constant. (5.3. 5)

The horizon velocity $V = \frac{dR_{CCH}}{dT} = \frac{d}{dT} \left[T^p \int_0^T \frac{dt}{t^p} \right] = \frac{T^p}{T^p} + \frac{R_{CCH}}{T^p} (pT^{p-1}) = 1 + \frac{p}{T} R_{CCH}$

But $\frac{p}{T}$ is the current Hubble constant so horizon velocity $V = 1 + H(T)R_{CCH}$ (5.3. 6)

Now the receding velocity of a comoving galaxy on the horizon is $V' = H(T)R_{CCH}$ and thus from Eq.(5.3. 6) the horizon velocity is always $V = 1 + V'$. In other words the horizon is moving at light velocity relative to comoving coordinates instantaneously on the horizon as measured by a central observer. Now clocks tick at the same rate in all comoving galaxies but clocks moving at almost the horizon light velocity (relative to comoving coordinates instantaneously on the horizon) will tick extremely slowly or as $1/\gamma$ from Eq.(5.3. 3) as special relativity applies locally in this case. Thus Planck modes on the receding horizon will obey Eq's.(5.3. 3) as seen in all comoving coordinates. Let us now imagine an infinity of frames all travelling at various relativistic velocities relative to comoving coordinates instantaneously on the horizon and radially as seen by central observers. We can think of these as spherical shells on the horizon *all of one Planck length thickness as measured by observers moving radially with them*. Transverse dimensions do not change for all radially moving observers and the effective surface area of all these shells is $4\pi R_{CCH}^2$. The internal volume of all these shells as measured in rest frames by observers moving radially with them as each of these observers measures their thickness as one Planck length is

$$\text{Rest frame internal shell volume } V = 4\pi R_{CCH}^2 \Delta R = 4\pi R_{CCH}^2 \quad (5.3. 7)$$

Now it is simpler from here to use zero point quanta available, where before redshifting a single zero point quanta has Planck energy ($k'=1$) and k energy after redshifting (k' before redshifting where $k'=1$ for these Planck energy modes and k after.) The final outcome is identical if we use energies but it is clearer this way. The density of Planck energy zero point modes in this shell is $k'^2 dk' / \pi^2$ and at energy $k'/2$ per mode this is

$$\frac{k'^2 dk'}{2\pi^2} \text{ quanta, which we will write as zero point quanta density } \frac{k'^3}{2\pi^2} \frac{dk'}{k'}. \quad (5.3. 8)$$

Now at Planck energy $k'=1$ and we are redshifting to k where from Eq's.(5.3. 3) $k = k' \sqrt{\varepsilon/2}$ & $dk = dk' \sqrt{\varepsilon/2}$. Thus $dk'/k' = dk/k$. As $k=1$ Eq. (5.3. 8) becomes

$$\text{Planck Energy Zero Point Quanta Density before redshifting} = \frac{1}{2\pi^2} \frac{dk}{k} \quad (5.3. 9)$$

(Equation (5.3. 12) makes clearer why we use dk/k .) Now multiply density by volume ie. Eq's. (5.3. 7) & (5.3. 9) to get the total Planck energy zero point quanta inside the rest frame

shell as $4\pi R_{CCH}^2 \cdot \frac{1}{2\pi^2} \frac{dk}{k}$. Two thirds of these quanta are transverse and one third radial so only $1/6$ of these quanta are available for redshifting radially inwards.

After redshifting to wavenumber k these quanta have radius $R' \approx \tilde{\lambda}_c = \frac{1}{k} = \frac{1}{k_{\min}} \frac{k_{\min}}{k} = \frac{R_{CCH}}{\Upsilon} \frac{k_{\min}}{k}$

and thus occupy spherical volume $V' \approx \frac{4\pi R_{CCH}^3}{3\Upsilon^3} \left[\frac{k_{\min}}{k} \right]^3$. Using $\Upsilon = k_{\min} R_{CCH}$ the effective

$$\text{density becomes } \approx \frac{1}{6} \left[4\pi R_{CCH}^2 \frac{1}{2\pi^2} \frac{dk}{k} \right] \frac{3\Upsilon^3}{4\pi R_{CCH}^3} \left[\frac{k}{k_{\min}} \right]^3 \approx \frac{\Upsilon^2}{4\pi^2} dk \left[\frac{k}{k_{\min}} \right]^2$$

These quanta are half scalar and *half the vector required to build infinite superpositions.*

$$\text{Density of vector quanta available after redshifting } \rho_k \approx \frac{\Upsilon^2}{8\pi^2} \left[\frac{k}{k_{\min}} \right]^2 dk \quad (5.3. 10)$$

Now an observer at the centre of all this sees space being added inside the horizon at the rate of the horizon velocity $V = 1 + H(T)R_{CCH}$ as in Eq. (5.3. 6). *We will conjecture that the space added in one unit of Planck time inside the expanding horizon also creates the source of these zero point quanta that we can borrow.* Thus Eq. (5.3. 10) becomes

$$\text{Density of vector quanta available } \rho'_k \approx \frac{\Upsilon^2 V}{16\pi^2} \left[\frac{k}{k_{\min}} \right]^2 dk = \frac{\Upsilon^2 (1 + H \cdot R_{CCH})}{16\pi^2} \left[\frac{k}{k_{\min}} \right]^2 dk \quad (5.3. 11)$$

5.3.2 Plotting available and required zero point quanta

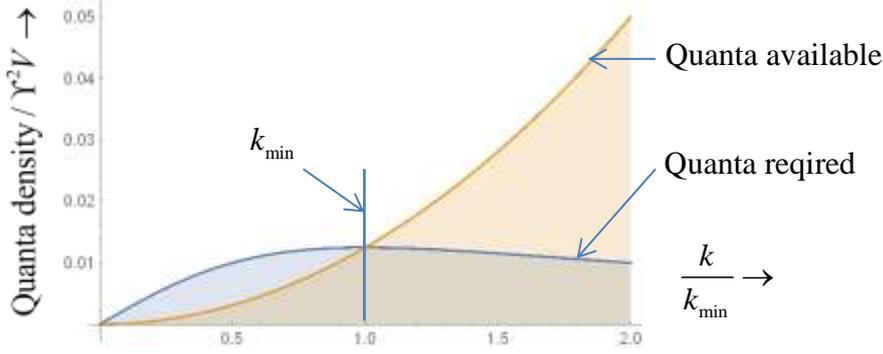


Figure 5.3. 1

Figure 5.3. 1 plots Eq's.(5.3. 2) & (5.3. 11) and when $k = k_{\min}$ we can equate these

$$\text{Quanta available} \approx \frac{\Upsilon^2 V}{8\pi^2} \left[\frac{k}{k_{\min}} \right]^2 dk = \text{Quanta required} \approx \frac{281\alpha_G}{\pi^2} \frac{kk_{\min}}{k^2 + k_{\min}^2} dk \quad (5.3. 12)$$

When $k = k_{\min}$ $\alpha_G \approx \frac{\Upsilon^2 V}{1124}$

Equation (5.2. 10) the average density of the universe $\rho_U \approx \frac{25.26\Upsilon^2}{2\pi R_{CCH}^2}$ allows us to solve the present value of $\Upsilon = k_{\min} R_{CCH}$. Using the 9 year WMAP (March 2013) data for Baryonic and Dark Matter density and radius $R_{CCH} \approx 2.7 \times 10^{61}$ Planck lengths ($\approx 46 \times 10^9$ light years) puts $\rho_U \times R_{CCH}^2 \approx 0.37$ in Planck units. Thus $\rho_U \times R_{CCH}^2 \approx (12.63)\Upsilon^2 / \pi \approx 0.37$ yields

$$\text{The current value for } \Upsilon = k_{\min} R_{CCH} \approx 0.303 \quad (5.3. 13)$$

The current Horizon Hubble velocity $V = 1 + H(T)R_{CCH} \approx 4.35$ and putting this and $\Upsilon \approx 0.303$ into Eq. (5.3. 12) we can solve the approximate graviton coupling constant α_G .

$$\alpha_G \approx \frac{\Upsilon^2 V}{1,124} \approx \frac{1}{2,800} \quad (5.3. 14)$$

The actual value for α_G is less important than the form of this equation as provided Eq. (5.2. 10) $\rho_U \times R_{CCH}^2 \approx (12.63)\Upsilon^2 / \pi$ is true (or in other words *all comoving observers measure constant virtual graviton density ρ_G as in Eq.(5.2. 11)*) GR is still true locally regardless of graviton coupling α_G . The normal gravitational constant (big) G is directly related to the metric change of GR, and if GR is true locally then G will not change, as it is independent of graviton coupling α_G . Because Eq. (5.3. 14) depends on the actual present values for Υ & V it must be approximate. The above analysis is based on a conjectured potential source of cosmic wavelength quanta that can only be borrowed if preons are born with infinite wavelength, but as we will see exponential expansion seems to follow naturally from Eq.

(5.3. 14) and it is hard to imagine any other large enough source. It also strongly suggests that if fundamental particles are in fact built from infinite superpositions that borrow quanta from zero point vector fields, then graviton coupling α_G between Planck masses must be way less than 1. So are there possible consequences of this?

5.3.3 Could virtual gravitons repel mass minutely but metric changes attract mass?

In both classical physics and quantum mechanics like charges repel each other and opposites attract, but gravity is different. Is it possible there could be a minute repulsive component between Planck masses of $\approx 3.5 \times 10^{-4}$ of the attraction due to the metric change? In section 5.3.6 we suggest an infinitesimal change to GR with affect only at cosmic scale. A small repulsive component due to virtual graviton coupling would only become dominant at distances of the order of the horizon radius. It could thus be a factor in the exponential expansion which is a solution to the above equations. It might also make a small change of ≈ 70 nanoseconds in the ≈ 200 microseconds Shapiro maximum time delay for light passing close to the sun. (It could minutely change the gravitational potential in the metric but not the spatial term). Even 200 nanoseconds would be almost impossible to measure however as variations in height of radar reflecting planet terrain would cause greater errors than this.

5.3.4 A possible exponential expansion solution and scale factors

Let the scale factor be a then density $\rho \propto \frac{1}{a^3}$ and Eq. (5.2. 10) tells us the average density of the universe $\rho_U \approx \frac{(12.63)\Upsilon^2}{\pi R_{CCH}^2}$ so that $\rho_U = \frac{1}{K} \frac{\Upsilon^2}{R_{CCH}^2} = \frac{1}{a^3}$ where $K = \frac{\pi}{12.63}$ is a constant.

$$\text{Thus } a^3 = KR^2\Upsilon^{-2} \rightarrow a = K'R^{2/3}\Upsilon^{-2/3} \text{ where } R = R_{CCH} \quad (5.3. 15)$$

The Hubble parameter H is

$$H = \frac{\dot{a}}{a} = \frac{(2/3)KR^{-1/3}\Upsilon^{-2/3} dR/dt}{KR^{2/3}\Upsilon^{-2/3}} - \frac{(2/3)KR^{2/3}\Upsilon^{-5/3} d\Upsilon/dt}{KR^{2/3}\Upsilon^{-2/3}} = \frac{2}{3} \left[\frac{1}{R} \frac{dR}{dt} - \frac{1}{\Upsilon} \frac{d\Upsilon}{dt} \right]$$

Thus the Hubble Horizon velocity @ R_{CCH} is $V' = H \cdot R = \frac{2}{3} \left[\frac{dR}{dt} - \frac{R}{\Upsilon} \frac{d\Upsilon}{dt} \right]$ (5.3. 16)

We can also write Eq.(5.3. 14) $\Upsilon^2 V \approx 2800\alpha_G = \text{a constant}$ and $\Upsilon^2 dV + 2\Upsilon d\Upsilon V = 0$, thus

$$\frac{1}{2V} \frac{dV}{dT} = -\frac{1}{\Upsilon} \frac{d\Upsilon}{dT}. \text{ Also Eq. (5.3. 6) tells us that the Horizon velocity } V = \frac{dR_{CCH}}{dt} = \frac{dR}{dt}.$$

Equation (5.3. 6) also tells us that $V' = H \cdot R = V - 1$ so we can write Eq. (5.3. 16) as

$$\left[3(V-1) - 2V = -\frac{2R}{\Upsilon} \frac{d\Upsilon}{dt} = \frac{2R}{2V} \frac{dV}{dt} \right] \rightarrow V-3 = \frac{R}{V} \frac{dV}{dt} \rightarrow \frac{dV}{dt} = \frac{V}{R} (V-3) \quad (5.3. 17)$$

We will look for an exponential increase of the horizon velocity so $dV / dt > 0$ and $3 \leq V \leq \infty$. Let us try first a simple $V = 3Exp(bt)$ with $V > 3$ for all values of b & $t > 0$.

Also simply put $R = \int_0^t V dt = \int_0^t 3Exp(bt) dt$ thus $R = \frac{3[Exp(bt) - 1]}{b}$.

Putting this value for R plus $V = 3Exp(bt)$ & $V - 3 = 3[Exp(bt) - 1]$ into Eq. (5.3. 17)

$$\frac{dV}{dt} = \frac{V}{R}(V - 3) = 3Exp(bt) \cdot \frac{b}{3[Exp(bt) - 1]} \cdot 3[Exp(bt) - 1] = 3bExp(bt).$$

But $V = 3Exp(bt)$ and again $\frac{dV}{dt} = \frac{d}{dt}[3Exp(bt)] = 3bExp(bt)$. Thus Eq's. (5.2. 10) & (5.3. 14)

are consistent with $V = 3Exp(bt)$ for positive b regardless of the value of graviton coupling α_G

$$\text{A possible expansion solution is } V = 3Exp(bt) \text{ \& } R = \frac{3[Exp(bt) - 1]}{b}, b > 0. \quad (5.3. 18)$$

But is this consistent with the local special relativity requirement for R_{CCH} ? In other words

does $R @ \text{time } T = a(T) \int_0^T \frac{dt}{a(t)} = \frac{3[Exp(bT) - 1]}{b}$? Now Eq. (5.3. 15) tells us the scale factor

$a^3 = KR^2\Upsilon^{-2} \rightarrow a = K'R^{2/3}\Upsilon^{-2/3}$ but Eq.(5.3. 14) says $\Upsilon^2 \propto 1/V$ so the scale factor $a \propto V^{1/3}R^{2/3}$.

From Eq. (5.3. 18), ignoring the constant factors 3 & b, $V \propto Exp(bt)$ & $R \propto [Exp(bt) - 1]$

$$\text{The scale factor } a(t) \propto Exp(bt)^{1/3}[Exp(bt) - 1]^{2/3} \quad (5.3. 19)$$

$$\text{Thus } R = a(T) \int_0^T \frac{dt}{a(t)} = Exp(bT)^{1/3}[Exp(bT) - 1]^{2/3} \int_0^T \frac{dt}{Exp(bt)^{1/3}[Exp(bt) - 1]^{2/3}} = \frac{3[Exp(bT) - 1]}{b}$$

and Eq. (5.3. 18) appears to be a consistent exponential expansion for both V and R .

5.3.5 Possible values for b and plotting scale factors

This simple exponential expansion starts at the Big Bang and is very different to the current cosmology models that keep a constant horizon velocity until Dark Energy starts to take effect. This continuous exponential increase could well lead to slightly different values for the radius R_{CCH} and also possibly the age $T \approx 13.8 \times 10^9$ years. (Some recent observations [1] have also been questioning the leading current dark energy explanations of acceleration). Current cosmology models put the Hubble parameter as $H = \dot{a} / a \approx 1/T$ at present (based on $T \approx 13.8 \times 10^9$ years). It also simplifies the plots below if we put $T \approx 13.8 \times 10^9 \text{ years} = 1$ with R_{CCH} or radius R becoming multiples of $T = 1$.

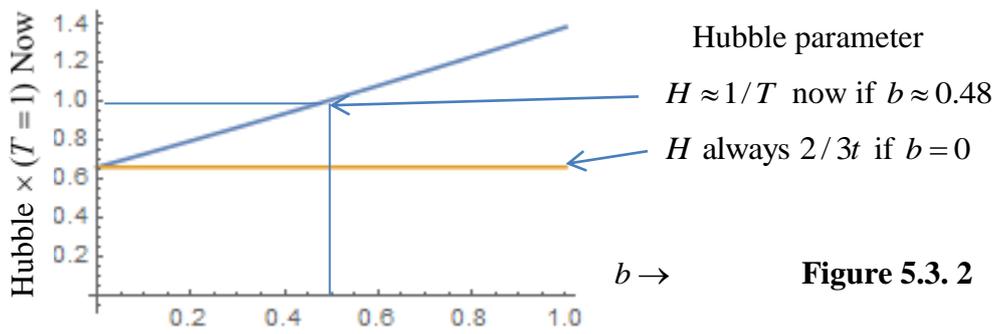


Figure 5.3. 2

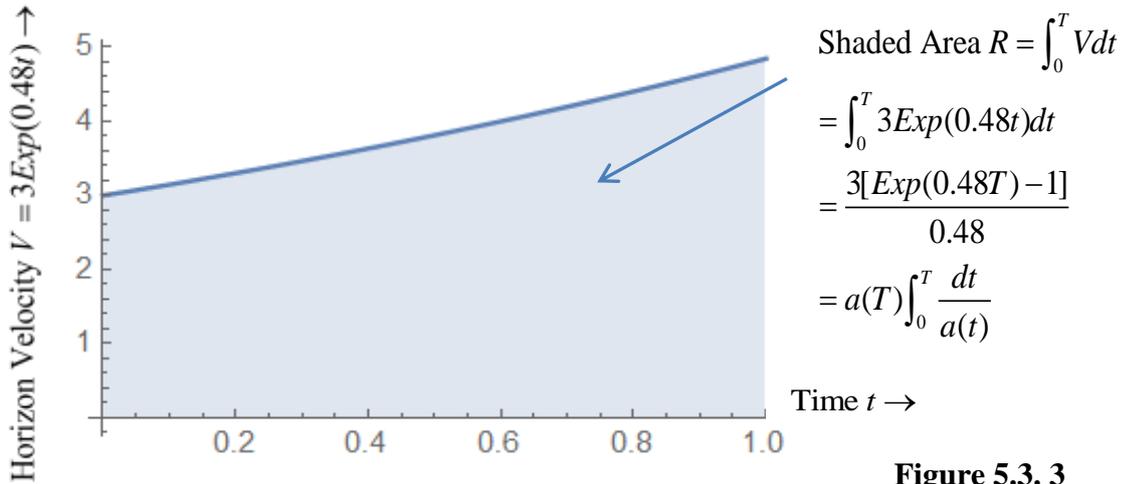


Figure 5.3. 3

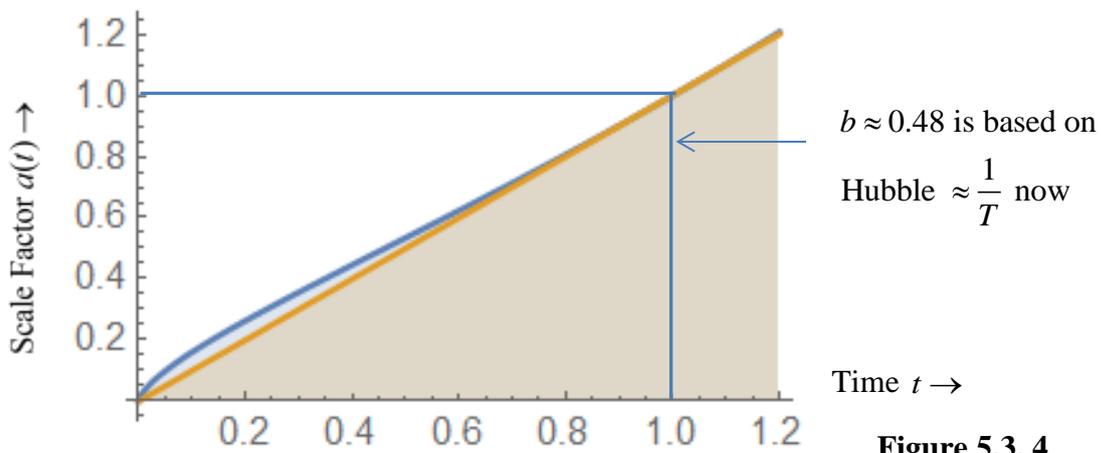


Figure 5.3. 4

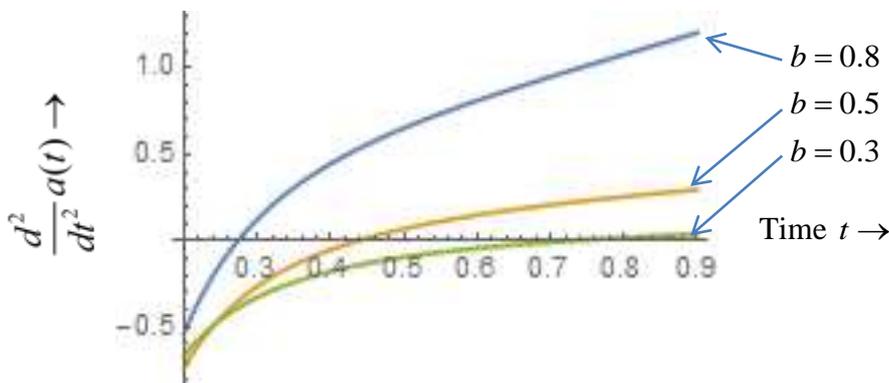


Figure 5.3. 5

Using Eq. (5.3. 6) $V = 1 + H(T)R$, Figure 5.3. 2 plots the Hubble parameter by time ($T = 1$) now as a function of the exponential time coefficient b showing if $b = 0$ that H always $= 2/(3t)$ as in current cosmology at critical density with no dark energy. Also if $H \approx 1/T$ now it shows that $b \approx 0.48$. This yields $R \approx 3.85T$ or $\approx 15\%$ greater than current cosmology. Figure 5.3. 3 plots horizon velocity & Figure 5.3. 4 the scale factor based on $b \approx 0.48$, but of course the actual value of b or rate of change with time must be in agreement with the redshifts currently observed when looking back towards the big bang. These could well change b and radius R . Figure 5.3. 5 plots the transition to positive acceleration of the scale factor showing the effect of changing the value of b .

5.3.6 An infinitesimal change to General Relativity effective at cosmic scale

Section 5 is based on energy in the zero point fields being limited. We argued that a uniform mass density throughout the cosmos has a uniform graviton density ρ_G . At this mass density the zero point energy density available equals that required. To maintain this required delicate balance (see Figure 5.3. 1) we argued that around any mass concentration the curvature of space expands space locally so as to maintain this uniform limiting graviton density ρ_G at all points. In other words *our conjecture only works if the local curvature of space depends on the difference between the local mass density and the uniform background*. Compared to General Relativity this is an infinitesimal change except at cosmic scale. GR says the curvature of space depends on local mass density whereas we argue that it depends on the difference between local mass density and the average background. This automatically guarantees the universe to be flat on average. All our arguments in Section 5 start with flat space on average. The equations of GR would look almost identical except the Energy Momentum Tensor $T_{\mu\nu}$ in comoving coordinates requires T_{00} the mass/energy density to change from ρ to $\rho - \rho_U$ where the density of the universe ρ_U is as in Eq. (5.2. 10).

$$\text{In comoving coordinates } T_{00} \text{ changes from } \rho \text{ to } \rho - \rho_U \text{ in the Energy Momentum Tensor } T_{\mu\nu} \quad (5.3. 20)$$

5.3.7 Non comoving coordinates.

With this change General Relativity is equivalent to maintaining constant graviton density ρ_G in comoving coordinates, and if Eq.(5.3. 20) is true it should also be a tensor equation *and should be true in all coordinates*. So let us see what happens if we move relative to comoving coordinates? The centre of momentum of all galaxies or mass is fixed relative to local comoving coordinates. In coordinates moving at velocity β' (relative to comoving coordinates) the average density of the universe ρ_U becomes $\gamma'^2 \rho_U$ where $\gamma' = (1 - \beta'^2)^{-1/2}$ (using gamma prime γ' to distinguish it from the metric change γ below and as in Eq.(5.2. 12)) as all mass increases as $\gamma' = (1 - \beta'^2)^{-1/2}$ and all volume elements shrink as γ'^{-1} . Similarly the number density of all gravitons ρ_G becomes $\gamma' \rho_G$. Also k_{\min} we now measure as $\gamma' k_{\min}$.

Similarly the horizon radius R_{CCH} we now measure as R_{CCH} / γ' so that $\Upsilon = (\gamma' k_{\min}) \cdot (R_{CCH} / \gamma') = k_{\min} R_{CCH}$ remains constant. (This change in the measured, or calculated, values of k_{\min} and R_{CCH} has implications that we discuss further at the end of section 6.2) We can repeat section 5.2 to show that if a mass concentration m_1 as in Figure 5.2. 1 is also moving at the same velocity β' relative to comoving coordinates then the local increase in gravitons $\Delta\rho_G$ due to m_1 , also becomes $\gamma'\Delta\rho_G$ as in Eq.(5.2. 6). So as in Eq.(5.2. 16)

$$\frac{\Delta V}{V} \approx \frac{\Delta\rho_G}{\rho_G} \approx \frac{\Delta\gamma}{\gamma} \approx \gamma^2 \frac{\Delta m_1}{r_1} \text{ becomes } \frac{\Delta V}{V} \approx \frac{\gamma'\Delta\rho_G}{\gamma'\rho_G} \approx \frac{\Delta\gamma}{\gamma} \approx \gamma^2 \frac{\Delta m_1}{r_1} \text{ in non comoving coordinates.}$$

In other words if we are moving at constant velocity β' relative to comoving coordinates far from any mass concentration we measure graviton density $\gamma'\rho_G$ and this does not change as we approach any such mass concentration. This is true regardless of the velocity of this mass relative to us. The curvature of space around this mass guarantees this. Thus General Relativity appears to be equivalent to maintaining the appropriate constant graviton average density $\gamma''\rho_G$ in any coordinates moving at *any velocity* β'' . If we now think of the mass in the universe as a dust of density ρ_U essentially at rest in comoving coordinates we can define a tensor $T_{\mu\nu}$ (Background). In these comoving coordinates $T_{\mu\nu}$ (Background) has only one non zero term T_{00} (Background) = ρ_U . In any other coordinates this same $T_{\mu\nu}$ (Background) tensor is transformed by the usual tensor transformations that apply in GR. If these coordinates for example move at velocity β' (relative to comoving coordinates) T'_{00} (Background) = $\gamma'^2 \rho_U$ = $\gamma'^2 T_{00}$ (Background). This all seems to suggest that *maintaining a constant background density of gravitons* ρ_G in comoving coordinates (or its transformed value in any others) is equivalent to the infinitesimally modified Einstein field equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} [T_{\mu\nu} - T_{\mu\nu}(\text{Background})] \quad (5.3. 21)$$

This modification is of course only relevant in the extreme case as $T_{\mu\nu}$ approaches $T_{\mu\nu}$ (Background). Far from mass concentrations $T_{\mu\nu} = T_{\mu\nu}$ (Background) so the curvature of space is zero and as we said above an immediate consequence is that Ω *does not need to be one for the causally connected universe to be flat on average*. If Dark Energy is not required and $\Omega \neq 1$ there can be more (or less) Dark Matter possibly increasing (or decreasing) ρ_U , Υ & α_G as in Eq's. (5.3. 13) & (5.3. 14).

6 Further consequences of Infinite Superpositions

6.1 Low frequency Infinite Superposition cutoffs

In section 4.2 when we introduced gravity, for the lower limit in our integrals we assumed $k_{\min} = 0$, and then in section 5 showed that there is a lower limit $k_{\min} > 0$. It turns out that for massive $N = 1$ superpositions the effect of this is negligible in comparison to the high frequency cutoff $k_{\text{cutoff}} < \infty$, which we showed gravity can address in section 4.2. For infinitesimal rest mass $N = 2$ superpositions we cannot however ignore the effect of $k_{\min} > 0$.

6.1.1 Quantifying the approximate effect of $k_{\min} > 0$ on infinite superpositions

If we look again at section 4.2.1 we can repeat what we did there as follows. Initially to illustrate these effects we will consider only $N = 1$ superpositions where we can say that

When $K_{nk\text{Cutoff}} \rightarrow \infty$ & (for $N = 1$ only) $K_{nk\min} \rightarrow 0$ and thus

$$\left[\frac{-1}{1 + K_{nk}^2} \right]_{K_{nk\min}}^{K_{nk\text{cutoff}}} = \frac{1}{1 + K_{nk\min}^2} - \frac{1}{1 + K_{nk\text{Cutoff}}^2} \approx 1 - \left[\frac{1}{K_{nk\text{Cutoff}}^2 + K_{nk\min}^2} \right] \approx 1 - \varepsilon'' \approx \frac{1}{1 + \varepsilon''} \quad (6.1.1)$$

Our earlier infinitesimal $\varepsilon \rightarrow \varepsilon'' \approx \frac{1}{K_{nk\text{Cutoff}}^2 + K_{nk\min}^2}$ and from Eq. (3.1.11) $K_{nk}^2 = \frac{n^2 s}{2} \tilde{\lambda}_c^2 k^2$.

For spin $1/2$ fermions for example $\langle n^2 s / 2 \rangle \approx 9$. Also $k_{\text{cutoff}}^2 \approx 1 / L_P^2$ and $k_{\min}^2 \approx 1 / R_H^2$ so that

$$\varepsilon'' \approx \frac{1}{K_{nk\text{Cutoff}}^2 + K_{nk\min}^2} \approx \frac{L_P^2}{9\tilde{\lambda}_c^2} + \frac{9\tilde{\lambda}_c^2}{R_{CCH}^2} \approx \frac{(L_P R_{CCH})^2 + (9\tilde{\lambda}_c^2)^2}{9\tilde{\lambda}_c^2 R_{CCH}^2}$$

The ratio of the extra contribution $\Delta\varepsilon$ to ε (where $\varepsilon'' = \varepsilon + \Delta\varepsilon$) is $\frac{\Delta\varepsilon}{\varepsilon} \approx \left[\frac{9\tilde{\lambda}_c^2}{L_P R_{CCH}} \right]^2$ (6.1.2)

(Where ε is the original $1 / K_{nk}^2$ cutoff of Eq. (4.2.2)). Equation (6.1.2) is for spin $1/2$, but the numerical factor 9 only changes slightly for spins 1 & 2. In Planck units $L_P R_{CCH} \approx 10^{61}$, but for electrons say $\tilde{\lambda}_c^2 \approx 6 \times 10^{44}$, so the effect is of order $\Delta\varepsilon / \varepsilon \approx 10^{46} / 10^{61} \approx 10^{-25}$ which we have been ignoring. We cannot ignore this however in the case of infinitesimal rest masses as we will see.

6.2 Infinitesimal Masses and $N = 2$ Superpositions

Looking again at angular momentum and rest masses in section 3.2 the key factor in our final integrals is in Eq. (6.1.1). Using Eq. (3.1.12) we can rewrite Eq. (6.1.1) as

$$\left[\frac{-1}{1 + K_{nk}^2} \right]_{K_{nk\min}}^{K_{nk\text{cutoff}}} = \frac{1}{\gamma_{nk\min}^2} - \frac{1}{\gamma_{nk\text{Cutoff}}^2} \quad (6.2.1)$$

With massive $N=1$ superpositions as above the difference between $\gamma_{nk\min}^2$ & 1 is vanishingly small, i.e. $(\gamma_{nk\min}^2 - 1) \rightarrow 1/\infty$ and as in section 6.1.1 this first term is of much less significance than the $\gamma_{nk\text{cutoff}}^2$ term. Now define an approximate equality between N & $\langle \gamma_{k\min} \rangle^2$ using Eq. (3.1. 12) as follows

$$N \approx \left[\langle \gamma_{k\min} \rangle^2 = 1 + \langle K_{k\min} \rangle^2 \right] \quad (6.2. 2)$$

In section 3.2 we derived angular momentum and rest masses for only massive or what we called $N=1$ particles. To get integral angular momentum we had to assume in deriving Eq. (3.2. 6) that the minimum value of K_{nk} or $K_{nk\min} = 0$. For massive $N=1$ particles such as the fermions the error in this assumption (as in section 6.1.1) is $\approx 10^{-25}$ times smaller than ε , which for an electron is already $\varepsilon \approx 10^{-45}$ due to the high frequency cutoff @ $\approx 10^{18.31} \text{ GeV}$. (We allowed for this $\varepsilon \approx 10^{-45}$ when we included gravity in section 4.2.) From section 6.1.1 above we approximated $K_{nk\min}^2$ as $\approx 9\lambda_c^2 / R_H^2$ for a spin $1/2$ fermion. So we can express Eq. (6.2. 2) in terms of this approximation for *fermions with non infinitesimal mass*

$$N \approx \left[\langle \gamma_{k\min} \rangle^2 = 1 + \frac{9\lambda_c^2}{R_H^2} \approx 1 \right] \quad \text{as} \quad \frac{9\lambda_c^2}{R_H^2} \rightarrow 0 \quad (6.2. 3)$$

$$\text{For example an electron has } \frac{9\lambda_c^2}{R_H^2} \approx 10^{-77}$$

For the massive particles it appears we can safely say that $N=1$. Even if neutrino masses were as low as 10^{-4} eV then $\langle \gamma_{k\min} \rangle^2 - 1 \approx 10^{-59}$. If the mass is too small however Eq. (6.2. 1) tells us we cannot get the correct angular momentum unless something else changes. *Infinitesimal increases above 1 of the order of $\approx 10^{-50}$ or so can be handled perhaps by a small change in the actual high frequency cutoff details, but this probably does not allow massive particles to be much less than sub micro electron volts.* So if massive particles are a group with $N=1$, then it would not seem unreasonable to imagine there could possibly be another group with $N=2 = 1 + \langle K_{k\min} \rangle^2$ implying that $\langle K_{k\min} \rangle^2 = 1$. Repeating the derivation of Eq. (3.2. 6) but with $N=2 = 1 + \langle K_{k\min} \rangle^2$ and for clarity and simplicity let $K_{nk\text{cutoff}} \rightarrow \infty$.

$$\mathbf{L}_z(\text{Total}) = s \cdot (N=2)m\hbar \int_{K_{nk\min}}^{\infty} \frac{K_{nk}^2}{(1+K_{nk}^2)^2} \frac{dK_{nk}}{K_{nk}} = sm\hbar \left[\frac{-1}{1+K_{nk}^2} \right]_{K_{nk\min}}^{\infty} \quad (6.2. 4)$$

$$\mathbf{L}_z(\text{Total}) = sm\hbar \left[\frac{1}{1+K_{nk\min}^2} \right] = sm\hbar \left[\frac{1}{(N=2)} \right] = \frac{sm\hbar}{2} \quad \text{as previously.}$$

Provided we have doubled the probability of superpositions as in Eq. (2.1. 4) from $s \cdot (N=1)dk/k$ to $s \cdot (N=2)dk/k$, the final angular momentum results in Eq's. (3.2. 6) &

(6.2. 4) are identical. The same is true for rest mass calculations. For multiple integer n infinite superpositions if $N = 2$ then the expectation value $\langle K_{k_{\min}} \rangle^2 = 1$. We thus conjecture that all $N = 2$ infinite superpositions have $\langle K_{k_{\min}} \rangle^2 = 1$.

From Table 4.3. 1

$N = 2$ infinitesimal rest mass spin 1 superpositions have $\langle n \rangle \approx 3.98$

$N = 2$ infinitesimal rest mass spin 2 superpositions have $\langle n \rangle \approx 3.33$

Using Eq's. (3.1. 11) and (5.1. 10)

$$\begin{aligned} \langle K_{k_{\min}} \rangle^2 &= \frac{\langle n \rangle^2 s}{2} \tilde{\lambda}_c^2 k_{\min}^2 \approx \frac{15.82}{2} \tilde{\lambda}_c^2 k_{\min}^2 = 1 \text{ or } \tilde{\lambda}_c \approx 0.355 \frac{R_{CCH}}{\Upsilon} \text{ for Spin 1} \\ &\approx \frac{11.09 \times 2}{2} \tilde{\lambda}_c^2 k_{\min}^2 = 1 \text{ or } \tilde{\lambda}_c \approx 0.300 \frac{R_{CCH}}{\Upsilon} \text{ for Spin 2} \end{aligned} \quad (6.2. 5)$$

Using the value for $\Upsilon \approx 0.303$ from Eq. (5.3. 13) based on WMAP data which also puts the horizon radius at $\approx 46 \times 10^9$ light years $R_{CCH} \approx 2.7 \times 10^{61}$ Planck lengths.

Spin	Compton Wavelength $\tilde{\lambda}_c$	Infinitesimal Rest Mass
1	$\approx 1.18 R_{CCH}$	$\approx 3.82 \times 10^{-34}$ eV.
2	$\approx 0.99 R_{CCH}$	$\approx 4.56 \times 10^{-34}$ eV.

Table 6.2 1 Infinitesimal rest masses of $N = 2$ photons, gluons & gravitons.

These Compton wavelengths and rest masses are the present values, changing slowly but exponentially with cosmic time T . They are based on WMAP data where $\Omega = 1$ and could be slightly different if $\Omega \neq 1$ and the Dark/Baryonic matter ratio is different as explained at the end of section 5.3.6. They also depend on the actual value of b in the exponential expansion $V = 3Exp(bt)$. These infinitesimal rest masses limit the range of virtual photons, gluons and gravitons to approximately the horizon. The graviton rest masses above are close to recent proposals explaining the accelerating expansion of the cosmos [2] [3]. They are also based on the value of k_{\min} measured (or calculated) in comoving coordinates. If we move at velocity β relative to comoving coordinates (as we said in section 5.3.7) $k_{\min} \rightarrow \gamma k_{\min}$ and contrary to what intuition would tell us the infinitesimal rest masses above become $m'_0 = \gamma m_0$. Of course these are infinitesimal rest masses and not the massive we are familiar with; they are also virtual and cannot be measured, but this still conflicts with our picture of what we have always called reality. There have been a huge number of experiments in the last few decades however confirming the counterintuitive Copenhagen interpretation that the result of a quantum experiment can depend on the act of measurement.

6.2.1 Cutoff behaviours for $N = 1$ & $N = 2$ superpositions

Equation (6.2. 1) can be written for both $N = 1$ & $N = 2$ superpositions using the results of sections 4.2 & 6.2 as follows

$$\left[\frac{-1}{1 + K_{nk}^2} \right]_{K_{nk \min}}^{K_{nk \text{cutoff}}} = \frac{1}{\left[\gamma_{nk \min}^2 = 2 \right]} - \frac{1}{\gamma_{nk \text{Cutoff}}^2} = \frac{1}{2(1 + \varepsilon'')} \quad \text{when } N = 2 \quad (6.2. 6)$$

$$\left[\frac{-1}{1 + K_{nk}^2} \right]_{K_{nk \min}}^{K_{nk \text{cutoff}}} = \frac{1}{\left[\gamma_{nk \min}^2 \approx 1 \right]} - \frac{1}{\gamma_{nk \text{Cutoff}}^2} = \frac{1}{1 + \varepsilon''} \quad \text{when } N = 1$$

(We should be using expectation values, but for clarity we simply imply them.) We have shown in section 6.2 that $\langle 1/\gamma_{k \min}^2 \rangle = 1/2$ when $N = 2$, but in reality it is Eq. (6.2. 6) that must be true. In section 4.2 we showed that for $N = 1$ superpositions the primary coupling of gravity to preons infinitesimally increased the interaction probability by ε' to $(1 + \varepsilon')$ where

$$\text{from Eq. (4.2. 4)} \quad \varepsilon' = \frac{m_0^2 \chi'_G \cdot G}{2shc(8 + 8\sqrt{\alpha_{EMP}})^2} = \varepsilon = \frac{1}{K_{nk}^2 \text{cutoff}} = \frac{2m_0^2 c^2}{sn^2 \hbar^2 (k_{\text{cutoff}})^2}.$$

In the $N = 1$ case this meant that any deficits due to a non infinite cutoff were exactly balanced by the contribution from gravity, but in the $N = 2$ case this infinitesimal correction is out by a factor of two. However Eq. (6.2. 6) tells us that exactness can be maintained in the $N = 2$ case by an infinitesimal change from $\langle 1/\gamma_{k \min}^2 \rangle = 1/2$ to $\langle 1/\gamma_{k \min}^2 \rangle \approx 1/2$. Thus both $N = 1$ & $N = 2$ superpositions can cut off at Planck energy as in section 4.2.2. The low frequency cutoff for all superpositions is at $k_{\min} \approx \Upsilon / R_{CCH}$ if they are to affect gravity.

6.2.2 Virtual particle pairs emerging from the vacuum and space curvature

For almost a century it has been a puzzle why spacetime is not massively curved by Planck scale zero point energy densities. However space appears to be flat on average regardless of this massive Planck scale zero point energy density so *something must be different*. In section 5.2.1 we argued that the curvature of space is consistent with a constant graviton background density ρ_G as in Eq. (5.2. 11). We calculated this graviton density ρ_G assuming gravitons only couple to the average density ρ_U of energy, plus baryonic and dark matter in the universe. If long wavelength gravitons coupled to all virtual pairs emerging from the vacuum there is just not enough zero point energy at cosmic wavelengths to build them. We argued that spacetime warps around any departure from a uniform background mass density ρ_U .

So let us similarly conjecture the possibility that secondary interaction gravitons (as distinct from primary interaction gravitons as in section 4.2.1) only couple to any departure from the uniform background of virtual pair creation. If real gravitons in gravitational waves signal spacetime how to behave, then virtual gravitons coupling to departures from uniform backgrounds also somehow signal space how to expand (as in section 5.3). However even if there are no $\approx k_{\min}$ gravitons coupling to virtual pairs, there must still be sufficient quanta available in the zero point vector fields to build the virtual particles themselves. It would

initially seem that this requirement would exceed that for gravitons. Particle lifetimes are key here. About 99% of graviton superpositions have lifetimes of $\approx T$ the age of the Universe where the uncertainty principle only allows energies $\Delta E \approx k_{\min} \approx R_{CCH}^{-1} \approx T^{-1}$. Available zero point energies are limited for only such long lifetime superpositions. On the other hand most virtual pairs have extremely short lifetimes where the uncertainty principle allows much greater supplies of zero point energies to build their superpositions. Because of this we will ignore this issue when applying Eq. (5.3. 12).

6.2.3 The primary to secondary graviton coupling ratio χ_G

In Eq. (4.2. 14) we found $\chi'_G \approx 318.3$ as the ratio between the primary graviton coupling to a bare Planck mass and the *normal measured gravitational constant* G . Equation (5.3. 14) defined graviton coupling between Planck masses α_G . If $\alpha_G = 1$ as we had expected, the ratio between primary and secondary graviton coupling (as defined for colour and electromagnetism in Eq. (3.3. 2)) would be $\chi_G = \alpha_G^{-1} \chi'_G = \chi'_G \approx 318.3$. But we found in section 5.3.2 that the graviton coupling constant between Planck masses was $\alpha_G \approx 1/2,800$ implying

$$\text{The primary to secondary graviton coupling ratio } \chi_G = \alpha_G^{-1} \chi'_G \approx 2,800 \times 318.3 \approx 890,000 \quad (6.2. 7)$$

However this is obviously very approximate. Equation (6.2. 7) can also be interpreted as the *primary graviton coupling to preons* is $\approx (318.3)G$ and the *secondary graviton coupling* is $\approx G/2,800$. To solve graviton superpositions we can use Eq. (3.3. 16) which is the *gravitational interaction probability* between fermions and we can now put on the RHS the coupling ratio $\chi_G \approx 890,000$ in the same way as we did for Eq.(3.3. 21)

$$\frac{[2s_{1/2}N_1c_{6a} * c_{6a}(1-c_{6a} * c_{6a})]^2 [2s_2N_2c_{4c} * c_{4c}(1-c_{4c} * c_{4c})]^2}{q^4} = \frac{4(\chi_G^{-1})^2}{q^4}$$

$$\text{Also } 2s_{1/2} = 1, 2s_1 = 2, N_1 = 1 \ \& \ N_2 = 2 \ \text{so } [c_{6a} * c_{6a}(1-c_{6a} * c_{6a})][8c_{4c} * c_{4c}(1-c_{4c} * c_{4c})] = 2\chi_G^{-1}$$

$$c_{4c} * c_{4c}(1-c_{4c} * c_{4c}) = \frac{\chi_G^{-1}}{4[c_{6a} * c_{6a}(1-c_{6a} * c_{6a})]} \approx \left[\frac{1}{890,000} \right] \frac{1}{4[c_{6a} * c_{6a}(1-c_{6a} * c_{6a})]}$$

$$\text{But from Eq. (4.4. 1) } c_{6a} * c_{6a}(1-c_{6a} * c_{6a}) = \sqrt{2/\chi_G} \approx \sqrt{2/50.4053} \approx 0.199194$$

$$c_{4c} * c_{4c}(1-c_{4c} * c_{4c}) \approx \frac{1}{4 \times 890,000 \times 0.199194} \approx 1.4 \times 10^{-6}. \text{ Using Eq.(4.4. 3), } \sum c_n * c_n \cdot n^4 \approx 170.95$$

for spin 2, $N = 2$ we get the infinitesimal mass graviton superposition values in Table 4.3. 1. The probability of a graviton, of the same mass/energy as say photons, gluons or fermions etc exchanging gravitons, (using the same procedure as in Eq. (3.3. 16)) is $\approx 10^{-8}$ of the probability of photons, gluons or fermions exchanging gravitons. *This is consistent with gravitational energy not being included in the Einstein tensor and why we said in section*

1.1.1 that gravitons may not emit gravitons. This implies that the gravitational constant does not run with wavenumber k at high energies as the other coupling constants do. This is why we can use the normal gravitational constant G as the secondary gravitational coupling constant G_s where we put the primary gravitational coupling to bare preons as $G_p = \chi'_G G_s = \chi'_G G$ in Eq.(4.2. 3).

6.2.4 $N=1$ & $N=2$ Bosons and the Higg's mechanism

In the Standard Model the Higg's mechanism adds mass to zero mass photons but here we say it adds mass to infinitesimal mass photons but not only does it do that, it also converts them from from $N = 2$ to $N = 1$, and also from $n = 3, 4, 5$ to $n = 4, 5, 6$ superpositions.

6.3 Black Holes, the Firewall Paradox and possible Spacetime Boundaries

Several recent papers [14] [15] [16] [17] [18] have discussed the BH firewall paradox. In section 5.2.2 we use the fact that outside observers see infalling mass remaining on the horizon. In fact if we look carefully at the analyses in sections 5.2.1 & 5.2.2 we see they strongly suggest that GR cutoff at the BH horizon; one of the possible firewall paradox implications. The equations we derived do not work inside the horizon. Our argument that comoving observers see a constant graviton density being consistent with GR will not work inside the horizon. Is it possible that the horizon of a Black Hole could well be a spacetime boundary?

6.4 Dark matter possibilities

Table 4.3. 1 shows a spin 2, $N=1$ neutral massive graviton type superposition that exchanges infinitesimal mass $N=2$ graviton superpositions $\approx 10^8$ more strongly than $N=2$ gravitons exchange $N=2$ gravitons. It may possibly be only detected via these graviton interactions.

6.5 Higgs Boson

It is not clear if the Higgs boson is a spin zero superposition so it is not in Table 2.2. 1; but if it is, it would be some superposition of infinite superpositions with a total angular momentum vector summing to zero just as two spin $1/2$ fermion superpositions can for example.

6.6 Constancy of fundamental charge

It has always been fundamental that the electromagnetic charge of protons and electrons is precisely equal and opposite to get a neutral universe. In section 4.2 we showed that the probability of superpositions was $sN \cdot dk(1+\varepsilon)/k$ where the infinitesimal ε is proportional to rest mass squared and thus different for various particles. We used this probability to determine interaction coupling strengths in section 3.3. This suggests that the probability of

virtual photon emission is also proportional to the probability $sN \cdot dk(1+\varepsilon)/k$ of each superposition, and would not be precisely equal for electrons and protons due to small variations in ε of the order of $\approx 10^{-45}$ between electrons and quarks. If however we look closely at Eq.(4.2. 3) and the following equations, by adding the amplitude for gravity at right angles we effectively added the probabilities of spin 2 gravity generated superpositions to those of spin 1 colour and electromagnetic superpositions. If somehow only those superpositions generated by spin 1 electromagnetic and colour interact with spin 1 photons this would cancel any minute difference in charge. If this is not so then there are infinitesimal differences in charge of the order of $\approx 10^{-45}$ which would surely have shown up in some form by now unless there are minute differences in the total number of electrons and protons.

6.7 Feynman's Strings

Over a century ago there were various models of the electron. The Abraham-Lorentz was probably the most well-known [19] [20]. All these models suffered from the problem that the electromagnetic mass in the field was 4/3 times the relativistic mass. In 1906 Poincare showed that if the bursting forces due to charge were balanced by stresses (or forces) in the same rest frame as the particle, these would cancel the extra 1/3 figure restoring covariance [21]. In chapter 29 Volume II of his famous lectures on physics, Feynman, probably jokingly, suggested that if the electron is held together by strings that their resonances could explain the muon mass; he just may have been right [22]. The equations for infinite superpositions in this paper apply equally to all massive particles. Also, as infinite superpositions are held together by interactions with zero point forces in the same rest frame, *could these zero point interactions possibly be Feynman's strings?* If they hold the virtual preons in orbit, it would seem that they should also be able to balance any bursting forces due to electric charge. However this paper suffers from the same problem as the Standard Model. There is nothing in it suggesting the quantization of mass of the massive particles; but it does however suggest the mass of infinitesimal rest mass particles.

7 Conclusions

If fundamental particles are built from infinite superpositions then why do we never see any sign of them? It is important to remember that all components of infinite superpositions are virtual and only complete infinite superpositions can behave as real particles. If infinite superpositions could be somehow decomposed into their virtual components this would destroy the resulting equivalent real particle. Could it be that particle conservation laws controlling the behaviour of fundamental particles somehow prevent any sign of their virtual components? Also the viability of this paper depends on primary interactions where spin zero preons can borrow mass from some Higgs type scalar zero point field, and energy from colour and electromagnetic zero point vector fields. *The behaviour of these primary*

interactions is very different to the secondary interactions that the SM is all about. The SM rules applying to borrowing mass and energy from scalar and vector zero point fields may not apply to primary interactions; but the secondary interactions of QED, QCD etc of the SM applying to fundamental particles must equally apply to infinite superpositions. We have also not discussed gravitational waves. Just as a minute graviton coupling constant can lead to much larger changes in the metric around mass concentrations, we can imagine similarly small emissions of real $m = \pm 2$ gravitons leading to larger waves in the metric. Finally, this paper suggests that if fundamental particles can in fact be built from infinite superpositions:

- *Quantum mechanics may well rule the exponential expansion of space and the warping of spacetime around concentrations of mass/energy.*
- *The warping of spacetime around mass concentrations and the exponential expansion of space may possibly be the only evidence of infinite superpositions we will ever see.*
- *General Relativity (in an infinitesimally modified form affecting cosmic scale only) could well be a consequence of Quantum Mechanics.*
- *The interaction between gravitons is minute in relation to the apparent attraction between mass concentrations due to metric changes. This may cause a small repulsion that could be a factor determining the rate of the exponential expansion.*
- *General Relativity may not apply inside Black Holes, and the event horizon itself may possibly be a Spacetime Boundary, as infinite superpositions will not form inside them.*

8 Addendum

The finding in this paper suggesting consistency between an infinitesimally modified General Relativity (affecting cosmic scale only) and all observers seeing constant background graviton densities (appropriate to their velocities relative to comoving coordinates) only considered the gravitons interacting between a small mass (in relation to the rest of the cosmos) and the mass of the rest of the cosmos. For clarity and simplicity those gravitons emitted by the small mass itself were ignored. Their effect is small except close to black holes. This second order effect will be explored in a separate paper.

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