Predicting the Binding Energies of the 1s Nuclides with High Precision, Based on Baryons which are Yang-Mills Magnetic Monopoles

Jay R. Yablon[*]
Schenectady, New York 12309

Abstract: *We employ the thesis that baryons are Yang-Mills magnetic monopoles to predict the binding energies of the alpha ^{4}He nucleus to less than four parts in one million, of the ^{3}He helion nucleus to less than four parts in 100,000, and of the ^{3}H triton nucleus to less than seven parts in one million, all in AMU. Of special import, we exactly relate the neutron–proton mass difference – which pervades all aspects of nuclear physics and beta decay – to a function of the up quark, down quark, and electron masses, which in turn enables us to predict the binding energy for the ^{2}H deuteron nucleus most precisely of all, to just over 8 parts in ten million. The thesis that Baryons are Yang-Mills magnetic monopoles thereby appears to have ample, indeed irrefutable empirical confirmation, establishes a basis for finally "decoding" the mass of known data regarding nuclear masses and binding energies, and may lay the foundation for technologically realizing the theoretical promise of nuclear fusion.*

Contents

[*] email: jyablon@nycap.rr.com

1. Introduction

In sections 11 and 12 of [1] we applied a pure gauge field Lagrangian $\mathcal{L}_{\text{gauge}}$ to specify the energy of the Yang-Mills magnetic monopoles according to [11.7] of [1], part of which is reproduced below:

$$E = -\iiint \mathcal{L}_{\text{gauge}}\, d^3x = \tfrac{1}{2}\,\text{Tr}\iiint F_{\mu\nu}F^{\mu\nu}\, d^3x\,. \tag{1.1}$$

We then made use in (1.1) of the field strength tensors for protons and neutrons, [11.3] and [11.4] of [11], respectively,

$$\text{Tr}F^{\mu\nu}{}_{\text{P}} = -i\left(\frac{\overline{\psi}_d\left[\gamma^\mu \vee \gamma^\nu\right]\psi_d}{"\rho_d - m_d"} + 2\frac{\overline{\psi}_u\left[\gamma^\mu \vee \gamma^\nu\right]\psi_u}{"\rho_u - m_u"}\right), \tag{1.2}$$

$$\text{Tr}F^{\mu\nu}{}_{\text{N}} = -i\left(\frac{\overline{\psi}_u\left[\gamma^\mu \vee \gamma^\nu\right]\psi_u}{"\rho_u - m_u"} + 2\frac{\overline{\psi}_d\left[\gamma^\mu \vee \gamma^\nu\right]\psi_d}{"\rho_d - m_d"}\right), \tag{1.3}$$

to deduce three relationships that yielded remarkable concurrence with empirical data:

First, we found in [11.22] of [1] that the mass of the electron is related to the masses of the up and down quarks according to:

$$m_e = 0.510998928\ MeV = \frac{3}{(2\pi)^{\frac{3}{2}}}(m_d - m_u), \tag{1.4}$$

where the divisor $(2\pi)^{\frac{3}{2}}$ results as a natural consequence of a three-dimensional Gaussian integration. Second and third, we found in [12.12] and [12.13] that if one *postulates* the mass of the up quark to be equal to deuteron (^{2}H nucleus) binding energy based on a) empirical concurrence within experimental errors and b) regarding the nucleons or nuclei to be bound *resonant cavities* with binding energies determined in relation to their up and down quark content, then the latent binding energies (energies available for binding) intrinsic to the proton and neutron, respectively, are:

$$B_{\text{P}} = 2m_u + m_d - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right)/(2\pi)^{\frac{3}{2}} = \mathbf{7.640679MeV} \tag{1.5}$$

$$B_{\text{N}} = 2m_d + m_u - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right)/(2\pi)^{\frac{3}{2}} = \mathbf{9.812358MeV}\,. \tag{1.6}$$

So for a nucleus with an equal number of protons and neutrons, the average binding energy per nucleon is 8.726519 MeV. Not only does this explain why a typical nucleus beyond the very lightest (which we shall be studying in detail here) has a binding energy in exactly this vicinity, but when applied to Fe^{56} with 26 protons and 30 neutrons, which has the distinction of using a higher percentage of this available binding energy than any other nucleus, we find that *maximum available* binding energy is *predicted* to be (see [12.14]):

$$B_{\max}(\text{Fe}^{56}) = 26\times 7.640679MeV + 30\times 9.812358MeV = \mathbf{493.028394MeV}\,. \tag{1.7}$$

This contrasts remarkably with the actual, *observed* binding energy **492.253892 MeV**. That is, precisely 99.8429093% of the *available* binding energy *predicted* by this model of nucleons as Yang-Mills magnetic monopoles goes into binding together the Fe^{56} nucleus, with the small balance of 0.1570907% serving to confine the quarks within each nucleon.

However, in deriving (1.4) through (1.6) we glossed over an aspect of (1.1) which, when carefully considered, requires us to amend the usual Yang-Mills magnetic monopole Lagrangian (1.1) in a slight but important way. This amendment will provide some further insights which will allow us to theoretically derive the observed binding energies for all of the triton (^{3}H nucleus), helion (^{3}He nucleus) and very importantly, the alpha particle (^{4}He nucleus), all with extremely close precision in relation to the empirical data.

2. The Lagrangian of Nuclear Binding Energies

The Lagrangian used in (1.1), because of suppression of the Yang-Mills matrix indexes, actually has an ambiguous mathematical meaning, and can be either an ordinary matrix multiplication, or a tensor (outer) product. The latter, outer product, is the most general bilinear operation that can be performed on $F_{\mu\nu}F^{\mu\nu}$, while the former represents a contraction which reduces the Yang-Mills rank by 2. When carefully considered, this provides an opportunity for developing a nuclear Lagrangian based on the t'Hooft monopole Lagrangian in [2.1] of [2].

Now, if we know that $\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ as we do from the terms in [11.7] of [1] that were omitted in (1.1) above, and also given that $TrT^iT^j = \frac{1}{2}\delta^{ij}$, then with explicit indexes $A,B,C,D = 1,2,3$ for the 3x3 Yang-Mills matrices of the $SU(3)_{C'}$ isospin-modified color group developed in section 8 of [1], an explicit appearance of Yang-Mills indexes would cause (1.1) to be written as:

$$\begin{aligned} E &= -\iiint \mathfrak{L}_{\text{gauge}}\, d^3x = \tfrac{1}{2}\,\text{Tr}\iiint F_{\mu\nu} \otimes F^{\mu\nu}\, d^3x = \tfrac{1}{2}\,\text{Tr}\iiint F_{\mu\nu\,AB}F^{\mu\nu}{}_{BD}\, d^3x \\ &= \tfrac{1}{2}\,\text{Tr}\iiint F_{AB}\cdot F_{BD}\, d^3x = \tfrac{1}{2}\iiint F_{AB}\cdot F_{BA}\, d^3x \end{aligned}, \qquad (2.1)$$

where we suppress the spacetime indexes using $F\cdot F = F_{\mu\nu}F^{\mu\nu}$ to focus attention on the contractions of the Yang-Mills indexes. That is, in the fourth and fifth terms above, we perform a contraction over the "*B*" index, which means that $F_{AB}\cdot F_{BD}$ is an *inner* product formed with ordinary matrix multiplication, and is a contraction of the most general bilinear Yang Mills tensor, the fourth rank (3x3x3x3) $F_{AB}\cdot F_{CD}$, down to rank two. In the sixth, final term, we write the trace as $\text{Tr}F_{AB}\cdot F_{BD} = F_{AB}\cdot F_{BA}$ using yet a second index contraction.

We point this out because (1.4) through (1.7) which successfully match the empirical nuclear binding data, and most particularly which lead to (1.5), (1.6) and (1.7), are in fact based not only on (2.1), but also taking the *tensor outer product* of $F_{AB}\cdot F_{BD}$, that is, on taking (carefully contrast the Yang-Mills indexes as between the final terms in (2.1) and (2.2):

$$E = -\iiint \mathcal{L}_{\text{gauge}} d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint F_{\mu\nu} \otimes F^{\mu\nu} d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint F_{\mu\nu\,AB} F^{\mu\nu}{}_{CD} d^3x$$
$$= \tfrac{1}{2}\mathrm{Tr}\iiint F_{AB} \cdot F_{CD} d^3x = \tfrac{1}{2}\iiint F_{AA} \cdot F_{BB} d^3x \qquad (2.2)$$

Here, in the final terms, we use $\mathrm{Tr}F_{AB} \cdot F_{CD} = F_{AA} \cdot F_{BB}$, as opposed to $\mathrm{Tr}F_{AB} \cdot F_{BD} = F_{AB} \cdot F_{BA}$, which highlights the notational ambiguity in (1.1) as well as the difference between the outer and inner matrix products.

Now, in general, the trace of a product of two square matrices is *not* the product of traces. The only circumstance in which the "trace of a product" equals the "product of traces" is when one forms a tensor product using the most general bilinear operation:

$$\mathrm{Tr}(A \otimes B) = \mathrm{Tr}(A)\mathrm{Tr}(B). \qquad (2.3)$$

Specifically, to obtain the terms $m_d + 4\sqrt{m_u m_d} + 4m_u$ and $m_u + 4\sqrt{m_u m_d} + 4m_d$ in (1.5) and (1.6), we are must use (2.2), while to obtain $2m_u + m_d$ and $2m_d + m_u$ in the same expressions, we instead must use (2.1). So (1.5) and (1.6) are formed by a linear combination of both inner and outer products. And because (1.5) and (1.6) predict binding energies per nucleon in the range of 8.7 MeV and yield an extremely close match to the ^{56}Fe binding energies, nature herself appears to be telling us that we need to combine inner and outer products in this way in order to match up with empirical data. This, in turn, gives us important data for how to construct our Lagrangian.

To see this all most vividly, we start with [11.8] and [11.9] from [1] as reproduced below:

$$E_{\mathrm{P}} = -\tfrac{1}{2}\iiint\left(\frac{\overline{\psi}_d\left[\gamma^\mu \vee \gamma^\nu\right]\psi_d}{"\not{p}_d - m_d"} + 2\frac{\overline{\psi}_u\left[\gamma^\mu \vee \gamma^\nu\right]\psi_u}{"\not{p}_u - m_u"}\right)\times\left(\frac{\overline{\psi}_d\left[\gamma_\mu \vee \gamma_\nu\right]\psi_d}{"\not{p}_d - m_d"} + 2\frac{\overline{\psi}_u\left[\gamma_\mu \vee \gamma_\nu\right]\psi_u}{"\not{p}_u - m_u"}\right)d^3x, \qquad (2.4)$$

$$E_{\mathrm{N}} = -\tfrac{1}{2}\iiint\left(\frac{\overline{\psi}_u\left[\gamma^\mu \vee \gamma^\nu\right]\psi_u}{"\not{p}_u - m_u"} + 2\frac{\overline{\psi}_d\left[\gamma^\mu \vee \gamma^\nu\right]\psi_d}{"\not{p}_d - m_d"}\right)\left(\frac{\overline{\psi}_u\left[\gamma_\mu \vee \gamma_\nu\right]\psi_u}{"\not{p}_u - m_u"} + 2\frac{\overline{\psi}_d\left[\gamma_\mu \vee \gamma_\nu\right]\psi_d}{"\not{p}_d - m_d"}\right)d^3x. \qquad (2.5)$$

Using these in (2.2) following the development in section 11 and [12.12] and [12.13] of [1], we rewrite $m_d + 4\sqrt{m_u m_d} + 4m_u$ and $m_u + 4\sqrt{m_u m_d} + 4m_d$, respectively, as the Yang-Mills matrix *outer products*:

$$E_P = \tfrac{1}{2}\mathrm{Tr}\iiint F_{P\,\mu\nu} \otimes F_P{}^{\mu\nu} d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint F_{P\,AB} \cdot F_{P\,CD} d^3x = \tfrac{1}{2}\iiint F_{P\,AA} \cdot F_{P\,BB} d^3x$$
$$= \frac{1}{(2\pi)^{\frac{3}{2}}}\mathrm{Tr}\left[\begin{pmatrix}\sqrt{m_d} & 0 & 0\\ 0 & \sqrt{m_u} & 0\\ 0 & 0 & \sqrt{m_u}\end{pmatrix}\otimes\begin{pmatrix}\sqrt{m_d} & 0 & 0\\ 0 & \sqrt{m_u} & 0\\ 0 & 0 & \sqrt{m_u}\end{pmatrix}\right], \qquad (2.6)$$
$$= \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d + 4\sqrt{m_u m_d} + 4m_u\right) = 1.715697\,MeV$$

$$E_N = \tfrac{1}{2}\mathrm{Tr}\iiint F_{N\,\mu\nu} \otimes F_N{}^{\mu\nu} d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint F_{N\,AB} \cdot F_{N\,CD} d^3x = \tfrac{1}{2}\iiint F_{N\,AA} \cdot F_{N\,BB} d^3x$$

$$= \frac{1}{(2\pi)^{\frac{3}{2}}}\mathrm{Tr}\left[\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}\right] \quad . \qquad (2.7)$$

$$= \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u + 4\sqrt{m_u m_d} + 4m_d\right) = 2.226696 MeV$$

The above connect the energy and Lagrangian $E = -\iiint \mathcal{L}_{\text{gauge}} d^3x$ to a very-transparent matrix format, and in turn, to the energy numbers that were in part responsible for empirically-matching the Fe^{56} binding energies.

Further, in this form, we also see that the simple sums $\Sigma m_p = 2m_u + m_d$ and $\Sigma m_n = 2m_d + m_u$ of the quark masses in a proton *p* or neutron *n* are similarly given by the Yang-Mills matrix *inner products*:

$$\Sigma E_p = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\mathrm{Tr}\iiint F_{P\,\mu\nu} F_P{}^{\mu\nu} d^3x = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\mathrm{Tr}\iiint F_{P\,AB} \cdot F_{P\,BD} d^3x = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\iiint F_{P\,AB} \cdot F_{P\,BA} d^3x$$

$$= \mathrm{Tr}\left[\begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix}\begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix}\right] = 2m_u + m_d = 9.356376 \text{ MeV} \quad , \qquad (2.8)$$

$$\Sigma E_n = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\mathrm{Tr}\iiint F_{N\,\mu\nu} F_N{}^{\mu\nu} d^3x = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\mathrm{Tr}\iiint F_{N\,AB} \cdot F_{N\,BD} d^3x = \tfrac{1}{2}(2\pi)^{\frac{3}{2}}\iiint F_{N\,AB} \cdot F_{N\,BA} d^3x$$

$$= \mathrm{Tr}\left[\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix}\right] = 2m_d + m_u = 12.039054 \text{ MeV} \quad . \qquad (2.9)$$

These expressions use the ordinary matrix product which appear in (2.1), and differ from (2.6) and (2.7) only insofar as how the indexes are contracted. The factor of $(2\pi)^{\frac{3}{2}}$, recall, originates from the three-dimensional Gaussian integration.

This means that we can reproduce equations (1.5) and (1.6) for the latent binding energy of a proton and neutron by combining (2.6) with (2.8), and (2.7) with (2.9), in *linear combinations of inner and outer Yang-Mills matrix products*, as follows:

$$B_P = \Sigma E_P - E_P = \tfrac{1}{2}\mathrm{Tr}\iiint\left((2\pi)^{\frac{3}{2}}F_{P\mu\nu}F_P{}^{\mu\nu} - F_{P\mu\nu}\otimes F_P{}^{\mu\nu}\right)d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint\left((2\pi)^{\frac{3}{2}}F_{PAB}\cdot F_{PBD} - F_{PAB}\cdot F_{PCD}\right)d^3x$$
$$= \tfrac{1}{2}\iiint\left((2\pi)^{\frac{3}{2}}F_{PAB}\cdot F_{PBA} - F_{PAA}\cdot F_{PBB}\right)d^3x = 2m_u + m_d - \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_d + 4\sqrt{m_u m_d} + 4m_u\right)$$
$$= \mathrm{Tr}\left[\begin{pmatrix}\sqrt{m_d} & 0 & 0\\ 0 & \sqrt{m_u} & 0\\ 0 & 0 & \sqrt{m_u}\end{pmatrix}\begin{pmatrix}\sqrt{m_d} & 0 & 0\\ 0 & \sqrt{m_u} & 0\\ 0 & 0 & \sqrt{m_u}\end{pmatrix} - \frac{1}{(2\pi)^{\frac{3}{2}}}\begin{pmatrix}\sqrt{m_d} & 0 & 0\\ 0 & \sqrt{m_u} & 0\\ 0 & 0 & \sqrt{m_u}\end{pmatrix}\otimes\begin{pmatrix}\sqrt{m_d} & 0 & 0\\ 0 & \sqrt{m_u} & 0\\ 0 & 0 & \sqrt{m_u}\end{pmatrix}\right] \quad (2.10)$$
$$= 9.356376\ MeV - 1.715697 MeV = \mathbf{7.640679MeV}$$

$$B_N = E_N - \Sigma E_N = \tfrac{1}{2}\mathrm{Tr}\iiint\left((2\pi)^{\frac{3}{2}}F_{N\mu\nu}F_N{}^{\mu\nu} - F_{N\mu\nu}\otimes F_N{}^{\mu\nu}\right)d^3x = \tfrac{1}{2}\mathrm{Tr}\iiint\left((2\pi)^{\frac{3}{2}}F_{NAB}\cdot F_{NBD} - F_{NAB}\cdot F_{NCD}\right)d^3x$$
$$= \tfrac{1}{2}\iiint\left((2\pi)^{\frac{3}{2}}F_{NAB}\cdot F_{NBA} - F_{NAA}\cdot F_{NBB}\right)d^3x = 2m_d + m_u - \frac{1}{(2\pi)^{\frac{3}{2}}}\left(m_u + 4\sqrt{m_u m_d} + 4m_d\right)$$
$$= \mathrm{Tr}\left[\begin{pmatrix}\sqrt{m_u} & 0 & 0\\ 0 & \sqrt{m_d} & 0\\ 0 & 0 & \sqrt{m_d}\end{pmatrix}\begin{pmatrix}\sqrt{m_u} & 0 & 0\\ 0 & \sqrt{m_d} & 0\\ 0 & 0 & \sqrt{m_d}\end{pmatrix} - \frac{1}{(2\pi)^{\frac{3}{2}}}\begin{pmatrix}\sqrt{m_u} & 0 & 0\\ 0 & \sqrt{m_d} & 0\\ 0 & 0 & \sqrt{m_d}\end{pmatrix}\otimes\begin{pmatrix}\sqrt{m_u} & 0 & 0\\ 0 & \sqrt{m_d} & 0\\ 0 & 0 & \sqrt{m_d}\end{pmatrix}\right] \quad (2.11)$$
$$= 12.039054\ MeV - 2.226696 MeV = \mathbf{9.812358MeV}$$

This now provides a fully-covariant, Yang-Mills matrix expression for the intrinsic, latent binding energies of the proton and neutron, contracted down to the scalars which specify these binding energies. And it is from these, that we are now clued into how we can amend the Lagrangian in (1.1) to provide a foundation for considering nuclear binding energies in general.

Contrasting (2.10) and (2.11) with (2.1) and (2.2), we see that the general form of a Lagrangian for the *latent* nuclear binding energy of a nucleon (which may be a proton or neutron or any other baryon) is:

$$\begin{aligned}\mathfrak{L}_{\text{binding}} &= \tfrac{1}{2}\mathrm{Tr}\left((2\pi)^{\frac{3}{2}}F_{\mu\nu}F^{\mu\nu} - F_{\mu\nu}\otimes F^{\mu\nu}\right) = \tfrac{1}{2}\mathrm{Tr}\left((2\pi)^{\frac{3}{2}}F_{AB}\cdot F_{BD} - F_{AB}\cdot F_{CD}\right)\\ &= \tfrac{1}{2}\left((2\pi)^{\frac{3}{2}}F_{AB}\cdot F_{BA} - F_{AA}\cdot F_{BB}\right)\end{aligned}. \quad (2.12)$$

Using this, we now start to amend the t'Hooft Lagrangian [9.2] of [1], reproduced below:

$$\mathfrak{L} = -\tfrac{1}{4}F^a_{\mu\nu}F_a{}^{\mu\nu} - \tfrac{1}{2}D_\mu\phi_a D^\mu\phi^a - \tfrac{1}{2}\mu^2\phi_a\phi^a - \tfrac{1}{8}\lambda\left(\phi_a\phi^a\right)^2. \quad (2.13)$$

First, we apply $TrT^iT^j = \tfrac{1}{2}\delta^{ij}$ together with $F^{\mu\nu} = T^iF_i^{\mu\nu}$ and $\Phi = T^a\phi_a$ to rewrite (2.13) in the Yang-Mills matrix form:

$$\begin{aligned}\mathfrak{L} &= -\tfrac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \mathrm{Tr}\left(D_\mu\Phi D^\mu\Phi\right) - \mu^2\mathrm{Tr}\left(\Phi\Phi\right) - \tfrac{1}{2}\lambda\left(\mathrm{Tr}\left(\Phi\Phi\right)\right)^2\\ &= -\tfrac{1}{2}\mathrm{Tr}\left(F_{\mu\nu\,AB}F^{\mu\nu}{}_{BD}\right) - \mathrm{Tr}\left(D_\mu\Phi_{AB}D^\mu\Phi_{BD}\right) - \mu^2\mathrm{Tr}\left(\Phi_{AB}\Phi_{BD}\right) - \tfrac{1}{2}\lambda\left(\mathrm{Tr}\left(\Phi_{AB}\Phi_{BD}\right)\right)^2,\\ &= -\tfrac{1}{2}F_{\mu\nu\,AB}F^{\mu\nu}{}_{BA} - D_\mu\Phi_{AB}D^\mu\Phi_{BA} - \mu^2\Phi_{AB}\Phi_{BA} - \tfrac{1}{2}\lambda\left(\Phi_{AB}\Phi_{BA}\right)^2\end{aligned} \quad (2.14)$$

with [9.4] of [11] also written in the compacted matrix form:

$$D_\mu \Phi = \partial_\mu \Phi - i\left[G_\mu, \Phi\right]. \tag{2.15}$$

Now, we compare (2.14) closely with (2.12), especially the term $-\frac{1}{2} F_{\mu\nu\,AB} F^{\mu\nu}{}_{BA}$ in (2.14) with $\frac{1}{2}(2\pi)^{\frac{3}{2}} F_{AB} \cdot F_{BA}$ in (2.12). Based on this, we *construct* a Lagrangian such that the leading (pure gauge) terms specify the nuclear binding energies, that is, we choose to make $\frac{1}{2}\left((2\pi)^{\frac{3}{2}} F_{AB} \cdot F_{BA} - F_{AA} \cdot F_{BB}\right)$ the leading Lagrangian term, because we know from (12.10) and (12.11) that this yields latent binding energies very much in accord with what is empirically observed in nuclear physics. Thus, we take (2.14), introduce a factor of $-(2\pi)^{\frac{3}{2}}$ in front of all the ordinary matrix products, subtract off a term $F_{AA} \cdot F_{BB}$, introduce similarly-contracted terms everywhere else, and so fashion the Lagrangian:

$$\begin{aligned} \mathcal{L} = (2\pi)^{\frac{3}{2}} \Big[& \tfrac{1}{2} F_{\mu\nu\,AB} F^{\mu\nu}{}_{BA} + D_\mu \Phi_{AB} D^\mu \Phi_{BA} + \mu^2 \Phi_{AB} \Phi_{BA} + \tfrac{1}{2} \lambda \left(\Phi_{AB} \Phi_{BA}\right)^2 \Big] \\ & - \tfrac{1}{2} F_{\mu\nu\,AA} F^{\mu\nu}{}_{BB} - D_\mu \Phi_{AA} D^\mu \Phi_{BB} - \mu^2 \Phi_{AA} \Phi_{BB} - \tfrac{1}{2} \lambda \left(\Phi_{AA} \Phi_{BB}\right)^2 \end{aligned}, \tag{2.16}$$

It is readily seen that the pure gauge terms $F_{\mu\nu} F^{\mu\nu}$ in the above are identical to (2.12), which means that these terms now represent the empirically-observed latent nuclear binding energies. However, in constructing this Lagrangian, we carry the same index structure forward to all the remaining terms and thus extend this understanding to the vacuum terms as well.

The value of all of this can be seen from (2.10) and (2.11). For a nucleus with *Z* protons and *N* neutrons, which therefor has *A*=*Z*+*N* nucleons, we may write the *available*, latent binding energy ${}^{A}_{Z}B$ as:

$$\begin{aligned} {}^{A}_{Z}B &= \tfrac{1}{2} Z \cdot \iiint \left((2\pi)^{\frac{3}{2}} F_{P\,AB} \cdot F_{P\,BA} - F_{P\,AA} \cdot F_{P\,BB}\right) d^3x + \tfrac{1}{2} N \cdot \iiint \left((2\pi)^{\frac{3}{2}} F_{N\,AB} \cdot F_{N\,BA} - F_{N\,AA} \cdot F_{N\,BB}\right) d^3x \\ &= Z \cdot \mathbf{7.640679MeV} + N \cdot \mathbf{9.812358MeV} \end{aligned}. \tag{2.17}$$

This simply restates in more formal terms, the results found in sections 11 and 12 of [1]. But, it ties the formal, invariant, theoretical expressions based on the general form $\mathcal{L} \propto -\frac{1}{2}\mathrm{Tr}(F \cdot F)$ with energies $E = -\iiint \mathcal{L} d^3x$, to a very practical formula for deriving real, numeric, empirically-accurate nuclear binding energies.

On the foregoing basis, we now show how to derive not only the *available* binding energies (designated *B*) via (2.17), but the *observed* binding energies (which will be designated throughout as B_0 with a "0" subscript) for several basic nuclides. Specifically, we now derive ${}^{3}_{1}B_0$ for the ^{3}H triton, ${}^{3}_{2}B_0$ for the ^{3}He helion, and most importantly given that it is a fundamental building block of the larger nuclei and many decay process, ${}^{4}_{2}B_0$ for the ^{4}He alpha, all extremely closely to the empirical data. We also lay a foundation for doing the same with larger nuclei.

3. Foundation for Deriving Observed Binding Energies of the 1s Nuclides

Now, it is our goal to derive the *observed, empirical* binging energies for all nuclides with $Z \leq 2$; $N \leq 2$, on a *totally theoretical* basis. Using a nuclear shell model similar to what is used for electron structure, all of these nuclides have nucleons in the 1s shell and so we refer to them as the 1s nuclides. We thereby embark on the undertaking set forth at the end of [1], to understand in detail, how *collections* of Yang-Mills magnetic monopoles – which monopole collections we now understand to be nuclei when the monopoles are protons and neutrons – organize and structure themselves.

The nuclear weights (masses ${}^{A}_{Z}M$) of the nuclides of immediate interest are set forth below in Table 1 (again, *A*=*Z*+*N*). Because we wish to do very precise calculations, and because nuclide masses are known much more precisely in u (atomic mass units, AMU) than in MeV due to the "relatively poorly known electronic charge" [3], we shall work in AMU. When helpful for illustration, we shall convert over to MeV via 1 u = 931.494 061(21) MeV/c2, but only after a calculation is complete. The data for these nuclides (and the electron mass below) is from [3] and / or [4], and is generally known to ten-digit precision in AMU with experimental errors specified at the eleventh and twelfth digits. For other nuclides not listed at these sources, we make use of a very helpful online compilation of atomic weights and isotopes at [5]. Vertical columns list isotopes, horizontal rows list isotones, and diagonal lines link isobars of like-*A*. The nuclides with border frames are the *stable* nuclides. $M(n)={}^{1}_{0}M = 1.008664916000\ u$ is the mass of the neutron, and $M(p)={}^{1}_{1}M = 1.008664916000\ u$ is the mass of the proton.

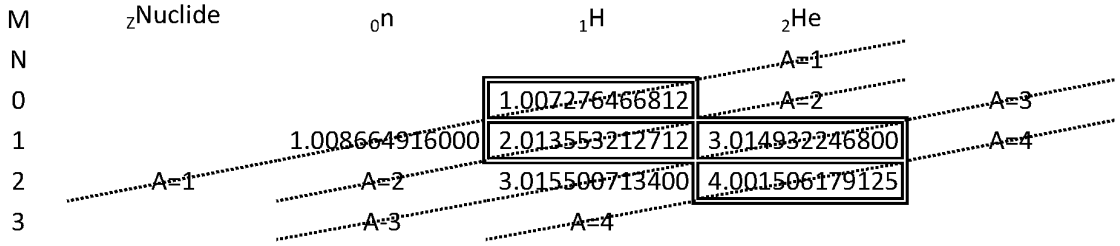

Table 1: Nuclear Weights (${}^{A}_{Z}M$) of 1s Nuclides (AMU)

The *observed* binding energies B_0 are readily calculated from the above using the proton and neutron masses $M(p)={}^{1}_{1}M$ and $M(n)={}^{1}_{0}M$ via ${}^{A}_{Z}B_0 = Z\cdot{}^{1}_{1}M + N\cdot{}^{1}_{0}M - {}^{A}_{Z}M$, and are given by:

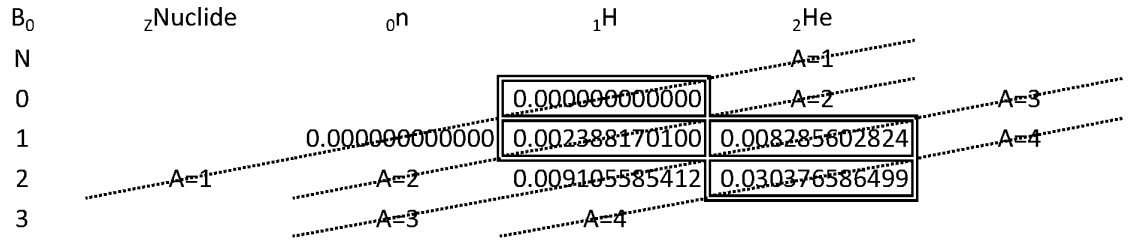

Table 2: Empirical Binding Energies (${}^{A}_{Z}B_0$) of 1s Nuclides (AMU)

Now let's get down to business. We already showed in [12.9] of [1] that by identifying the mass of the up quark with the deuteron binding energy by *defining* via hypothesis that $m_u \equiv B_{H^2} = \mathbf{2.224566\ MeV}$, we can not only establish very precise masses for the up and down quarks but also can explain the confluence of confinement and fission and fusion at ^{56}Fe in a very profound way, wherein 99.8429093% of the *available* binding energy predicted by this model of nucleons as Yang-Mills magnetic monopoles goes into binding the Fe^{56} nucleus and only the remaining 0.1570907% is used to confine the quarks. And, we established that in some manner, nucleons will fuse based on some form of "resonant cavity" analysis based on the quark content of the nucleons. So we now write this identification of the up mass with the deuteron binding energy, in the notations to be employed here, and in AMU, as:

$$m_u \equiv {}^2_1B_0 = B_0({}^2_1H) = 0.0023881701\ 00\ u\ . \tag{3.1}$$

In AMU, the electron mass is:

$$m_e = 0.000548579909\ u\ . \tag{3.2}$$

We then use (1.4) (see also [12.10] of [1]) with (3.1) and (3.2) to obtain the down quark mass:

$$m_d = \frac{(2\pi)^{\frac{3}{2}}}{3} m_e + m_u = 0.0052681432\ 99\ u\ . \tag{3.3}$$

It will also be helpful in the discussion following to use the mass construct:

$$\sqrt{m_u m_d}\ = 0.0035470018\ 76\ u\ , \tag{3.4}$$

because this expression appears frequently in the earlier discussion, starting with (1.5) and (1.6).

We then use the foregoing in (1.5) and (1.6) to calculate in AMU, the *latent, available* binding energy of each of the proton and neutron, designated by *B* without the "0" subscript:

$$B(p) = {}^1_1B = 2m_u + m_d - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right)/(2\pi)^{\frac{3}{2}} = 0.008202607332\ u \tag{3.5}$$

$$B(n) = {}^1_0B = 2m_d + m_u - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right)/(2\pi)^{\frac{3}{2}} = 0.010534000622\ u\ . \tag{3.6}$$

Via (2.17), (3.5) and (3.6) are used to calculate generally, the *latent, available* binding energy:

$$\begin{aligned} {}^A_Z B &= Z\cdot\left(2m_u + m_d - \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}}\right) + N\cdot\left(2m_d + m_u - \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}}\right) \\ &= Z\cdot 0.008202607332\ u + N\cdot 0.010534000622\ u \end{aligned} \tag{3.7}$$

in AMU, for *any* nuclide *Z*, *N*. For the nuclides in Tables 1 and 2, this *theoretically-available, latent* binding energy, is *predicted* to be:

B	$_Z$Nuclide	$_0$n	$_1$H	$_2$He	
N				A=1	
0			0.008202607332	A=2	A=3
1		0.010534000622	0.018736607954	0.026939215286	A=4
2	A=1	A=2	0.029270608576	0.037473215908	
3		A=3	A=4		

Table 3: Theoretically Available Binding Energies ($^{A}_{Z}\mathbf{B}$) of 1s Nuclides (AMU)

Taking the *ratio* of the *empirical* values in Table 2 over the *theoretical* values in Table 3 yields:

B_0/B(%)	$_Z$Nuclide	$_0$n	$_1$H	$_2$He	
N				A=1	
0			0.000000000%	A=2	A=3
1		0.000000000%	12.746010942 1%	30.756659895 4%	A=4
2	A=1	A=2	31.108288672 4%	81.062128677 7%	
3		A=3	A=4		

Table 4: Used-to-Available Binding Energies ($^{A}_{Z}\mathbf{B}_0/^{A}_{Z}\mathbf{B}(\%)$) of 1s Nuclides (%)

So we see, for example, that the ^{4}He alpha nucleus uses about 81.06% of its total available binding energy to bind itself together, with the remaining 18.94% retained to confine the quarks inside each nucleon. The *free* proton and neutron, of course, use 100% of this latent energy to bind their quarks, but as soon as they start to fuse together, they release some of this energy and the negative of this energy goes into the mass loss and binds together the nuclei. The deuteron releases about 12.74% of what is available to bind, while the isobars with A=3 use about 31% of what is available for binding with the balance reserved for quark confinement.

As a point of comparison, for ^{56}Fe, which has the highest percentage of used-to-available binding energy, the nuclear weight $^{56}_{26}M = 55.92067442\,u$ (cf. Table 1), the empirical binding energy is $^{56}_{26}B_0 = 0.52846119\,u$ (cf. Table 2), the available binding energy is $^{56}_{26}B = 0.52928781\,u$ (cf. Table 3), and the used-to-available percentage is $^{56}_{26}B_0/^{56}_{26}B(\%) = 99.843825\%$ (cf. Table 4). No nuclide has a higher such percentage than ^{56}Fe. While ^{62}Ni has a larger empirical binding energy *per nucleon*, its used-to-available percentage is lower, because the calculation in (3.7) literally and figuratively, *weights the neutrons more heavily than the protons* by a ratio of:

$$\frac{B(n)}{B(p)} = \frac{^1_0B}{^1_1B} = \frac{0.010534000622\,u}{0.008202607332\,u} = 1.284225880325 \tag{3.8}$$

The above ratio also explains, at least in part, why heavier nuclides tend to have a greater number of neutrons than protons: As a nucleon grows larger, because the neutrons carry an energy available for binding which is about 28.42% larger than that of the proton, neutrons will in general find it easier to bind into a large nucleus by a factor of 28.42%. Simply put: neutrons

bring more available binding energy to the table than protons and so are more welcome at the table. The nuclides running from ^{31}Ga to ^{48}Cd tend to have stable isotopes with neutron-to-proton number ratios (*N*/*Z*) roughly in the range of (3.8). Additionally, and likely for the same reason, this is the range in which, beginning with ^{41}Nb and ^{42}Mo, and as the N/Z ratio grows even larger than (3.8), one begins to see nuclides which become theoretically unstable with regard to spontaneous fission.

Next, we subtract Table 2 form table 3, to obtain the unused (*U*) binding energy ${}^{A}_{Z}U$ for each nuclide. Of course, for the proton and neutron, all of this energy is unused. This yields:

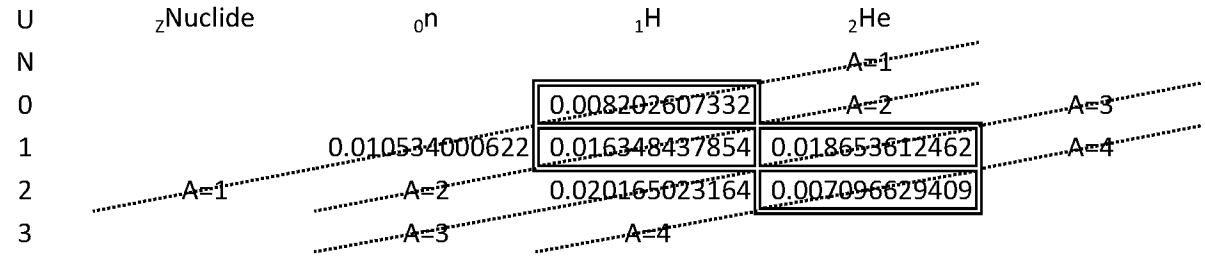

Table 5: Unused Latent Binding Energies (${}^{A}_{Z}U$) of 1s Nuclides (AMU)

Finally, to lay the groundwork for predicting the observed binding energies B_0 in Table 2, let us return to (2.6) and (2.7), remove the trace, and define two (3x3)x(3x3) tensor (outer) product matrices, one for the proton ($E_{P\,AB\,CD}$) and one for the neutron ($E_{N\,AB\,CD}$), according to:

$$(2\pi)^{\frac{3}{2}} E_{P\,AB\,CD} = \tfrac{1}{2}(2\pi)^{\frac{3}{2}} \iiint F_{P\,AB} \cdot F_{P\,CD}\, d^3x = \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_d} & 0 & 0 \\ 0 & \sqrt{m_u} & 0 \\ 0 & 0 & \sqrt{m_u} \end{pmatrix}. \tag{3.9}$$

$$(2\pi)^{\frac{3}{2}} E_{N\,AB\,CD} = \tfrac{1}{2}(2\pi)^{\frac{3}{2}} \iiint F_{N\,AB} \cdot F_{N\,CD}\, d^3x = \left[\begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \otimes \begin{pmatrix} \sqrt{m_u} & 0 & 0 \\ 0 & \sqrt{m_d} & 0 \\ 0 & 0 & \sqrt{m_d} \end{pmatrix} \right]. \tag{3.10}$$

From the above, one can readily deduce that the eighteen diagonal outer product *components* (nine for the proton and nine for the neutron) are (with $E_{P\,AB\,CD} = E_{N\,AB\,CD} = 0$ otherwise):

$$\begin{aligned} &E_{N\,11\,11} = E_{P\,22\,22} = E_{P\,33\,33} = E_{P\,22\,33} = E_{P\,33\,22} = m_u / (2\pi)^{\frac{3}{2}} \\ &E_{P\,11\,11} = E_{N\,22\,22} = E_{N\,33\,33} = E_{N\,22\,33} = E_{N\,33\,22} = m_d / (2\pi)^{\frac{3}{2}} \\ &E_{P\,11\,22} = E_{P\,11\,33} = E_{P\,22\,11} = E_{P\,33\,11} = E_{N\,11\,22} = E_{N\,11\,33} = E_{N\,22\,11} = E_{N\,33\,11} = \sqrt{m_u m_d} / (2\pi)^{\frac{3}{2}} \end{aligned} \quad . \tag{3.11}$$

This is why (3.1), (3.3) *and* (3.4) will be of special interest in the development following. With the foregoing, we now have all the ingredients we need to closely deduce the empirical binding energies in Table 2 on totally theoretical grounds. We start with the alpha, ^{4}He.

4. Prediction of the Alpha Nuclide Binding Energy to 3 parts in One Million

The alpha particle is the ^{4}He nucleus. It is highly stable, and is central to many aspects of nuclear physics insofar as many other nuclei will decay into more stable states by releasing alpha particles via so-called alpha decay. In this way, it is a bedrock building block of nuclear physics. The *unused* binding energy for the alpha particle is ${}^4_2U = 0.007096629409\ u$, as shown in Table 5. Looking over the mass numbers developed in section 3, we see that this is *very close* to being twice the value of $\sqrt{m_u m_d}$ in (3.4), that is, that $2\sqrt{m_u m_d} = 0.007094003752\ u$. In fact, these energies are equal to 2.26 parts *per million!* Might this be an indication that the alpha particle uses all of its available binding energy, less $2\sqrt{m_u m_d}$, for nuclear binding, with the balance of $2\sqrt{m_u m_d}$ retained to confine the quarks inside each of its four nucleons? First, let's look at the numbers, then let's examine the theoretical reasons why this might make sense.

If in fact this numerical coincidence is not just a coincidence but has real physical meaning, then this would mean that the empirical binding energy ${}^4_2B_0$ of the alpha is *predicted* to be (3.7) for ${}^4_2B$, less $2\sqrt{m_u m_d}$, that is:

$$\begin{aligned} {}^4_2B_{0\,\mathrm{Predicted}} &= 2\cdot\left(2m_u + m_d - \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}}\right) + 2\cdot\left(2m_d + m_u - \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}}\right) - 2\sqrt{m_u m_d}, \quad (4.1)\\ &= \mathbf{0.030379212155\ u} \end{aligned}$$

where we have calculated using m_u and m_d from (3.1) and (3.3). In contrast, as we see from Table 2, the empirical ${}^4_2B_0 = \mathbf{0.030376586499\ u}$. The difference:

$${}^4_2B_{0\,\mathrm{Predicted}} - {}^4_2B_0 = 0.0303792121\ 55\ u\text{-}0.0303765864\ 99\ u = \mathbf{0.00000262\ 5656\ u} \qquad (4.2)$$

is extremely small, with these two values, as noted just above, differing from one another by less than 3 parts in 1 million! So, let us regard (4.1) to be a correct prediction of the alpha binding energy, at least to first, dominant order. Now, let's discuss the theoretical reasons why this makes sense.

In [1], a key hypothesis was to identify the mass of the down quark with the deuteron binding energy, see (3.1) here in which we again reviewed that identification. Beyond the numerical concurrence, a theoretical explanation for this, is that in some fashion the nucleons are *resonant cavities*, and so the energies that they will tend to release (or retain) during fusion will be very closely tied to the masses / wavelengths of the contents of these cavities. But, of course, these "cavities" contain up quarks and down quarks, and their masses are given in (3.1) and (3.3) together with $\sqrt{m_u m_d}$ in (3.4), and so these will specify preferred "harmonics" to determine the precise energies which are released for nuclear binding, or retained for quark confinement.

We also see that *components* of the outer products $E_{P\,AB\,CD} = \frac{1}{2}\iiint F_{P\,AB} \cdot F_{P\,CD} d^3x$ and $E_{N\,AB\,CD} = \frac{1}{2}\iiint F_{N\,AB} \cdot F_{N\,CD} d^3x$ in (3.9) and (3.10), times $(2\pi)^{\frac{3}{2}}$ which is naturally supplied by Gaussian integration, take on one of four values: m_u, m_d, $\sqrt{m_u m_d}$, and 0, see (3.11). So, in trying to make a theoretical fit to empirical binding data, and in an effort to not stray from the discipline imposed by the outer products $E_{AB\,CD} = \frac{1}{2}\iiint F_{AB} \cdot F_{CD} d^3x$, we *require* that empirical binding energies be calculated *only* from the outer products $E_{AB\,CD} = \frac{1}{2}\iiint F_{AB} \cdot F_{CD} d^3x$ for the proton and neutron, using *only* some combination of a) the *components* of this outer product and b) *index contractions* of this outer product, see again the discussion in section 2. So the ingredients that we shall use to do this numerical fitting, will be restricted to a) the *latent, available* nuclide binding energies as calculated from (3.7), b) the three energies m_u, m_d, $\sqrt{m_u m_d}$ and quantized multiples thereof, c) any of the foregoing with a $(2\pi)^{\frac{3}{2}}$ coefficient or divisor, as suitable, and d) the rest mass of the electron m_e. This fitting involves essentially poring over the numerical nuclear binding data, and seeing if it can be arrived at closely using *only* the foregoing ingredients. In the case of the alpha, (4.1) meets all of these criteria. In fact, rewritten using (2.6) through (2.9) and (3.11), we find that (4.1) can be expressed *entirely* in terms of the outer tensor product $E_{AB\,CD} = \frac{1}{2}\iiint F_{AB} \cdot F_{CD} d^3x$, as:

$$
\begin{aligned}
{}^4_2B_{0\,\mathrm{Pr}\,edicted} &= 2\cdot\left((2\pi)^{\frac{3}{2}} E_{P\,AB\,BA} - E_{P\,AA\,BB}\right) + 2\cdot\left((2\pi)^{\frac{3}{2}} E_{N\,AB\,BA} - E_{P\,AA\,BB}\right) - (2\pi)^{\frac{3}{2}}\left(E_{P1122} + E_{N1122}\right) \\
&= 2\cdot\left(2m_u + m_d - \frac{m_d + 4\sqrt{m_u m_d} + 4m_u}{(2\pi)^{\frac{3}{2}}}\right) + 2\cdot\left(2m_d + m_u - \frac{m_u + 4\sqrt{m_u m_d} + 4m_d}{(2\pi)^{\frac{3}{2}}}\right) - 2\sqrt{m_u m_d} \quad . \qquad (4.3)
\end{aligned}
$$

This totally theoretical expression yields the alpha binding energy to 2.26 parts per million.

In this light, (4.3) tells us that the alpha binding energy is actually the 11 22 *component* of a (3x3)x(3x3) outer product $E_{AB\,CD}$, in linear combination with invariant traces of $E_{AB\,CD}$. This is reminiscent, for example, of the Maxwell tensor $-4\pi T^{\mu\nu} = F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}$, which has some components with both a component structure and a trace term just like (4.3) (e.g. $-4\pi T^{00} = F^{0\alpha}F^{0}{}_{\alpha} - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}$) (we analogize $F^{0\alpha}F^{0}{}_{\alpha}$ to $E_{P\,22\,33}$ and $F^{\alpha\beta}F_{\alpha\beta}$ to $(2\pi)^{\frac{3}{2}} E_{AB\,BA} - E_{AA\,BB}$ in (4.3)), and which has other components that do *not* include the trace term (e.g., $-4\pi T^{01} = F^{0\alpha}F^{1}{}_{\alpha} - \eta^{01}\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} = F^{0\alpha}F^{1}{}_{\alpha}$, where $\eta^{\mu\nu}$ filters out the trace). This latter analogy allows us to represent (3.1) for the deuteron as a component *without* a trace term, thus:

$$
{}^2_1B_{0\,\mathrm{Predicted}} = m_u = 0 + (2\pi)^{\frac{3}{2}} E_{N\,11\,11} \,. \qquad (4.4)
$$

So we now start to think about the individual, observed nuclear binding energies as *components of a (3x3)x(3x3) fourth rank Yang Mills tensor* of which (4.3) and (4.4) are two

samples. Thus, as we proceed to examine many different nuclides, we will want to see what patterns may be discerned as to how each nuclide fits into this tensor.

Physically, the alpha particle of course contains two protons and two neutrons, and at the quark level, six up quarks and six down quarks. It is seen that in (4.1), the up quarks enter in a completely symmetric fashion relative to the down quarks, i.e., that (4.1) is invariant under the interchange $m_u \leftrightarrow m_d$. The factor of 2 in front of $\sqrt{m_u m_d}$ of course means that two components of the outer product are also involved. The deuteron, per (4.4), uses only one member of the m_u, m_d, $\sqrt{m_u m_d}$ "component toolkit" from (3.11), i.e., m_u, while (4.4) uses two members of this toolkit, i.e., $2\sqrt{m_u m_d}$. Further, while each component of m_u, m_d, $\sqrt{m_u m_d}$ in the (3.11) toolkit is associated with several different components of the outer product, we have as a preliminary matter hypothesized an association $2\sqrt{m_u m_d} = E_{P11\,22} + E_{N11\,22}$, so that the neutron pair and the proton pair each contribute $1\sqrt{m_u m_d}$ to (4.3), and (4.3) thereby remains absolutely symmetric under $p \leftrightarrow n$ and $u \leftrightarrow d$ interchange. The choice of the $E_{11\,22}$ elements appears to be somewhat arbitrary given (3.11), and should be revisited once we study other nuclides not yet considered and seek to understand the more general Yang-Mills tensor structure of which the individual nuclide energies are components.

One other physical observation is also particularly noteworthy: Below in Figure 1, we have included the well-known "per-nucleon" binding graph to facilitate discussion. One of the great mysteries of nuclear physics, is how, exactly, to account for the great "chasm" between the ^{2}H, ^{3}H and ^{3}He nuclides, and the alpha nuclide ^{4}He for which we have now predicted the binding energy to within a fraction of a percent. Contrasting (4.1) for ^{4}He with (4.4) for ^{2}H, we see that for the latter deuteron, we "start at the bottom" with ${}^1_1B_0 = 0$ for ^{1}H, and then "add" ${}^2_1B_0 = 0 + m_u$ worth of energy to bind the proton and the neutron together into ^{2}H. But for the alpha, we "start at the top," with the total latent binding energy ${}^4_2B = 0.037473215908\,u$, and then subtract off $2\sqrt{m_u m_d}$, to obtain the empirical result ${}^4_2B_0 = 0.037473215908\,u - 2\sqrt{m_u m_d}$. But as we learned in section 12 of [1], any time we do *not* use some of the latent energy for nuclear binding, that unused energy remains behind to confine the quarks. So what we learn is that for the alpha particle, a total of $2\sqrt{m_u m_d} = 0.007094004\,u$ is *held in reserve* to confine the quarks, while the balance is *released* to bind the nucleons to one another.

Now to the point: for some nuclides, (e.g. the deuteron) the question is: how much energy is *released* from quark confinement to bind the nucleons? This is a "bottom to top" approach. For other nuclides (e.g., the alpha), the question is: how much energy is *reserved* out of the theoretical maximum available, to confine the quarks. This is a "top to bottom" approach. For "top to bottom" nuclides, there is an invariant trace in the tensors. For "bottom to top" nuclides there is not. Using the Maxwell tensor analogy just discussed, one might suppose that somewhere there is a Kronecker delta $\delta^A{}_B$ and / or $\delta^{AB}{}_{CD}$ which filters out the trace from some "off-diagonal" terms and leaves the trace intact for other "on-diagonal" terms. In this way, the "bottom to top" nuclides are off-diagonal elements, and the "top to bottom" nuclides are "on

diagonal." In either case, however, the "resonance" for nuclear binding is established by the components of the $E_{N\,AB\,CD}$, which are m_u, m_d, $\sqrt{m_u m_d}$ in some combination and / or integer multiple. And, as regards Figure 1 below, the chasm leading up to ^{4}He, is explained on the basis that each of ^{2}H, ^{3}H and ^{3}He are "bottom to top" "off-diagonal" nuclides, while ^{4}He is the first "top to bottom" "on-diagonal" nuclide.

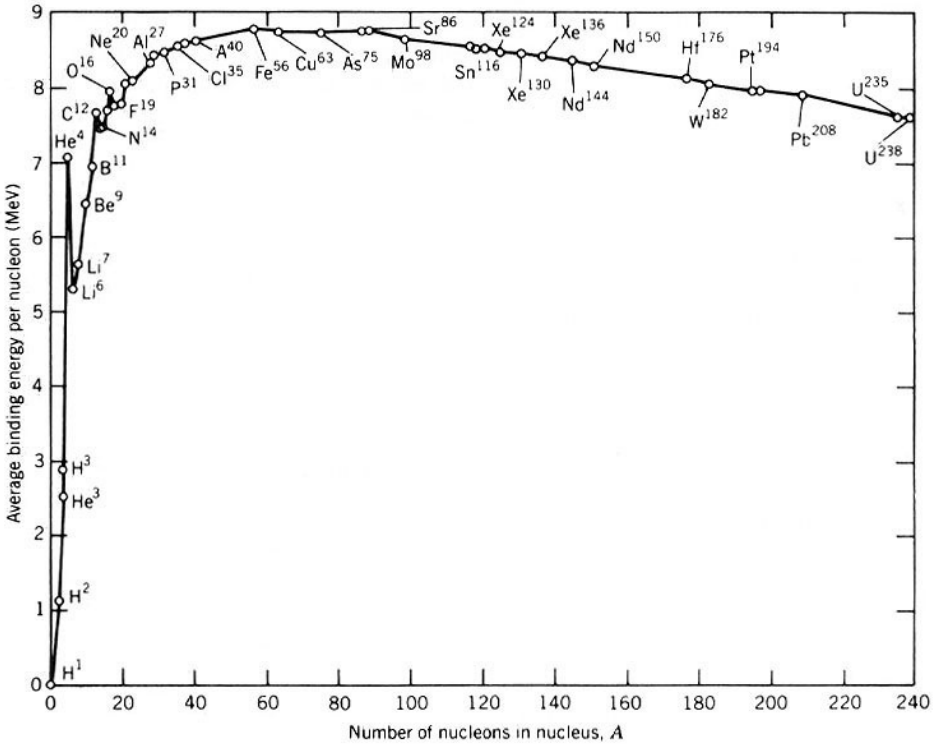

Figure 1

Let us now peek ahead at some higher energy nuclides, namely, $_3$Li and $_4$Be with Z=3 and Z=4, because they deepen the lessons learned above for the alpha. Using a nuclear shell model similar to what is used for electron structure, all the nucleons in the ^{4}He alpha are in 1s shells. The two protons are spin up and down each with 1s, and as are the two neutrons. As soon as we add one more nucleon, by exclusion, we must jump up to the 2s shell, which admits four more nucleons and so can reach up to ^8_4Be before we must make a first incursion into the 2p shell. The four additional nuclides we shall wish to briefly examine are shown in Table 6 below:

B_0	$_Z$Nuclide	$_0$n	$_1$H	$_2$He	$_3$Li	$_4$Be
N				A=1		
0			0.000000000000	A=2	A=3	
1		0.000000000000	0.002388170100	0.008285602824	A=4	
2	A=1	A=2	0.009105585412	0.030376586499	A=5	A=6
3		A=3	A=4	A=5	0.034347094	0.040365106
4				A=6	0.042130255	0.060654752
				A=7	A=8	

Table 6: Empirical Binding Energies ($^A_Z B_0$) of Selected 1s and 2s Nuclides (AMU)

We note immediately from the above – which has been noticed by others before – that the binding energy ${}^{8}_{4}B_0 = 0.060654752\,u$ of ^{8}Be is almost twice as large as that of the alpha particle, to just under one part in ten thousand AMU. Specifically:

$$2\cdot{}^{4}_{2}B_0 - {}^{8}_{4}B_0 = 2\cdot 0.030376586499\,u - 0.060654752\,u = \mathbf{0.000098421\,u}\,. \tag{4.5}$$

This is part of the explanation as to why the ^{8}Be is unstable and invariably decays almost immediately into two alpha particles of ^{4}He. (It is ^{9}Be which is the stable Be isotope.) But what is of particular interest here, is to subtract off the alpha ${}^{4}_{2}B_0 = 0.030376586499\,u$ from each of the Li and Be isotopes shown in the above, and compare them side by side with the non-zero binding energies from H and He. The result of this exercise is shown in Table 7 below.

B_0	$_Z$Nuclide	$_1$H	$_2$He	B_0-B_0(alpha)	$_Z$Nuclide	$_3$Li	$_4$Be
N				N			
1		0.002388170100	0.008285602824	3		0.003970507	0.009988519
2	A=2	0.009105585412	0.030376586499	4	A=6	0.011753668	0.030278165
3	A=3	A=4		5	A=7	A=8	

Table 7: Comparison of Alpha-subtracted 2s Binding Energies, with 1s Binding Energies (AMU)

Equation (4.5) is represented above by the fact that ${}^{8}_{4}B_0 - {}^{4}_{2}B_0 \cong {}^{4}_{2}B_0$. The chart on the left is a "1s square" and the chart on the right is a "2s square." But they are both "s-squares." What is of interest is that the remaining three nuclides in the Li, Be "square" are not dissimilar either from the pattern shown for the other three nuclides in the H, He "square." This means that three of the four nuclides in the 2s square start "at the bottom" "off-diagonal" just as in 1s, and the fourth, ^{8}Be starts "on diagonal" "at the top." But, in the 2s square, the "bottom" is ${}^{4}_{2}B_0 = 0.030376586499\,u$ from the alpha particle. So the complete 1s shell below the 2s shell provides a "platform," a "zero-prime energy" for examining binding energies in the 2s square.

Finally, before turning to ^{3}He in the next section, let us comment briefly on experimental errors and the precision of the foregoing. The prediction of the alpha in (4.1) to be ${}^{4}_{2}B_{0\,\mathrm{Predicted}} = 0.030379212155\,u$, in contrast to the empirical ${}^{4}_{2}B_0 = 0.030376586499\,u$, is an exact match, in AMU, through the fifth decimal place, but is still *not* within experimental errors. Specifically, the alpha mass listed in [4] and shown in Table 1 is 4.001506179125(62) u, which is accurate to *ten* decimal places in AMU. Similarly, the proton mass 1.007276466812(90) u and the neutron mass 1.00866491600(43) u used to calculate ${}^{4}_{2}B_0 = 0.030376586499\,u$ are accurate to ten and nine decimal places respectively. So the match between ${}^{4}_{2}B_{0\,\mathrm{Predicted}}$ and the empirical ${}^{4}_{2}B_0$ beyond five decimals to under 3 parts per million is still not within the experimental errors, which are known to at least nine decimal places in AMU. Consequently, (4.1) must be regarded as a very close, but still *approximate* relationship for the observed alpha binding energy. Additionally, because (4.1) is based on (3.1), wherein the mass of the up quark is identified with

the deuteron binding energy $m_u \equiv {}^2_1B_0 = B_0({}^2_1H) = 0.0023881701\,00\ u$, the question must be considered whether this identification (3.1), while very close, is also still approximate.

Specifically, it is *possible* to make (4.1) for alpha into an *exact* relationship, within experimental errors, if we reduce the up quark mass by exactly ε=0.000000351251415 u (in the seventh decimal place), such that:

$$m_u = 0.0023878188\,49\ u \cong {}^2_1B_0 = B_0({}^2_1H) = 0.0023881701\,00\ u\ . \qquad (4.6)$$

That is, we can make (4.1) for the alpha into an *exact* relationship if we make (3.1) for the up quark into an *approximate* relationship, or vice versa, but not both. So what do we do? A further clue is provided by (4.5), whereby the *empirical* ${}^8_4B_0 / {}^4_2B_0 \cong 2$ is a close, but still approximate relationship. This seems to suggest, as one adds more nucleons to a system and makes empirical predictions such as (4.1) based on the up and down quark masses, that higher order corrections (at the sixth decimal place in AMU for alpha and the fifth decimal place in AMU for ${}^8_4B_0$) will still be needed. So because two body systems such as the deuteron can generally be modeled nearly-exactly, and because a deuteron will suffer less from "large *A=Z+N* corrections" than any other nuclide, it makes sense absent evidence to the contrary to regard (3.1) identifying the up quark mass with the deuteron binding energy to be an *exact* relationship, and to regard (4.1) for the alpha to be an *approximate* relationship that still requires some correction ε in the sixth decimal place. Similarly, as we develop other relationships which, in light of experimental errors, are also close but still approximate, we shall take the view that these relationships too, will require higher order corrections based on factors such as the complexities of a multi-body system, growing nuclide size, and the fact that the nuclear interaction drops off rapidly as between nucleons not immediately adjacent to one another in a nucleus. Thus, for the moment, we leave (3.1) intact as an exact relationship.

In section 9, however, we shall show why (3.1) is actually not an exact relationship but is only approximate to about 8 parts per *ten million* AMU. But this will be due not to the closeness of the alpha particle predicted versus observed energies, but due to our being able to develop a theoretical expression for the difference $M(n) - M(p)$ between the observed masses of the fee neutron and the free proton to *better than one part per million* AMU.

5. Prediction of the Helion Nuclide Binding Energy to 4 parts in 100,000

Now, we turn to the ${}^3_2He$ nucleus, sometimes referred to as the helion. In contrast with the alpha and the deuteron already examined which are integer-spin bosons, this nucleon is a half-integer spin fermion. Knowing that our ingredients for constructing binding energy predictions are m_u, m_d, $\sqrt{m_u m_d}$, knowing as pointed out after (4.4) that we will "start at the bottom" for this nuclide, and knowing already that the "components" in the (3.11) toolbox we have used so far are m_u for ${}^2_1B_0$ and $2\sqrt{m_u m_d}$ for ${}^4_2B_0$, it turns out after some exercises strictly with this toolbox of energies, that we can make a fairly close prediction by setting:

$$B_0(^3He)_{\text{Predicted}} = {}^3_2B_{0\ \text{Predicted}} \cong 2m_u + \sqrt{m_u m_d} = \mathbf{0.008323342076\ u}\,. \tag{5.1}$$

The empirical energy from Table 2, in comparison, is ${}^3_2B_0 = \mathbf{0.008285602824\ u}$, so that:

$${}^3_2B_{0\ \text{Predicted}} - {}^3_2B_0 = 0.008323342076\ u - 0.008285602824\ u = \mathbf{0.000037739252\ u}\,. \tag{5.2}$$

While not quite as close as (4.2) for the alpha particle, this is still a very close match to just under 4 parts in 100,000 AMU. But does this make sense in light of the outer products (3.9), (3.10)?

If we wish to write (5.1) in the manner of (4.3) and (4.4) in terms of the components of an outer product $E_{AB\,BA}$, then referring to (3.9), we find that:

$${}^3_2B_{0\ \text{Predicted}} = (2\pi)^{\frac{3}{2}} E_{P\,33\,AA} = 2m_u + \sqrt{m_u m_d} = \sqrt{m_u}\left(\sqrt{m_d} + 2\sqrt{m_u}\right). \tag{5.3}$$

So the expression $2m_u + \sqrt{m_u m_d}$ in (5.1) in fact has a very natural formulation which utilizes the trace $\sqrt{m_d} + 2\sqrt{m_u}$ (AA index summation) of one of the matrices in (3.9), times a $\sqrt{m_u}$ taken from the third (or possibly second) diagonal component of the other matrix in (3.9). The use in (5.3) of E_P from (3.9) rather than of E_N from (3.10), draws from the fact that we need the trace to be $\sqrt{m_d} + 2\sqrt{m_u}$, and not $\sqrt{m_u} + 2\sqrt{m_d}$ as would otherwise occur if we used (2.7). So here, the empirical data clearly causes us to choose components from E_P rather than from E_N.

6. Prediction of the Triton Nuclide Binding Energy to 3 parts in One Million, and the Proton – Neutron Mass Difference to 7 Parts in Ten Million

Now we turn to the ${}^3_1H$ triton nuclide, which as shown in Table 2, has a binding energy ${}^3_1B_0 = 0.009105585412\ u$. As with the alpha and the helion, we use the energies from components of the outer products $E_{AB\,CD}$ of section 2, see again, (3.11). However, following careful consideration of all possible combinations, there is no readily apparent combination of m_u, m_d, $\sqrt{m_u m_d}$ together with m_e and factors of $(2\pi)^{\frac{3}{2}}$ which yield a close match to well under 1 percent, to the observed binding energy ${}^3_1B_0 = 0.009105585412\ u$.

But all is not lost, and much more is found: When studying nuclear data, there are two interrelated ways to formulate that data. First, is to look at binding energies as we have done so far. Second, is to look a *nuclear weight loss*, conversely known as "mass excess." This formulation, mass excess, is very helpful when studying nuclear fusion and fission processes, and as we shall now see, it is this approach that enables us to match up the empirical binding data for the triton to the m_u, m_d, $\sqrt{m_u m_d}$, m_e and factors of $(2\pi)^{\frac{3}{2}}$ that we have already successfully employed for the deuteron, alpha, and helion. As a tremendous bonus, we will be able to derive a *strictly theoretical* expression for the *observed, empirical* difference:

$$M(n) - M(p) = {}^{1}_{0}M - {}^{1}_{1}M = 0.001388449188\ u \tag{6.1}$$

between the free, unbound neutron mass $M(n) = 1.008664916000\ u$ and the free, unbound proton mass $M(p) = 1.007276466812\ u$, see Table 1.

To begin with, let us consider a hypothetical fusion process in which we seek to fuse a ${}^{1}_{1}H$ nucleus (proton) with a ${}^{2}_{1}H$ nucleus (deuteron) to produce a ${}^{3}_{1}H$ nucleus (triton), plus whatever by-products emerge from the fusion. Because the inputs ${}^{1}_{1}H$ and ${}^{2}_{1}H$ each have a charge of +1, and the output ${}^{3}_{1}H$ also has a charge of +1, a positron will be needed to carry off the additional electric charge, and this will need to be balanced with a neutrino. Of course, there will be some fusion energy released. So in short, the fusion reaction we now wish to study is:

$${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + e^{+} + \nu + \text{Energy} \tag{6.2}$$

The question: how much energy is released?

As we can see, this process includes a β^{+} decay. If we neglect the neutrino mass, i.e., if we take $m_{\nu} \cong 0$, and since $m_{e^{+}} = m_{e}$, we can reformulate (6.2) using the nuclide masses in Table 1, as the *empirical* relationship:

$$\text{Energy} = {}^{1}_{1}M + {}^{2}_{1}M - {}^{3}_{1}M - m_{e} = \mathbf{0.004780386215\ u} \tag{6.3}$$

If we then return to our "toolbox" (3.11), we see that $2m_{u} = \mathbf{0.004776340200\ u}$. The difference:

$$\text{Energy} - 2m_{u} = 0.004780386215\ u - 0.004776340200\ u = -\mathbf{0.000004046015\ u}, \tag{6.4}$$

is four parts per million! So, we now regard $\text{Energy} \cong 2m_{u}$ to be very close relationship to the empirical data for the reaction (6.2). In context, for the deuteron, alpha and helion, our toolbox matched up to a binding energy. But for the triton, our toolbox instead matched up to a fission-release energy. A new player in this mix, which has not heretofore become directly involved in predicting binding energies, is the electron rest mass, which appears in (6.3). So, based on (6.4), we set $\text{Energy} = 2m_{u}$, and then rewrite (6.3), using ${}^{1}_{1}M = M(p)$, as:

$${}^{3}_{1}M_{\text{Predicted}} = {}^{1}_{1}M + {}^{2}_{1}M - 2m_{u} - m_{e} = M(p) + {}^{2}_{1}M - 2m_{u} - m_{e}\,. \tag{6.5}$$

Now, to translate between Table 1 and Table 2, we of course used:

$${}^{A}_{Z}B_{0} = Z \cdot {}^{1}_{1}M + N \cdot {}^{1}_{0}M - {}^{A}_{Z}M \tag{6.6}$$

which relates observed binding energy B_0 in general, to nuclear mass M in general. So let us now use (6.6) specifically for ${}^{3}_{1}B_0$ with Z=1 and N=2, and combine this with (6.5) using ${}^{1}_{0}M = M(n)$, to write:

$$ {}^{3}_{1}B_{0\,\text{Predicted}} = 1\cdot{}^{1}_{1}M + 2\cdot{}^{1}_{0}M - {}^{3}_{1}M = 2M(n) - {}^{2}_{1}M + 2m_u + m_e \tag{6.7} $$

Then, to take care of the remaining deuteron mass ${}^{2}_{1}M$ in the above, we use (6.6) a second time, now for ${}^{2}_{1}B_0$ with Z=1 and N=1, to write:

$$ {}^{2}_{1}B_{0\,\text{Predicted}} = {}^{1}_{1}M + {}^{1}_{0}M - {}^{2}_{1}M = M(p) + M(n) - {}^{2}_{1}M \tag{6.8} $$

We then combine (6.8) rewritten in terms of ${}^{2}_{1}M$, with (6.7) to obtain:

$$ {}^{3}_{1}B_{0\,\text{Predicted}} = M(n) - M(p) + {}^{2}_{1}B_{0\,\text{Predicted}} + 2m_u + m_e \tag{6.9} $$

Now all that is left is ${}^{2}_{1}B_{0\,\text{Predicted}}$. But this is just the deuteron binding energy that we have already found in (4.4), namely, ${}^{2}_{1}B_{0\,\text{Predicted}} = m_u$, and which we take to be an *exact* relationship, see the discussion at the end of section 4. So final substitution of ${}^{2}_{1}B_{0\,\text{Predicted}} = m_u$ into (6.9) yields:

$$ {}^{3}_{1}B_{0\,\text{Predicted}} = M(n) - M(p) + 3m_u + m_e . \tag{6.10} $$

So now, we do have a prediction for the triton binding energy, and it does include the electron rest mass, but it also includes the *difference* (6.1) between the free (unbound) neutron and proton masses. It would be highly desirable for many reasons beyond simply the present exercise, to express this relationship as well, on a completely theoretical basis.

To do this, we repeat the analysis just conducted, but now, we fuse two ${}^{1}_{1}H$ nuclei (protons) into a single ${}^{2}_{1}H$ nucleus (deuteron). Analogously to (6.2), we thus write:

$$ {}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu + \text{Energy} , \tag{6.11} $$

and we again ask, how much energy? This fusion, it is also noted, is the first step of the process by which the sun and stars produce their energy, and is the simplest of all fusions, and so is interesting from a wide variety of viewpoints.

As in (6.3), we first reformulate (6.11) using the nuclide masses in Table 1, as the empirical:

$$ \text{Energy} = {}^{1}_{1}M + {}^{1}_{1}M - {}^{2}_{1}M - m_e = 2M(p) - {}^{2}_{1}M - m_e = \mathbf{0.000451141003\ u} , \tag{6.12} $$

As a point of reference, this is equivalent to 0.420235 MeV, which will be familiar to anybody to who has studied hydrogen fusion. As before, we pore over the "toolbox" in (3.11), including $(2\pi)^{\frac{3}{2}}$ divisors, to discover that $2\sqrt{m_\mu m_d}/(2\pi)^{\frac{3}{2}} = \mathbf{0.000450424092\,u}$. Once again, we see a very close match, specifically:

$$\text{Energy} - 2\sqrt{m_\mu m_d}/(2\pi)^{\frac{3}{2}} = 0.000451141003\,u - 0.000450424092\,u = \mathbf{0.000000716911\,u}. \quad (6.13)$$

Here, the match is to *just over 7 parts in ten million*. This is a mere 0.000667798 MeV, which is a scant 0.1306848742% of the electron mass, and it is the closest match yet! So we take this to be a significant relationship as well, and use this to rewrite (6.12) as:

$$2\sqrt{m_\mu m_d}/(2\pi)^{\frac{3}{2}} = 2M(p) - {}^2_1M - m_e, \quad (6.14)$$

Now we need to reduce this expression. First, using (3.1), namely ${}^2_1B_0 = m_u$, we write (6.8) as:

$${}^2_1M = M(p) + M(n) - m_u. \quad (6.15)$$

Then we combine (6.15) with (6.14) and rearrange to write:

$$\left[M(n) - M(p)\right]_{\text{Predicted}} = m_u - m_e - 2\frac{\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} = \mathbf{0.001389166099\,u}. \quad (6.16)$$

<u>This is an extremely important relationship</u>, as it relates the difference (6.1) between the neutron and proton mass solely to the up, down, and electron masses. This is useful in a wide array of circumstances, including all forms of beta decay and the relationships between nuclear isobars (along the diagonal lines of like-*A* which are shown in the Tables here) which *by definition* convert one into the other via a beta decay which exchanges a neutron with a proton. Comparing (6.16) with (6.1), we see that:

$$\begin{aligned}&\left[M(n) - M(p)\right]_{\text{Predicted}} - \left[M(n) - M(p)\right]_{\text{Observed}} \\ &= 0.001389166099\,u - 0.001388449188\,u = \mathbf{0.000000716911\,u}\end{aligned}. \quad (6.17)$$

This is the exact same degree of accuracy, to just over *7 parts in ten million* AMU, which we saw in (6.13).

So now, taking (6.16) as a given relationship, we use this in (6.10) to write:

$$B_0({}^3H)_{\text{Predicted}} = {}^3_1B_{0\,\text{Predicted}} = 4m_u - 2\frac{\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} = \mathbf{0.009102256308\,u}. \quad (6.18)$$

As a result, we finally have a theoretical expression for the binding energy of the triton, totally in terms of the up and down quark masses. The empirical value ${}^{3}_{1}B_{0} = \mathbf{0.009105585412\ u}$ is shown in Table 2, and doing the comparison, we have:

$${}^{3}_{1}B_{0\,\text{Predicted}} - {}^{3}_{1}B_{0} = 0.009102256308\,u - 0.009105585412\,u = -\mathbf{0.000003329104u}\,. \tag{6.19}$$

We see that this result is accurate to just over three parts in one million!

As to the theoretical expression for (6.18) using components of an outer product $E_{AB\,BA}$ as in (4.3), (4.4) and (5.3), one way to write (6.18) is:

$${}^{3}_{1}B_{0\,\text{Predicted}} = (2\pi)^{\frac{3}{2}}\left(E_{P22\,22} + E_{P22\,33} + E_{P33\,22} + E_{P33\,33}\right) - E_{P11\,22} - E_{P11\,33} = 4m_u - 2\frac{\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}}\,. \tag{6.20}$$

As earlier noted, there will be some ambiguity in these tensor component assignments until we have developed a wider swathe of binding energies beyond the "1s square," and begun to discern the wider patterns. But we have now reached our goal of deducing precise theoretical expressions for all of the 1s binding energies, solely as a function of elementary fermion masses. In the process, we have also deduced a like-expression for the neutron-proton mass difference!

7. Excess Mass Predictions

Let us now aggregate some of the results so far. First of all, let us go back to (6.5), and use (6.15) and the neutron-proton mass difference (6.16) to rewrite (6.5) as:

$${}^{3}_{1}M_{\text{Predicted}} = M(p) + 2M(n) - 4m_u + 2\frac{\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}}\,. \tag{7.1}$$

Specifically, we have refashioned (6.5) to include one proton mass and two neutron masses, because this ${}^{3}_{1}H$ triton nuclide in fact contains one proton and two neutrons. Thus, the additional terms $-4m_u + 2\sqrt{m_\mu m_d}\,/(2\pi)^{\frac{3}{2}}$ represent a theoretical value of the mass excess, expressed as a mass loss (negative number). We see this is equal in magnitude and opposite in sign to binding energy (6.20).

Let us do a similar thing for the Helium nuclei. First we use (6.6) to write:

$${}^{3}_{2}B_{0} = 2\cdot{}^{1}_{1}M + {}^{1}_{0}M - {}^{3}_{2}M = 2M(p) + M(n) - {}^{3}_{2}M \tag{7.2}$$

We then place ${}^{3}_{2}M$ on the left side and use (5.1) to write:

$${}^{3}_{2}M = 2M(p) + M(n) - 2m_u - \sqrt{m_u m_d}\,. \tag{7.3}$$

Here, $-2m_u - \sqrt{m_u m_d}$ is helion mass loss, also equal and opposite to binding energy (5.1).

Next, we again use (6.6) to write:

$$ {}^4_2B_0 = 2\cdot{}^1_1M + 2\cdot{}^1_0M - {}^4_2M = 2\cdot M(p) + 2\cdot M(n) - {}^4_2M \tag{7.4} $$

Combining this with (4.1) then yields:

$$ {}^4_2M = 2M(p) + 2M(n) - 6m_u - 6m_d + \frac{10m_d + 10m_u + 16\sqrt{m_u m_d}}{(2\pi)^{\frac{3}{2}}} + 2\sqrt{m_u m_d} \tag{7.5} $$

The mass loss for the alpha – much larger than for the other nuclides we have examined – is given by the lengthier terms after $2M(p) + 2M(n)$. Again, this is equal and opposite to the alpha binding energy in (4.11), with terms consolidated in (7.5) above.

Finally, from (3.1), via (6.6), it is easy to deduce for the deuteron, that:

$$ {}^2_1M \equiv M(p) + M(n) - m_u , \tag{7.6} $$

with a mass loss represented simply by $-m_u$, again, equal and opposite the binding energy (3.1).

8. A Theoretical Review of the Solar Fusion Cycle, and a Possible Approach to Catalyzing Fusion Energy Release

As a practical exercise, let us now use all of the foregoing results to examine the solar fusion cycle. The first step in this cycle is (6.11), for the fusion of two protons into a deuteron. It was from (6.11) that we determined that an energy (6.12) is released in this fusion, which energy, in light of (6.14), now becomes:

$$ \text{Energy}\left({}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu + \text{Energy}\right) = 2\frac{\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} = 0.000451141003\,u . \tag{8.1} $$

This equates to the well-known 0.420235 MeV as noted earlier. The positron annihilates with an electron $e^+ + e^- \rightarrow \gamma + \gamma$ to produce an additional $2m_e$ worth of energy as well.

The second step in the solar fusion cycle is the reaction:

$$ {}^2_1H + {}^1_1H \rightarrow {}^3_2He + \text{Energy} \tag{8.2} $$

wherein the deuterons produced in (8.1) fuse with protons to produce helions. We write this reaction in terms of the masses as:

$$\text{Energy}={}_1^2M+{}_1^1M-{}_2^3M \tag{8.3}$$

The proton mass is ${}_1^1M$, and these other two masses have already been found, respectively, in (7.6) and (7.3). Thus, (8.3) may be reduced to:

$$\text{Energy}\left({}_1^2H+{}_1^1H\rightarrow{}_2^3He+\text{Energy}\right)=m_u+\sqrt{m_u m_d}=0.005935171976\,u\,, \tag{8.4}$$

which equates to 5.528577 MeV, also a well-known number in the study of solar fusion.

The final step in this cycle fuses helions together to produce alpha particles plus protons, which themselves are available to repeat the cycle starting at (8.1), according to:

$${}_2^3He+{}_2^3He\rightarrow{}_2^4He+{}_1^1H+{}_1^1H+\text{Energy} \tag{8.5}$$

The mass equivalent of this relationship is as follows:

$$\text{Energy}={}_2^3M+{}_2^3M-{}_2^4M-{}_1^1M-{}_1^1M \tag{8.6}$$

Here we again make use of ${}_1^1M=M(p)$, together with (7.3) and (7.5) to write:

$$\begin{aligned}&\text{Energy}\left({}_2^3He+{}_2^3He\rightarrow{}_2^4He+{}_1^1H+{}_1^1H+\text{Energy}\right)\\&=2m_u+6m_d-4\sqrt{m_u m_d}-\frac{10m_d+10m_u+16\sqrt{m_u m_d}}{(2\pi)^{\frac{3}{2}}}=0.013732528003\,u\,.\end{aligned} \tag{8.7}$$

This equates to 12.791768 MeV, also a well-known number from solar fusion studies.

Now, as is well known (see, e.g. [6]), the reaction (8.4) must occur twice to produce the two ${}_2^3He$ which are input to (8.7), and the reaction (8.1) must occur twice to produce the two ${}_1^2H$ which are in turn the input to (8.4). So pulling this all together from (8.1), (8.4), (8.7) and $e^+ + e^- \rightarrow \gamma+\gamma$, we may express the entire solar fusion cycle as:

$$\begin{aligned}&\text{Energy}\left(4\cdot{}_1^1H+2e^-\rightarrow{}_2^4He+\gamma(12.79MeV)+2\gamma(5.52MeV)+2\gamma(.42MeV)+4\gamma(e)+2\nu\right)\\&=\left(2m_u+6m_d-4\sqrt{m_u m_d}-\frac{10m_d+10m_u+16\sqrt{m_u m_d}}{(2\pi)^{\frac{3}{2}}}\right)+2\left(m_u+\sqrt{m_u m_d}\right)+2\left(2\frac{\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}}\right)+4(m_e)+2(m_\nu)\cdot\\&=4m_u+6m_d+4m_e-2\sqrt{m_u m_d}-\frac{10m_d+10m_u+12\sqrt{m_u m_d}}{(2\pi)^{\frac{3}{2}}}=26.733389\;MeV\end{aligned} \tag{8.8}$$

Above, in the top line, we show in detail each energy release from largest to smallest. In the middle line, we have segregated in separate parenthesis, each contribution that is shown in the

top line, including the neutrino mass presumed to be virtually zero. In the bottom line, we have consolidated terms.

The above shows at least two things. First, the total energy of approximately $26.73\ MeV$ known to be released during solar fusion is expressed entirely in terms of a theoretical combination of the up, down and electron masses, with nothing else added! Consequently, this is an *entirely theoretical* calculation of the known solar fusion energy release, *expressed totally as a function of elementary fermion masses*, and it portends the ability to do the same for other types of fusion as well, as the analysis of this paper is extended to larger nuclides Z>2, N>2.

Secondly, because the results throughout this paper seem to validate modeling nucleons as resonant cavities, this tells us how to catalyze "resonant fusion" in a more practical manner, because (8.8) tells us the precise resonances that go into releasing the total $26.73\ MeV$ of energy in the above. In particular, if one wished as a technological matter to facilitate fusion by creating an artificial "sun in a box," one would be inclined to amass a large store of helium, and subject that helium store to gamma radiation *at or near the specified discrete energies that appear in (8.8)*, so as to facilitate resonant cavity vibrations at or near the energies required for fusion to occur. Specifically, one would bath the helium with a combination of gamma radiation at the following energies / frequencies, some without, and some with, the Gaussian $(2\pi)^{\frac{3}{2}}$ divisor (we convert to wavelengths via $1F = 1/(197MeV)$):

$$
\begin{aligned}
&6m_d = 29.44MeV = 6.69F \\
&m_u = 2.22MeV = 88.56F \\
&2m_u(\text{harmonic}) = 4.45MeV = 44.28F \\
&4m_u(\text{harmonic}) = 8.90MeV = 22.14F \\
&\sqrt{m_u m_d} = 3.30MeV = 59.62F \\
&2\sqrt{m_u m_d}(\text{harmonic}) = 6.61MeV = 29.81F \\
&4\sqrt{m_u m_d}(\text{harmonic}) = 13.22MeV = 14.91F \\
&10m_d/(2\pi)^{\frac{3}{2}} = 3.12MeV = 63.23F \\
&10m_u/(2\pi)^{\frac{3}{2}} = 1.41MeV = 139.47F \\
&2\sqrt{m_u m_d}/(2\pi)^{\frac{3}{2}} = 0.42MeV = 469.53F \\
&4\sqrt{m_u m_d}/(2\pi)^{\frac{3}{2}}(\text{harmonic}) = .84MeV = 234.77F \\
&12\sqrt{m_u m_d}/(2\pi)^{\frac{3}{2}}(\text{harmonic}) = 2.52MeV = 78.26F \\
&16\sqrt{m_u m_d}/(2\pi)^{\frac{3}{2}}(\text{harmonic}) = 3.36MeV = 58.69F
\end{aligned}
\qquad . \qquad (8.9)\ (8.10)
$$

In the above, we have explicitly shown each basic frequency / energy which appears in the middle and bottom lines of (8.8) as well as harmonics that play a role in those equations. Also, one ought not to neglect the electron mass and its wavelength.

So, what do we learn? If the nucleons are treated as resonant cavities and the energies at which they fuse depend on the masses of their constituent quarks as is made very evident by (8.8), and given the particular energies and harmonics shown above which appear to play roles in solar fusion, the idea for harmonic fusion is to subject a helium store to high-frequency gamma radiation proximate at least one of the frequencies (8.10), with the view that these harmonic oscillations will catalyze fusion by perhaps reducing the amount of heat that is required. In present-day approaches, fusion reactions are triggered using heat generated from a fission reaction, and one goal would be to reduce or eliminate this need for such high heat and especially the need for any fissile trigger. That is, we at least wish to posit the possibility that providing the proper harmonics in (8.9) and (8.10) to a helium store can catalyze fusion better than known methods are able to do, with less heat and ideally little or no fission trigger required.

Of course, these energies in (8.9) and (8.10) are very high, and aside from the need to produce this radiation via known methods such as, but not limited to, Compton backscattering and any other methods which are known at present or may become known in the future for producing gamma radiation, it would also be necessary to provide substantial shielding against the health effects of such radiation. The highest energy component, $6m_d = 29.44MeV = 6.69F$, is extremely high and would be very difficult to shield (and to produce), but this resonance arises from (8.8) which is for the final ${}_2^3He + {}_2^3He \rightarrow {}_2^4He + {}_1^1H + {}_1^1H + \text{Energy}$ portion of the solar fusion cycle. If one were to forego this portion of the fusion cycle and focus only on fusing protons into deuterons according to ${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \nu + \text{Energy}$ in (8.1), then the only resonance needed is $2\sqrt{m_u m_d}/(2\pi)^{\frac{3}{2}} = 0.42MeV = 469.53F$. Not only is this easiest to produce because its energy is the lowest of all the harmonics in (8.9) and (8.10), but it is the easiest to shield and the least harmful to humans.

Certainly, a safe, reliable and effective method and associated hardware for producing energy via the fusion of protons into deuterons via the reaction (8.1), and perhaps further fusing protons and deuterons into helions as in (8.4), by introducing at least one of the harmonics in (8.9) and / or (8.10) into a Helium store perhaps in combination with other known methods, while insufficient to create the "artificial sun" modeled above if one foregoes the final alpha production in (8.7), would nonetheless represent a welcome, practical addition to the sources of energy available for all forms of peaceful human endeavor.

9. Recalibration of Masses and Binding Energies via an Exact Relationship for the Neutron – Proton Mass Difference

At the end of section 4, we briefly commented on experimental errors, and as between the alpha particle and the deuteron, we determined that is was more sensible to associate the binding energy of the deuteron *precisely* with the mass of the up quark, thus making the theoretically-predicted alpha binding energy a close but not exact approximation to its empirically observed value, rather than vice versa vis-à-vis the deuteron. But the prediction in (6.16) for the neutron – proton mass difference to just over 7 parts in ten million is a very different matter. This is even more precise by half an order of magnitude than the alpha mass prediction, and given the fundamental and pervasive nature of the relationship for $M(n) - M(p)$ anywhere and everywhere that beta-decay takes place, we now argue why (6.16) *should* be taken as an *exact* relationship

with all other relationships recalibrated accordingly, so that now the up quark mass will still be very close to the deuteron binding energy, but will no longer be *exactly* equal to this energy.

First of all, as just noted, the $M(n)-M(p)$ mass difference is the most precisely predicted relationship of all the relationships developed above, to under one part in one million AMU. Second, we have seen that all the other nuclear binding energies we have predicted are close approximations, but not exact, and would expect that this inexactitude will grow larger as we consider larger nuclides. So, rhetorically speaking, what should make the deuteron "special," as opposed to any other nuclide, that it gets to have an "exact" relation to some combination of elementary fermion masses while all the other nuclides do not? Yes, the deuteron should come *closest* to the theoretical prediction (namely the up mass) of all the nuclides, because it is the smallest composite nuclide. Closer than all other nuclides, *but still not exact*. After all, even the deuteron should suffer from the effects of "large *A=Z+N*," even if only to the very slightest degree of parts per ten million.

Third, if this is so, then we gain a new footing to be able to consider how the larger nuclides differ from the theoretical ideal, because even for this simplest *A*=2 deuteron nuclide, we will already have a precisely-known deviation which we may perhaps be able to extrapolate to larger nuclides for which this deviation certainly becomes enhanced. Fourth, in a basic sense, the deuteron, which is one proton fused to one neutron, has a mass which is a measure of "neutron *plus* proton," while $M(n)-M(p)$ is a measure of "neutron *minus* proton." So we are really faced with a choice between who gets to be exact and who must be only approximate: n+p, or n-p. Seen in this light, $M(n)-M(p)$ measures an energy feature of neutrons and protons in their native, unbound states, as separate and distinct entities, and thus is a function of these elemental nucleons in their purest form. In the deuteron, by contrast, we have a two-body system which is less-pure, so if we are to choose between one or the other, we should choose $M(n)-M(p)$ to be an *exact* relationship, with the chips then falling where they will for all other relationships, including the deuteron binding energy. Now, the deuteron is relegated to the same "approximate" status as all other compound poly-nuclides, and only the proton and neutron as distinct mono-nuclides get to enjoy an "exact" status.

Let us therefore do exactly that. Specifically, for the reasons given above, we now abandon our original hypothesis that the up quark mass is *exactly* equal to the deuteron binding energy, and in its place we substitute the hypothesis that (6.16) is an *exact* relationship, period. That is, we now define, by hypothesis, that the *exact* relationship which drives all the others, is:

$$\left[M(n)-M(p)\right]_{\text{Observed}} = \mathbf{0.001388449188\,u} \equiv m_u - m_e - 2\frac{\sqrt{m_\mu m_d}}{(2\pi)^{\frac{3}{2}}} = \left[M(n)-M(p)\right]_{\text{Predicted}}. \quad (9.1)$$

Then, we modify all the other relationships accordingly.

The simplest way make this adjustment is to modify the original hypothesis (3.1) to read:

$$m_u \equiv {}^2_1B_0 + \varepsilon = B_0({}^2_1H) + \varepsilon = 0.002388170100\,u + \varepsilon\,, \quad (9.2)$$

and to then substitute this into (9.1) with ε taken as the unknown. This is most easily solvable numerically, and it turns out that $\varepsilon = -0.000000830773\ u$, which is just over 8 parts in ten million u. That is, substituting $\varepsilon = -0.000000830773\ u$ into (9.2), then using (1.4) to derive the down quark mass, then substituting all of that into (9.1), will make (9.1) exact through all twelve decimal places (noting that the experimental errors are in the last two places).

As a consequence, the following critical energies developed earlier, become nominally adjusted starting at the sixth decimal place in AMU, as such (contrast (3.1), (3.3), (3.4), (3.5) and (3.6) respectively):

$$m_u = 0.0023873393\ 27\ u\ , \tag{9.3}$$

$$m_d = 0.0052673125\ 26\ u\ , \tag{9.4}$$

$$\sqrt{m_u m_d} = 0.0035461052\ 36\ u\ , \tag{9.5}$$

$$B_{\mathrm{P}} = 2m_u + m_d - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right)/(2\pi)^{\frac{3}{2}} = 0.008200606481\,u \tag{9.6}$$

$$B_N = 2m_d + m_u - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right)/(2\pi)^{\frac{3}{2}} = 0.010531999771\,u\ . \tag{9.7}$$

Additionally, this will slightly alter the binding energies that were predicted earlier. The new results are as follows (contrast (4.1), (5.1) and (6.18) respectively):

$$B_0(^4He) = {}^4_2B_{0\,\mathrm{Predicted}} = 0.0303730020\ 32\ u\ , \tag{9.8}$$

$$B_0(^3He)_{\mathrm{Predicted}} = 0.008320783890\,u\ . \tag{9.9}$$

$$B_0(^3H)_{\mathrm{Predicted}} = 0.009099047078\,u\ . \tag{9.10}$$

and, via (9.3) and this adjustment of masses,

$$B_0(^2H)_{\mathrm{Predicted}} = {}^2_1B_{0\,\mathrm{Predicted}} = m_u = 0.0023873393\,27\ u\ , \tag{9.11}$$

In (9.11), we continue to regard the predicted deuteron binding energy $B_0(^2H)_{\mathrm{Predicted}}$ as being equal to the mass of the up quark, but because the mass of the up quark has now changed slightly, the observed energy (which is $B_0(^2H) = .002388170100\ u$) will no longer be *exactly* equal to the predicted energy, but rather, we will now have $B_0(^2H) \neq B_0(^2H)_{\mathrm{Predicted}}$, with a difference of less than one part in a million AMU. The precise, theoretical exactitude now belongs to the $M(n) - M(p)$ difference specified in (9.1). As a bonus, the up and down masses now have a ten-digit precision in AMU, with experimental errors in the 11th and 12th digits.

One other point is worth noting. With an entirely theoretical expression now developed for the neutron–proton mass difference via (9.1), we start to assault the full, dressed proton and neutron masses themselves. Specifically, it would be extremely desirable to be able to specify the proton and neutron masses solely and exclusively as a function of the elementary up, down, and electron fermion masses. Fundamentally, by elementary algebraic principles, taking each of the proton and neutron masses as an unknown, we can deduce these masses if we have can find *two* independent equations, one of which contains an exact expression related to the *sum* of these masses, and the other which contains an exact expression related to the *difference* of these masses. Equation (9.1) achieves the first half of this objective: for the first time, we now have a theoretical expression for the *difference* between these masses. But we still lack an independent expression related to their sum.

Every effort should now be undertaken to find another relationship related to the sum of these masses. In all likelihood, that relationship, which must inherently explain the natural number just shy of 1840, between the masses of the nucleons and the electron, and / or similar ratios of about 420 and 190 involving the up and down masses, will need to emerge from an examination of Lagrangian terms we have neglected thus far, and / or the perturbations which as explained in section 11 of [1], have been set to zero throughout the course of this development. While analyzing binding energies and excess mass and nuclear reactions as we have done here is a very valuable exercise, the inherent limitation is that all of these analyses involve *differences*. What is needed to obtain the "second" of the desired two independent equations, are sums, not differences.

10. Summary and Conclusion

Summarizing the results developed here, we now have the following theoretical predictions for the binding energies shown in Table 2:

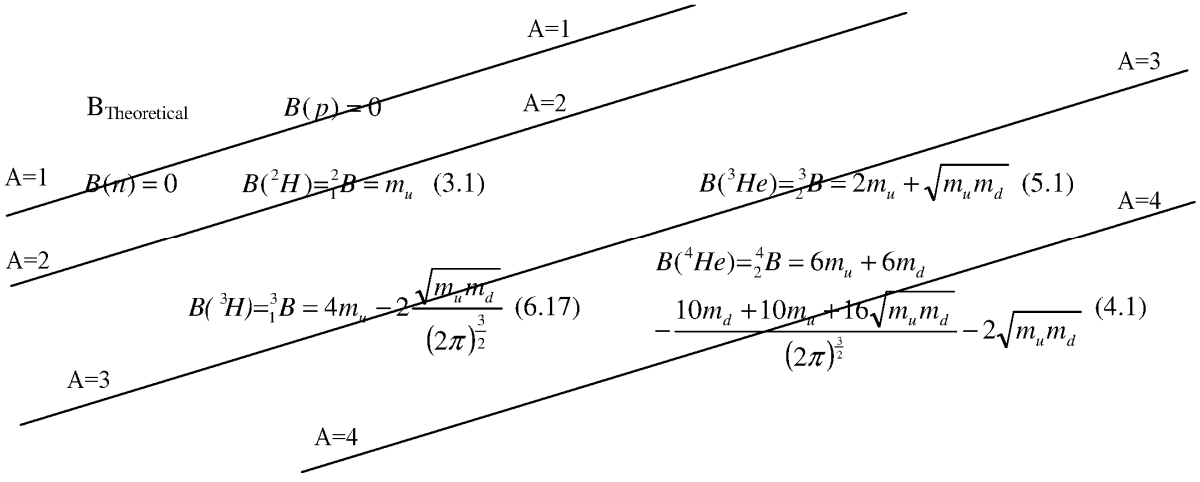

Table 8: Binding Energies (${}_Z^A B_0$) of 1s Nuclides (Theoretical, AMU)

Above, we have also referenced the equations in which these predictions are derived. The mass losses (excess masses) discussed in section 7 which were very helpful to the exercise of examining the solar fusion cycle in section 8, are simply the negative (positive) of the above.

Having just considered the $M(n)-M(p)$ mass difference, it is useful to also look at the difference between the ^{3}H and ^{3}He isobars, A=3 in the above. Given that ^{3}He is the stable nuclide and that ^{3}H undergoes β^- decay into ^{3}H, we may calculate the difference in binding energies to be:

$$B(^3He)-B(^3H)=-2m_u+\left(1+\frac{2}{(2\pi)^{\frac{3}{2}}}\right)\sqrt{m_u m_d}=-0.000778263189\,u\,. \tag{10.1}$$

Similar calculations may be carried out as between the isotopes and isotones in Table 8, and it is helpful to contrast the above to (the negative of) (9.1) which represents the most elementary β^- decay of a neutron into a proton.

The numerical values of these theoretical binding energies in Table 8, in AMU, from the updated (9.8) through (9.11), are predicted to be as follows:

$B_{predicted}$ N	$_Z$Nuclide	$_0$n	$_1$H	$_2$He
				A=1
0			0.000000000000	A=2
1		0.000000000000	0.002387339327	0.008320783890
2	A=1	A=2	0.009099047078	0.030373002032
3		A=3	A=4	

Table 9: Binding Energies ($^{A}_{Z}B_0$) of 1s Nuclides (Predicted, AMU)

These theoretical predictions should be carefully compared to the empirical values in Table 2. Indeed, subtracting each entry in Table 2 from each entry in Table 9, we find:

$B_{predicted-observed}$ N	$_Z$Nuclide	$_0$n	$_1$H	$_2$He
				A=1
0			0.000000000000	A=2
1		0.000000000000	-0.000000830773	0.000035181066
2	A=1	A=2	-0.000006538334	-0.000003584467
3		A=3	A=4	

Table 10: Predicted Minus Observed Binding Energies ($^{A}_{Z}B_0$) of 1s Nuclides (AMU)

This shows us how much each *predicted* binding energy (mass excess) differs from the *observed empirical* energies, in AMU.

As has been reviewed, every one of these predictions is accurate to under four parts in 100,000 AMU (^{3}He has this largest difference). Specifically: we have now used the thesis that Baryons are Yang-Mills magnetic monopoles to predict the binding energies of the alpha ^{4}He nucleus to under *four parts in one million*, of the ^{3}He helion nucleus to under *four parts in 100,000*, and of the ^{3}H triton nucleus to under *seven parts in one million*. And of special import, we have exactly related the neutron – proton mass difference – which pervades all aspects of

nuclear physics and beta decay – to the up quark, down quark, and electron masses, which in turn enables us to predict the binding energy for the ^{2}H deuteron nucleus most precisely of all, to just over *8 parts in ten million*.

The thesis that Baryons are Yang-Mills magnetic monopoles now appears to have ample, indeed irrefutable empirical confirmation, establishes a basis for finally "decoding" the abundance of known data regarding nuclear masses and binding energies, and may lay the foundation for technologically realizing the theoretical promise of nuclear fusion.

References

[1] Yablon, J. R., *Why Baryons Are Yang-Mills Magnetic Monopoles*, Hadronic Journal, Volume 35, Number 4, 401-468 (2012)
[2] t'Hooft, G., *Magnetic Monopoles in Unified Gauge Theories*, Nuclear Physics B79, 276-284, (1974)
[3] J. Beringer et al. (Particle Data Group), PR D86, 010001 (2012) (URL: http://pdg.lbl.gov)
[4] http://physics.nist.gov/cuu/Constants/index.html
[5] http://en.wikipedia.org/wiki/Table_of_nuclides_(complete)
[6] http://library.thinkquest.org/3471/fusion.html