Conditions for the Absence and Presence of Matter in General Relativity (GR) - Einstein Field Equations (EFEs)

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Abstract

In this paper the various “presence of matter” and “absence of matter” conditions, in gravitational field regions, of the vacuum of space-time, in the General Relativity (GR) Einstein Field Equations (EFEs) is discussed. The Static & Dynamic Vacuum Solutions of the (EFEs) and the various conditions are discussed using simplified ***Classical Mathematics and Mechanics***.

Predicted in the various “absence of matter” or “presence of matter” conditions of the (EFEs), “empty space” is shown to be filled with “Electromagnetic Heat Radiation Energy” or “Dark Expansion Space Energy” in gravitational field regions, of the vacuum of space-time, where there is no mass present in that location. Furthermore, it will be validated in the mathematics, of General Relativity (GR), and the Einstein Field Equations (EFEs), that the solution of the Black Hole Event Horizon, describes the maximum curvature, and the smallest, infinitesimal, most minimum space-time distance, of a gravitational field.

Keywords: General Relativity, Special Relativity, Einstein Field Equation, Gravitational Field, Heat Radiation Gravitation, Dark Energy Gravitation, Black Hole Event Horizon, Principle of Equivalence, Mass-Energy Equivalence, Vacuum Solution

1. Introduction – General Relativity – Einstein Field Equations

1.1. Mathematical Conditions for the Absence and Presence of Matter in Regions of Space-time – Part #1

This paper presents the various conditions in defense of the validity of the General Relativity Einstein Field Equations (EFEs), in the form of counter arguments based on the incorrect suppositions, described in a paper written by Steven Crothers, *"Proof of the Invalidity of the Black Hole and Einstein's Field Equations".*

In Steven Crothers's paper the "Proof of the Invalidity of the Black Hole and Einstein's Field Equations", he provides four (4) arguments that can be refuted, however, in this write-up, I will only address argument #1 - **The Invalidity of the Black Hole**.

In the first paragraph Crothers [1] writes:

> "Recall that according to Einstein his Principle of Equivalence and his laws of Special Relativity must hold in sufficiently small finite regions of his gravitational field and that these regions can be located anywhere in his gravitational field [1]."

I argue that this statement and sentence from a conceptual description of the "Einstein – Principle of Equivalence" is very vague. The sentence above is not wrong, so I will agree with this statement.

However, I will add for clarity that this should mean that the "Principle of Equivalence" is related to General Relativity (GR) which describes motion in frames of reference that are accelerated and contain gravitational fields.

The "Principle of Equivalence" further declares that there is no distinguishable difference between a mass body undergoing constant "Inertial" linear acceleration, free from gravity fields, and a mass body free falling, with a uniform "Gravitational" acceleration in a gravitational field.

Special Relativity (SR) in general, and for "Inertial Frames of Reference" declares that in any region of the vacuum of space-time, electromagnetic heat radiation (photons), travels at the constant speed of light (c_{Light}), and is isotropic in frames of reference that are non-accelerated and free from gravitational field.

Likewise, Special Relativity (SR) also predicts motion for "Non-Inertial Accelerated Frames of Reference" and these frames are considered equivalent to "Gravitational Field (GR) Frames of Reference"; Hence the "Einstein – Principle of Equivalence" .

Furthermore, Special Relativity (SR) in general, and for "Non-Inertial Accelerated Gravitational Field (GR) Frames of Reference" declares that in any region of the vacuum of space-time, electromagnetic heat radiation (photons), travels at the constant speed of light (c_{Light}), and is isotropic in frames of reference that are accelerated and homogeneous gravitational fields; Hence the "Einstein – Principle of Equivalence".

This "accelerated motion" equivalence principle holds true in sufficiently small finite regions of a uniformly accelerated gravitational field; and that these regions can be located anywhere in the uniformly accelerated gravitational field.

The **"Mass-Energy Equivalence"** in **Special Relativity (SR)**, refers to the "Inertial Mass" of a system body; and is the concept that the mass of a body is a measure of its energy content, ($E_{Energy} = m_{Mass} \cdot c_{Light}^2$). And likewise, makes the claims that, whenever any type of energy (E_{Energy}) is removed from a system, the mass (m_{Mass}) associated with the energy (E_{Energy}) is also removed, and the system therefore loses mass (m_{Mass}).

The **"Mass-Energy Equivalence"** in **Special Relativity (SR)**, which describes an "Inertial Mass" (m_{Net}) system body, with the "Rest Energy" content, ($E_{Rest\text{-}Energy} = m_{Mass} \cdot c_{Light}^2$), means that if a system body gives off the energy in the form of electromagnetic heat radiation mass-energy, it's "Inertial Mass" (m_{Net}) also diminishes.

$$\mathbf{m_{Net}} = \frac{\mathbf{E_{Rest\text{-}Energy}}}{\mathbf{c_{Light}^2}} = \mathbf{Inertial\ Mass}$$

The "**Principle of Equivalence**" in **General Relativity (GR)**, which describes a "Gravitational Mass" (m_{Net}) system body, means that if a system body gives off the "gravitational field" energy in the form of electromagnetic heat radiation mass-energy, it's "Gravitational Mass" (m_{Net}) and "gravitational field" also diminishes.

$$\mathbf{m_{Net}} = \frac{\mathbf{g_{Gravity} \cdot r^2}}{\mathbf{G}} = \mathbf{Gravitational\ Mass}$$

Thus, the **"Mass-Energy Equivalence"** of **Special Relativity (SR)**, combined with the "Equivalence Principle" of General Relativity (GR), makes the claims that the "gravitational" mass and the "inertial" mass is identical, or the same, for every object in nature.

$$\mathbf{m_{Net}} = \mathbf{Gravitational\ Mass} = \mathbf{Inertial\ Mass}$$

Therefore, the **"Mass-Energy Equivalence"** of **Special Relativity (SR)**, combined with the **"Equivalence Principle"** of **General Relativity (GR)**, results in the prediction that all forms of energy contribute to the gravitational field, created by an object with mass or energy.

In this concept, mass is a property of all energy, and energy is a property of all mass, and the two properties of "Mass" (m_{Mass}) and "Energy" (E_{Energy}), are connected, by the product of the "Gravitational Field Acceleration" ($g_{Gravity}$), and the geometric "Area" ($A_{Area} = 4\pi \cdot r^2$) of space; equal to the square of the distance (r^2), measured relative to the center of mass of the system.

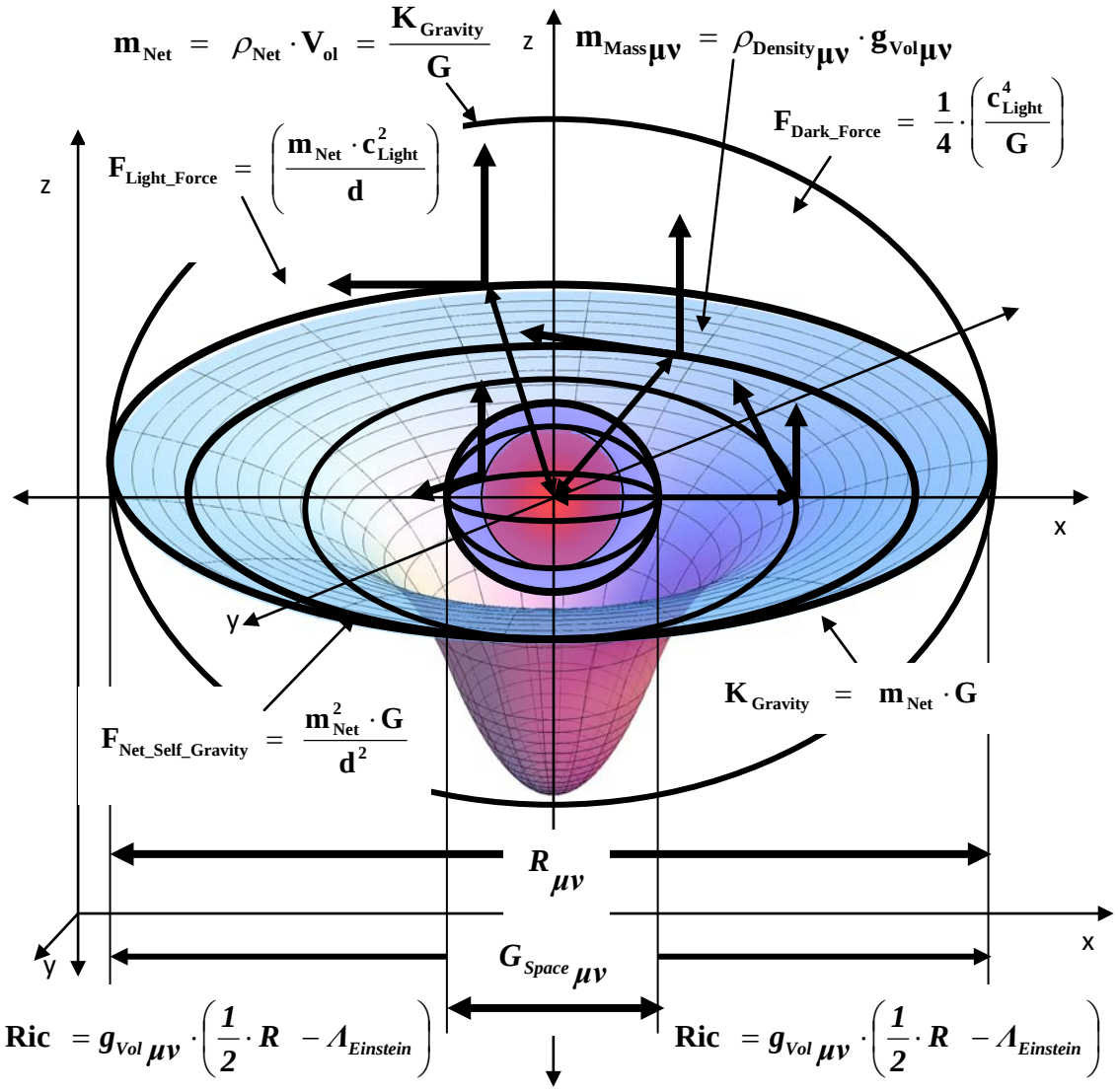
$$\mathbf{m_{Net}} = \frac{\mathbf{E_{Rest\text{-}Energy}}}{\mathbf{c^2_{Light}}} = \frac{\mathbf{g_{Gravity}} \cdot \mathbf{r^2}}{\mathbf{G}}$$

Furthermore, **Mass-Energy Equivalence"** of **Special Relativity (SR)**, combined with the **"Equivalence Principle"** of **General Relativity (GR)**, results in the prediction that the "gravitational field" energy, propagating in the form of "electromagnetic heat radiation" energy, is also a form of mass-energy.

$$\mathbf{m_{Net}} \propto \mathbf{E_{Rest\text{-}Energy}} \propto \mathbf{g_{Gravity}} \cdot \mathbf{r^2}$$

The proof of this is the observational fact, due to the gravitational attraction, of "electromagnetic heat radiation" in the form of "light" bending, as it passes by the sun. This bending of light, where there is large mass/matter and gravitational field energy, was observed; and confirmed that the energy carried by "light" is indeed, equivalent to a gravitational mass.

There is also the observational fact that the energy of photons increases when they fall in the gravitational field of the earth. The energy, and therefore the gravitational mass, of photons are proportional to their frequency as given by the Einstein/Planck energy relation.



1.2. Mathematical Conditions for the Absence and Presence of Matter in Regions of Space-time – Part #2

In the first paragraph Crothers [1] further writes:

> "Recall also that both The Principle of Equivalence and Special Relativity are defined in terms of the a priori presence of multiple arbitrarily large finite masses and photons [1]."

This is also a somewhat vague statement. However, I will also agree with this statement. And I will add for clarity, that you can claim priori presence of electromagnetic heat radiation (photons) due to mathematics of Special Relativity (SR) and the isotropy and constancy of the speed of light (c_{Light}), of the propagating photons, in a uniformly accelerated gravitational field.

And you can claim priori presence of large finite masses due to mathematics of General Relativity (GR), and the fact that large mass create gravitational fields, where there is no distinguishable difference between a mass body in constant "Inertial" linear acceleration free from gravity field, and a mass body free falling with a uniform "Gravitational" acceleration in a gravitational field.

The **Principle of Equivalence** makes equal the Special Relativity (SR) linear "Inertial" acceleration ($a_{Acceleration}$), equal to the General Relativity (GR) "Gravitational" acceleration ($g_{Gravity} = \frac{m_{Net} \cdot G}{r^2}$), of a gravitational field. Therefore, the Principle of Equivalence makes the following claims; ($a_{Acceleration} = g_{Gravity}$).

The **Principle of Equivalence** also makes the claims that the isotropy and the constancy of the speed of light (c_{Light}), remains constant and isotropic, for electromagnetic "heat radiation" propagating photons, in both "Inertial Frames" and "Gravitational Field" frames of reference.

And, likewise the **Principle of Equivalence** makes the following proportionally claims about the total quantity of matter/mass (m_{Net}) of a gravitational field system body, which is directly proportional to the total amount of "uniform acceleration" ($a_{Acceleration} = g_{Gravity} = \frac{m_{Net} \cdot G}{r^2}$), in a localized frame of reference. The frame of reference is "Inertial" for the considerations of Special Relativity (SR), and "Gravitational" for the considerations of General Relativity (GR). The essence of the "equivalence principle" is that it makes the claims, that wherever there is mass or matter there is a gravitational field.

$$a_{Acceleration} = g_{Gravity} \propto m_{Net}$$

1.3. Mathematical Conditions for the Absence and Presence of Matter in Regions of Space-time – Part #3

In the first paragraph of section #1, Crothers [1] further writes:

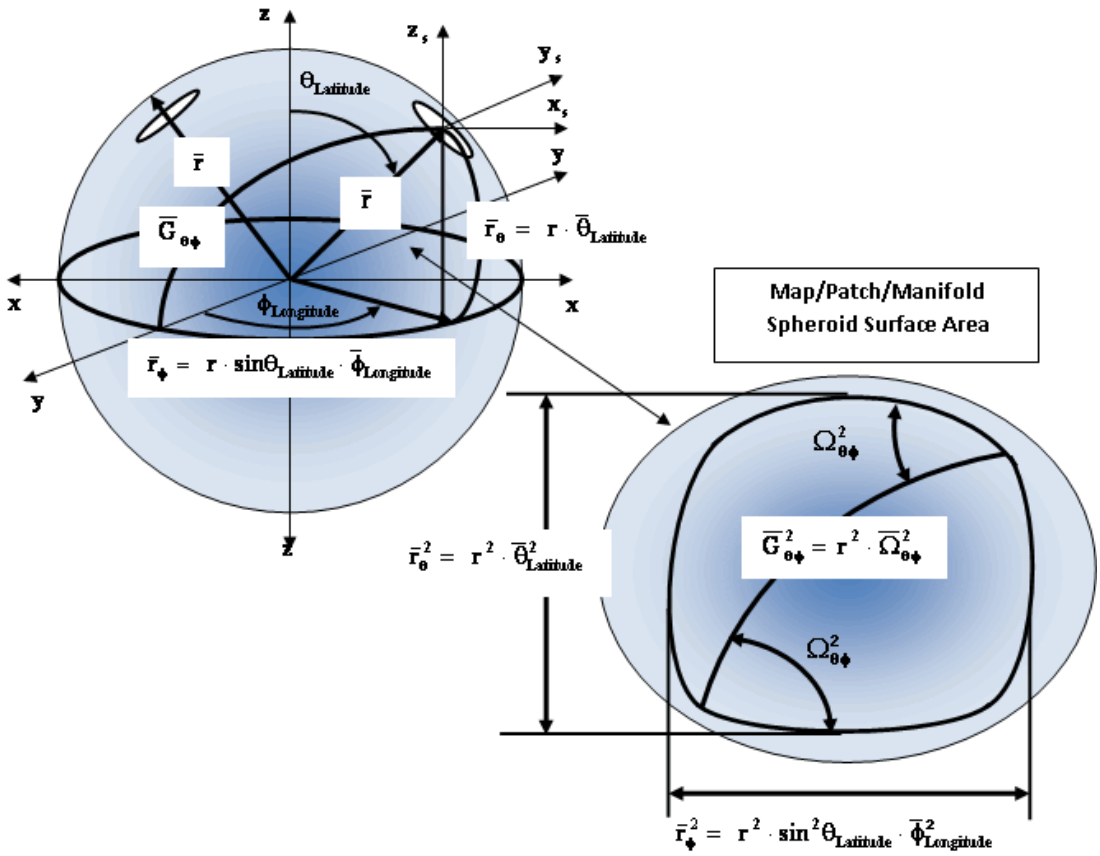
> "Now Einstein maintains that his field equations (in covariant components) ($R_{\mu\nu} = 0$) are "*The Field Equations of Gravitation in the Absence of Matter*" [2]; they pertain to the so-called 'static vacuum field'.
>
> "*Einstein made the assumption that in empty space (*$R_{\mu\nu} = 0$*)*" [3].
>
> "*The Einstein equations in the absence of matter are (*$R_{\mu\nu} = 0$*)*" [4].
>
> In contravariant components, "*…….the empty space field equations are (*$R_{\mu\nu} = 0$*)*" [5]."

The arguments made by Crothers about the Einstein Field Equation (EFE) for Gravitation above are the following:

1. Considering the Einstein Field Equation (EFE) for Gravitation – Static Vacuum Field solution only
2. The EFE "*makes the assumption that* ***in empty space*** *(*$R_{\mu\nu} = 0$*)*"
3. The EFE "*makes the assumption that* ***in the absence of matter*** *(*$R_{\mu\nu} = 0$*)*"

Now let's check if the above is true. Also, I will use the space curvature indices ($\theta\phi$) instead of abstract tensor indices ($\mu\nu$); and where ($R_{\mu\nu} = R_{\theta\phi}$).

The Map/Patch/Manifold Curvature ($\Omega_{Map\ \theta\phi} = \sqrt{\theta^2_{Latitude} + \phi^2_{Longitude} \cdot \sin^2(\theta_{Latitude})}$) describes the curvature of space, matter, energy, and any region of gravitational field, and is inherent in the descriptive mathematics of the Einstein Field Equation (EFE). Therefore using the space curvature indices ($\theta\phi$), instead of the abstract tensor indices ($\mu\nu$), gives real physical meaning to the EFEs.



Differential Angle Metrics that exist for a Euclidean Sphere

The equation below is the Einstein Field Equation (EFE) for Gravitation – **Dynamic Vacuum Field solution** – Einstein Tensor

In the **Dynamic Vacuum Field** solution, the Einstein Vacuum Density Cosmological Constant term ($\Lambda_{\text{Einstein}} \neq 0$) is ***not equal to zero***.

1.1

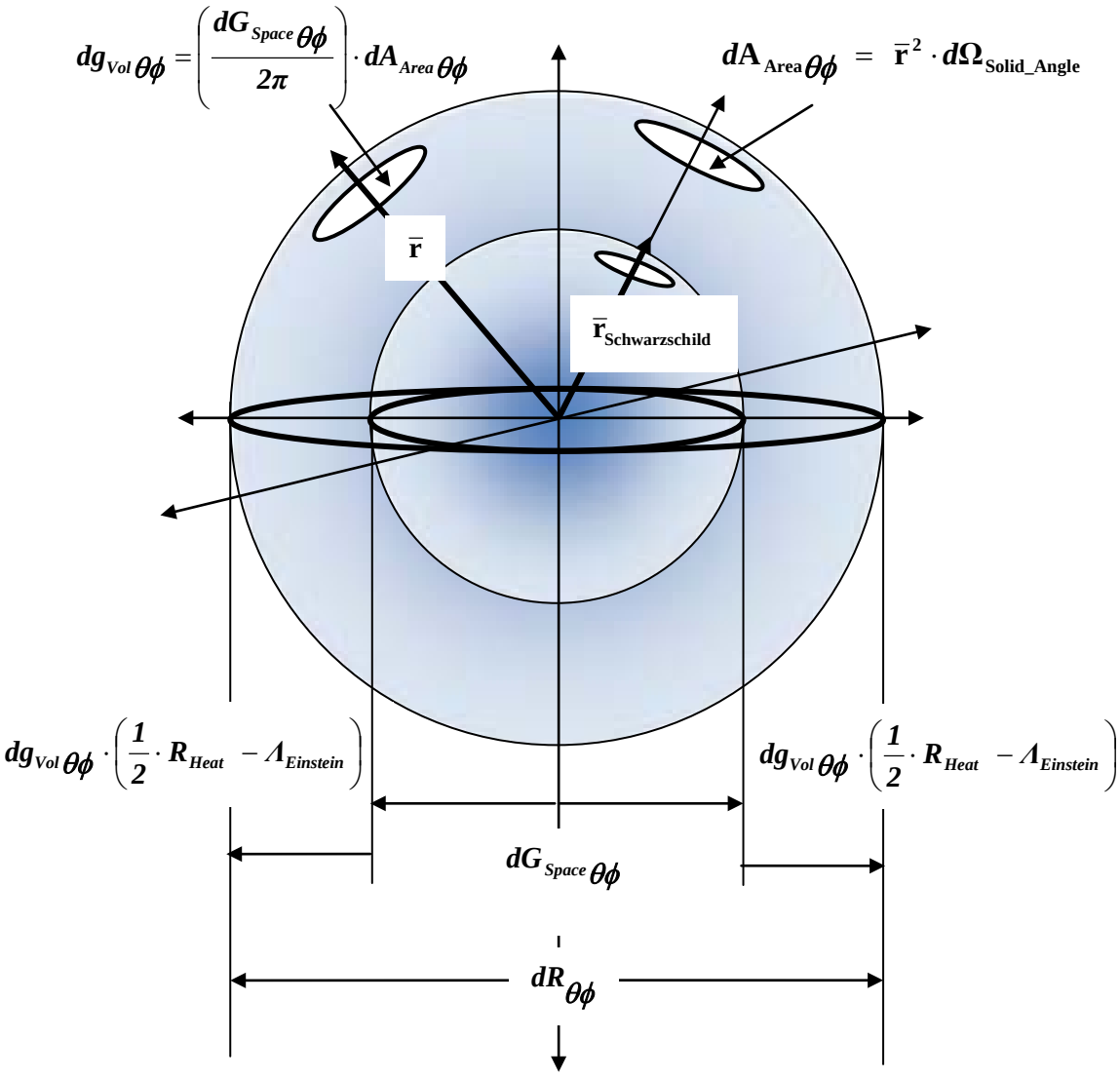
$$G_{Space\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c^4_{\text{Light}}}{G} \right)} \right) = \left[R_{\theta\phi} - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}} \right) \right] \rightarrow m$$

The equation below is the Einstein Field Equation (EFE) for Gravitation – **Static Vacuum Field solution** – Einstein Tensor

In the **Static Vacuum Field** solution, the Einstein Vacuum Density Cosmological Constant term ($\Lambda_{\text{Einstein}} = 0$) is ***equal to zero***.

1.2

$$G_{Space\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = \left[R_{\theta\phi} - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \right] \rightarrow m$$



Below the terms in the above Einstein Field Equation (EFE), are loosely defined.

Source of Gravitation – Einstein Tensor "Geodesic" Arc Length, Matter/Mass-Energy, strong curvature ***"inside internal body"*** field term

$$G_{Space\,\theta\phi} \quad \rightarrow m$$

Extended Field of Gravitation – Riemann Tensor "Geodesic" Arc Length (***Vacuum "Empty" Space***), weak curvature ***"outside external body"*** field term

$$R_{\theta\phi} \quad \rightarrow m$$

Four-Dimensional Space-time "Metric Tensor" – Volume Element term

$$g_{Vol\,\theta\phi} \quad \rightarrow m^3$$

Electromagnetic Heat Radiation Gravitation Inverse Square Distance Expansion/Contraction Density

$$\frac{1}{2} \cdot R_{Heat} \quad \rightarrow 1/m^2$$

Vacuum Energy Gravitation Inverse Square Distance Density - Einstein Cosmological Constant Term

$$\Lambda_{\text{Einstein}} \quad \rightarrow 1/m^2$$

Static Vacuum Solution – condition $(\Lambda_{\text{Einstein}} = 0)$ – **Ricci Tensor** – Heat Radiation Gravitation Expansion/Contraction "Geodesic" Arc Length curvature term

$$\mathbf{Ric} \quad = \quad g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \quad \rightarrow m$$

Dynamic Vacuum Solution – condition $(\Lambda_{\text{Einstein}} \neq 0)$ – **Ricci Tensor** – "Dark Energy" Gravitation Expansion/Contraction "Geodesic" Arc Length curvature term

$$\mathbf{Ric} \quad = \quad g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \quad - \quad \Lambda_{\text{Einstein}} \right) \quad \rightarrow m$$

Source of Gravitation – Four Dimensional Space-time **Stress Energy-Momentum** of Space-time

$$T_{Energy\,\theta\phi} \quad \rightarrow kg \cdot m^2/s^2$$

The “presence of mass/matter” kinetic energy term of the EFE is the **Stress Energy-Momentum** ($T_{Energy\,\theta\phi}$) of the vacuum of space-time, and it predicts that mass or matter in the form of energy is the ***“inside internal body”*** source of the gravitational field, which curves space-time ($G_{Space\,\theta\phi}$), strongly in the presence of the mass ($\mathrm{m}_{\mathrm{Net}}$).

1.3

$$T_{Energy\,\theta\phi} = \frac{1}{2}\cdot\left(\mathrm{m}_{\mathrm{Net}}\cdot\mathrm{c}_{\mathrm{Light}}^{2}\right)\cdot\left(\frac{\Omega_{Map\,\theta\phi}^{2}}{2\cdot\left(ln\left(\frac{\mathrm{c}_{\mathrm{Light}}^{2}\cdot\mathrm{r}}{2\cdot\mathrm{m}_{\mathrm{Net}}\cdot\mathrm{G}}\right)\right)^{2}+\Omega_{Map\,\theta\phi}^{2}}\right)^{2}\cdot\left(\frac{\Omega_{Map\,\theta\phi}}{2\pi}\right)\rightarrow kg\cdot m^{2}/s^{2}$$

Now, let’s set the weak curvature ***“outside external body”*** gravitational field, Riemann Tensor term ($R_{\theta\phi}=0$) equal to zero, in the above Einstein Field Equation (EFE) for Gravitation – **Static Vacuum Field solution**. In this condition the source of the gravitational field ($G_{Space\,\theta\phi}\neq 0$), where mass/matter is still present ($T_{Energy\,\theta\phi}\neq 0$) is not equal to zero.

1.4

$$G_{Space\,\theta\phi} = 2\pi\cdot\left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4}\cdot\left(\frac{\mathrm{c}_{\mathrm{Light}}^{4}}{\mathrm{G}}\right)}\right) = \left[R_{\theta\phi} - g_{Vol\,\theta\phi}\cdot\left(\frac{1}{2}\cdot R_{Heat}\right)\right]\rightarrow m$$

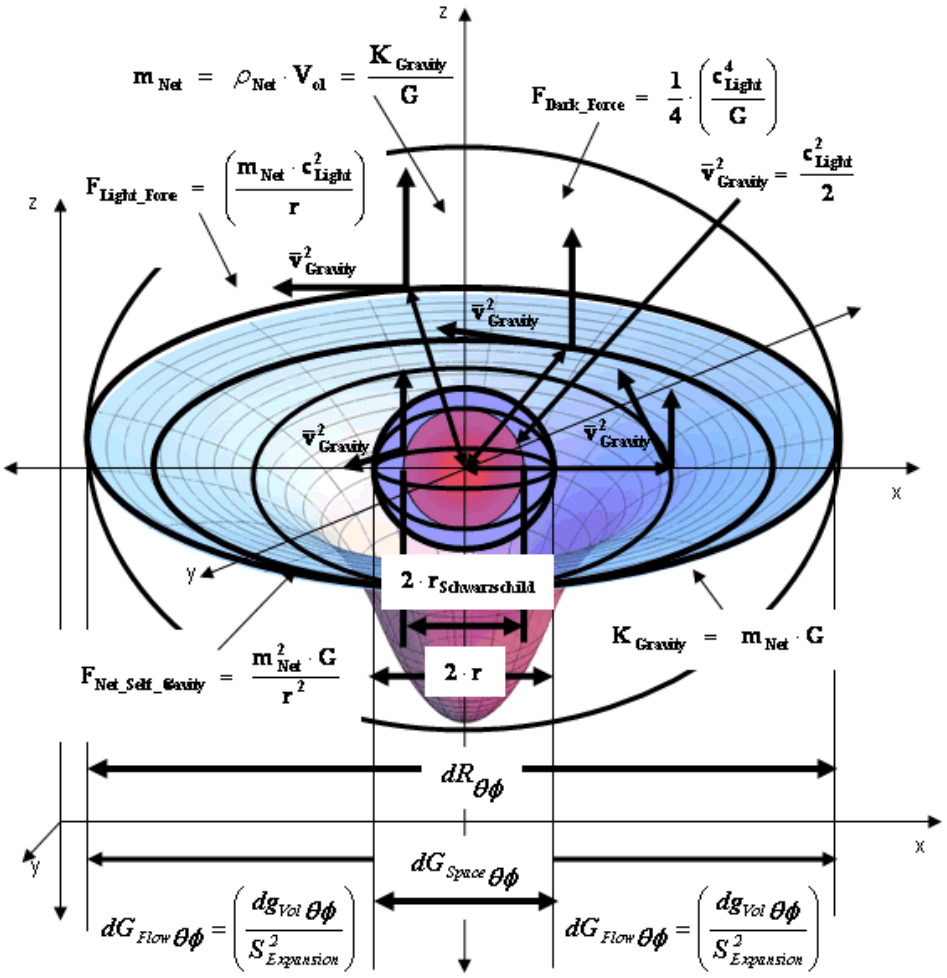
Letting, ($R_{\theta\phi}=0\rightarrow m$)

Yields the source of the curvature of the gravitational field ($G_{Space\,\theta\phi}\neq 0$)

1.5

$$G_{Space\,\theta\phi} = -\mathbf{Ric} = 2\pi\cdot\left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4}\cdot\left(\frac{\mathrm{c}_{\mathrm{Light}}^{4}}{\mathrm{G}}\right)}\right) = -\,g_{Vol\,\theta\phi}\cdot\left(\frac{1}{2}\cdot R_{Heat}\right)\rightarrow m$$

Now, what does setting Riemann Tensor ($R_{\theta\phi} = 0$), weak gravitational field curvature ***"outside external body"***, gravitational field term equal to zero, in the above Einstein Field Equation (EFE) for Gravitation mean?



The Riemann Tensor ($R_{\theta\phi}$) Extended Field of Gravitation "Geodesic" Arc Length, describes the weak gravitational field curvature, which is an ***"outside external body"*** gravitational field term, and is identical to describing a region of empty space in a gravitational field.

The Riemann Tensor ($R_{\theta\phi}$) describes curvature in a region of the vacuum of space-time, where mass is either present or absent; and where electromagnetic heat radiation, and the vacuum energy, are distributed and extended, in curved space and time, when there is no matter present.

The Source of Gravitation - Einstein Tensor ($G_{Space\,\theta\phi}$) Source Field of Gravitation "Geodesic" Arc Length, describes the strong gravitational field curvature, which is an ***"inside internal body"*** gravitational field term, and is identical to describing a region of space in a gravitational field that contains mass/matter and energy.

The maximum curvature most minimum distance of the source of the gravitational field ($G_{Space\,\theta\phi}$), and the most condensed infinitesimal space in given by the Black Hole, Schwarzschild Semi-Radius ($r_{Schwarzschild} = 2 \cdot \left(\frac{m_{Net} \cdot G}{c_{Light}^2} \right)$), which is also proportional to mass/matter (m_{Net}) in a localized region of space-time.

The Stress Energy-Momentum ($T_{Energy\,\theta\phi}$) source of the gravitational field is directly proportional to mass or matter (m_{Net}), and likewise is directly proportional to the source of curvature and gravitational ($G_{Space\,\theta\phi}$) space-time.

$$G_{Space\,\theta\phi} \quad \propto \quad \Omega_{Map\,\theta\phi} \quad \propto \quad m_{Net} \quad \propto \quad T_{Energy\,\theta\phi}$$

In essence, the Source of Gravitation Einstein Tensor ($G_{Space\,\theta\phi}$) describes curvature and the presence of mass/matter (m_{Net}) and Stress Energy ($T_{Energy\,\theta\phi}$) of a given gravitational field, in a region of the vacuum of space-time.

The Riemann Tensor ($R_{\theta\phi} = 0$), weak gravitational field curvature term means that the extended gravitational field and the curvature of space-time, that surrounds a net sum of matter/mass or energy, which extends from the Black Hole Event Horizon, source of gravity into infinity spaces of vacuum space, **is ignored or zero**.

However, the source of the gravitational field ($G_{Space\,\theta\phi} \neq 0$) is not equal to zero, and still has matter presence, and maximum curvature in that location, that **is not ignored and not zero**.

Therefore the above equation ($R_{\theta\phi} = 0$), means that the extended gravitational field which extends into infinity is removed, while there remains the presence of mass/matter (m_{Net}) and Stress Energy-Momentum ($T_{Energy\,\theta\phi} \neq 0$), which is the source of the curvature and of the gravitation field ($G_{Space\,\theta\phi} \neq 0$).

Therefore, according to the condition where the Riemann Tensor extended gravitational field "weak" curvature ($R_{\theta\phi} = 0$), is ***equal to zero***; and the Einstein Tensor ($G_{Space\,\theta\phi} \neq 0$), source of matter gravitational field "strong" curvature is ***not equal to zero***. This condition describes the curving of space-time, without extending the gravitational field into the vacuum of space, beyond the source of gravitation.

Starting with the **Static Vacuum Solution** ($\Lambda_{\text{Einstein}} = 0$), and setting the Riemann Tensor ($R_{\theta\phi} = 0$) equal to zero; in this condition, the condense matter that is being expanded and contracted by the **Ricci Tensor** - Heat Radiation Gravitation ($\mathbf{Ric}$), term, is balanced or in equilibrium with source of gravitation, and the curvature of space-time, ($G_{Space\,\theta\phi} = -\mathbf{Ric} \neq 0$), due to the presence of mass, in a localized region of the vacuum of space-time.

Ricci Tensor - Heat Radiation Gravitation ($\mathbf{Ric}$)

1.6

$$G_{Space\,\theta\phi} = -\mathbf{Ric} = -g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \rightarrow m$$

The above **Ricci Tensor** - Heat Radiation Gravitation ($\mathbf{Ric}$), equation means that the ***mass/matter energy*** in a localized region of the gravitational field, in the vacuum of space-time, is balanced by an equal and opposite amount of ***electromagnetic heat radiation energy***; ($G_{Space\,\theta\phi} = -\mathbf{Ric} \neq 0$), which exist in the exact same localized region of the gravitational field in the vacuum of space-time.

Starting with the **Dynamic Vacuum Solution** ($\Lambda_{\text{Einstein}} \neq 0$), and setting the Riemann Tensor ($R_{\theta\phi} = 0$) equal to zero; in this condition, the condense matter that is being expanded and contracted by the **Ricci Tensor** – Dark Energy Gravitation ($\mathbf{Ric}$), term, is balanced or in equilibrium with source of gravitation, and the curvature of space-time, ($G_{Space\,\theta\phi} = -\mathbf{Ric} \neq 0$), due to the presence of mass, in a localized region of the vacuum of space-time.

Ricci Tensor – Dark Energy Gravitation ($\mathbf{Ric}$)

1.7

$$G_{Space\,\theta\phi} = -\mathbf{Ric} = -g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}} \right) \rightarrow m$$

The above **Ricci Tensor** – Dark Energy Gravitation ($\mathbf{Ric}$), equation means that the ***mass/matter energy*** in a localized region of the gravitational field, in the vacuum of space-time, is balanced by an equal and opposite amount of ***Dark Expansion Energy***; which exist in the exact same localized region of the gravitational field in the vacuum of space-time. ($G_{Space\,\theta\phi} = -\mathbf{Ric} \neq 0$).

Based on the above description setting ($R_{\theta\phi} = 0$), describes **empty space of an extended gravitational field as being zero or ignored**.

However, there remains the presence of matter or mass of a gravitational field localized in a region of the vacuum of space-time, because ($G_{Space\,\theta\phi} \neq 0$).

Furthermore setting the Riemann Tensor ($R_{\theta\phi} = 0$),***"outside external body"*** extended gravitational field, equal to zero, does indeed describe the ***absence of matter and energy*** in an extended gravitational field, and away from the spherical surface of the Black Hole Event Horizon, Schwarzschild Semi-Major Radius ($r \geq r_{Schwarzschild} = 2 \cdot \left(\frac{m_{Net} \cdot G}{c^2_{Light}} \right)$).

However, there is the ***presence of matter and energy*** at the source of the gravitational field ($G_{Space\,\theta\phi} \neq 0$) still exists.

What this means is that Einstein and his statements concerning the mathematics of General Relativity, and his descriptions about the presence of matter and empty space, and the absence of matter and empty space, are indeed true!

1.4. Mathematical Conditions for the Absence and Presence of Matter in Regions of Space-time – Part #4

In the first paragraph Crothers [1] further writes:

> "*The law ($G_{Space\,\mu\nu} = 0$) in empty space, is chosen by Einstein for his law of gravitation*" [6].
>
> Note that in the case of [6], Eddington writes the Ricci tensor ($R_{\mu\nu} = 0$) as ($G_{Space\,\mu\nu} = 0$), but the meaning is precisely the same."

The arguments made by Crothers about the Einstein Field Equation (EFE) for Gravitation above are summarized in the following:

1. Considering the Einstein Field Equation (EFE) for Gravitation – Static Vacuum Field solution only
2. The EFE "*makes the assumption that **in empty space** ($G_{Space\,\mu\nu} = 0$)*"
3. The EFE "*makes the assumption that **in empty space** and **in the absence of matter** ($R_{\mu\nu} = G_{Space\,\mu\nu} = 0$)*"; have precisely the same meaning.

From the above explanation that I made earlier, it is determined that the Riemann Tensor "Geodesic" Arc Length gravitational field term ($R_{\theta\phi}$) is not the exact same meaning as the Source of Gravitation - Matter & Energy "Geodesic" Arc Length term ($G_{Space\,\theta\phi}$).

Condition #1 - Einstein Field Equation (EFE) – ($R_{\theta\phi} = G_{Space\,\theta\phi}$)

But let's investigate what happens when the Riemann Tensor "Geodesic" Arc Length gravitational field term ($R_{\theta\phi} = G_{Space\,\theta\phi}$) is equal to the Source of Gravitation - Matter & Energy "Geodesic" Arc Length ($G_{Space\,\theta\phi} = R_{\theta\phi}$) term.

In this ($R_{\theta\phi} = G_{Space}\theta\phi$) case the **Static Vacuum Solution** ($\Lambda_{\text{Einstein}} = 0$) – and the ***Ricci Tensor*** – "Heat Radiation" Gravitation Expansion/Contraction "Geodesic" Arc Length term ($\mathbf{Ric} = g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat}\right) = 0$) is equal to zero.

1.8

$$G_{Space}\theta\phi = 2\pi \cdot \left(\frac{T_{Energy}\theta\phi}{\frac{1}{4} \cdot \left(\frac{c_{\text{Light}}^4}{G} \right)} \right) = \left[R_{\theta\phi} - g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \right] \rightarrow m$$

Letting, $R_{\theta\phi} = G_{Space}\theta\phi \rightarrow m$

$$G_{Space}\theta\phi = 2\pi \cdot \left(\frac{T_{Energy}\theta\phi}{\frac{1}{4} \cdot \left(\frac{c_{\text{Light}}^4}{G} \right)} \right) = R_{\theta\phi} \rightarrow m$$

The **Static Vacuum Solution** in this condition ($\left(\frac{1}{2} \cdot R_{Heat}\right) = 0$) & ($\Lambda_{\text{Einstein}} = 0$)

Static Vacuum Solution – ***Ricci Tensor*** – Heat Radiation Gravitation Expansion/Contraction "Geodesic" Arc Length term is equal to zero; when the condition exists; ($R_{\theta\phi} = G_{Space}\theta\phi$).

1.9

$$\mathbf{Ric} = g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat}\right) = 0 \rightarrow m$$

The **Dynamic Vacuum Solution** in this condition, ($\Lambda_{\text{Einstein}} = \frac{1}{2} \cdot R_{Heat} \neq 0$)

Dynamic Vacuum Solution – ***Ricci Tensor*** – "Dark Energy" Gravitation Expansion/Contraction "Geodesic" Arc Length term is equal to zero; when the condition exists; ($R_{\theta\phi} = G_{Space}\theta\phi$).

1.10

$$\mathbf{Ric} = g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}}\right) = 0 \rightarrow m$$

The condition ($R_{\theta\phi} = G_{Space}\theta\phi \neq 0$), means that there is the **presence of mass-energy or matter** present, at the source of a gravitational field environment, where there is maximum curvature, and with no extended gravitational field considered beyond the source and curvature of the gravitational field in localized region of the vacuum of space-time.

The Static Vacuum condition ($\frac{1}{2} \cdot R_{Heat} = \Lambda_{\text{Einstein}} = 0$), means that there is an amount of Electromagnetic Heat Radiation Gravitation field, in the **absence of mass**, and in a localized region of the vacuum of space-time is equal to zero.

The Dynamic Vacuum condition ($\frac{1}{2} \cdot R_{Heat} = \Lambda_{\text{Einstein}} \neq 0$), means that there is an equal amount of Electromagnetic Heat Radiation Gravitation equal to the Vacuum Energy, in the **absence of mass** in a localized region of the vacuum of space-time.

The Dynamic Vacuum condition ($\frac{1}{2} \cdot R_{Heat} = \Lambda_{\text{Einstein}} \neq 0$), along with the condition ($R_{\theta\phi} = G_{Space}\theta\phi \neq 0$), also means that there is a separate and independent **heat radiation gravitational field** "empty space" environment, in the **absence of mass** or a matter presence environment. In that region of space-time, there is curvature, due to the Electromagnetic Heat Radiation Gravitation Energy, equal to the Vacuum Energy, in localized region of the vacuum of space-time.

Condition #2 - Einstein Field Equation (EFE) – ($R_{\theta\phi} = G_{Space}\theta\phi = 0$ **)**

Now let's investigate what happens to the EFE when the Riemann Tensor "Geodesic" Arc Length gravitational field term ($R_{\theta\phi} = G_{Space}\theta\phi = 0$) is equal to the Source of Gravitation - Matter & Energy Arc Length term ($G_{Space}\theta\phi = R_{\theta\phi} = 0$), and is equal to zero.

In this ***absence of matter*** condition, the Einstein Tensor, source of matter, curvature, and gravitation ($G_{Space}\theta\phi = 0$) is equal to zero, and the Stress Energy-Momentum ($T_{Energy}\theta\phi = 0$) is also equal to zero.

In this EFE conditional case ($R_{\theta\phi} = G_{Space\,\theta\phi} = 0$), the **Dynamic Vacuum Solution** ($\Lambda_{\text{Einstein}} \neq 0$), and the ***Ricci Tensor*** – "Dark Energy" Gravitation Expansion/Contraction "Geodesic" Arc Length term is also equal to zero. ($\mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}}\right) = 0$).

1.11

$$G_{Space\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{\text{Light}}^4}{G}\right)}\right) = \left[R_{\theta\phi} - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}}\right)\right] \rightarrow m$$

Letting, ($R_{\theta\phi} = G_{Space\,\theta\phi} = 0 \quad \rightarrow m$)

The **Static Vacuum Solution** in this condition ($\left(\frac{1}{2} \cdot R_{Heat}\right) = 0$) & ($\Lambda_{\text{Einstein}} = 0$)

Static Vacuum Solution – ***Ricci Tensor*** – Heat Radiation Gravitation Expansion/Contraction "Geodesic" Arc Length term is equal to zero; when the EFE condition exists; ($R_{\theta\phi} = G_{Space\,\theta\phi} = 0$).

1.12

$$\mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat}\right) = 0 \quad \rightarrow m$$

The above EFE condition ($R_{\theta\phi} = G_{Space\,\theta\phi} = 0$), in the **Static Vacuum Solution** ($\Lambda_{\text{Einstein}} = \frac{1}{2} \cdot R_{Heat} = 0$) means that there is the **absence of matter** or no mass-energy or matter present, and also there is the **absence of heat radiation energy** in a local environment. Where the Ricci Tensor ($\mathbf{Ric} = 0$) is equal to zero. And this condition is ***invalid; and makes no physical sense***.

This above **Static Vacuum Solution** ($\Lambda_{\text{Einstein}} = \frac{1}{2} \cdot R_{Heat} = 0$), condition ***cannot exist*** where the condition ($R_{\theta\phi} = G_{Space\,\theta\phi} = 0$), exist concurrently!

The above **Static Vacuum Solution** condition ($\Lambda_{\text{Einstein}} = \frac{1}{2} \cdot R_{Heat} = 0$) along with the EFE condition ($R_{\theta\phi} = G_{Space}\theta\phi = 0$), compels the ***Ricci Tensor*** – Heat Radiation Gravitation Expansion/Contraction "Geodesic" Arc Length term to equal to zero ($\mathbf{Ric} = g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat}\right) = 0$); ***and makes no physical sense***.

However, above **Dynamic Vacuum Solution** ($\Lambda_{\text{Einstein}} = \frac{1}{2} \cdot R_{Heat} \neq 0$), condition ***can exist***, where the condition ($R_{\theta\phi} = G_{Space}\theta\phi = 0$), exist concurrently!

The **Dynamic Vacuum Solution** condition, ($\Lambda_{\text{Einstein}} = \frac{1}{2} \cdot R_{Heat} \neq 0$)

Dynamic Vacuum Solution – Ricci Tensor – "Dark Energy" Gravitation Expansion/Contraction "Geodesic" Arc Length term is equal to zero; when the condition exists; ($R_{\theta\phi} = G_{Space}\theta\phi = 0$).

1.13

$$\mathbf{Ric} = g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}}\right) = 0 \quad \rightarrow m$$

The above EFE condition ($R_{\theta\phi} = G_{Space}\theta\phi = 0$), along with the **Dynamic Vacuum Solution** condition, ($\Lambda_{\text{Einstein}} = \frac{1}{2} \cdot R_{Heat} \neq 0$), means that there is the **absence of matter** or no mass or matter energy present, but only electromagnetic heat radiation gravitation energy ($\frac{1}{2} \cdot R_{Heat}$) equal to the vacuum energy ($\Lambda_{\text{Einstein}}$), is present in a localized region of a gravitational field, in the vacuum of space-time.

1.14

$$\frac{1}{2} \cdot R_{Heat} = \Lambda_{\text{Einstein}} \quad \rightarrow 1/m^2$$

This above condition and equation describes a universe, which is void or absence of matter. Such that a universe in the above case is filled with Electromagnetic Heat Radiation Gravitation equal to Vacuum Energy Gravitation.

Condition #3 - Einstein Field Equation (EFE) – ($G_{Space}\theta\phi = 0$)

Next, consider what happens when there is an **absence of matter** condition; the Source of Gravitation - Matter & Energy "Geodesic" Einstein Tensor ($G_{Space}\theta\phi = 0$) is equal to zero; and the Riemann Tensor ($R_{\theta\phi} \neq 0$).

1.15

$$G_{Space}\theta\phi = 2\pi \cdot \left(\frac{T_{Energy}\theta\phi}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = \left[R_{\theta\phi} - g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \right] \rightarrow m$$

When there is the absence of matter in the vacuum of space-time, the source of gravitation geodesic arc length ($G_{Space}\theta\phi = 0$), is equal to zero. Also, the Stress Energy-Momentum ($T_{Energy}\theta\phi = 0$), and the Matter/Mass (m_{Net}) at the source is equal to zero; and the Riemann Tensor ($R_{\theta\phi} \neq 0$).

In this **Static Vacuum Solution** ($\Lambda_{Einstein} = 0$), and where the source of gravitation geodesic arc length ($G_{Space}\theta\phi = 0$), is equal to zero, the ***Ricci Tensor*** ($R_{\theta\phi} = \mathbf{Ric} = g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat} \right)$) Heat Radiation Gravitation term, is equal to the Riemann Tensor "Geodesic" Arc Length ($R_{\theta\phi} \neq 0$), and weak curvature, in an extended gravitational field.

1.16

$$R_{\theta\phi} = \mathbf{Ric} = g_{Vol}\theta\phi \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \rightarrow m$$

In the above **absence of matter** condition and equation, means that there is an absence of mass/matter in a region of space-time, where the Matter & Energy "Geodesic" Arc Length, Einstein Tensor term ($G_{Space}\theta\phi = 0$) is equal to zero.

Likewise in the above **absence of matter** condition the non-zero Riemann Tensor ($R_{\theta\phi} \neq 0$) term is equal to Ricci Tensor ($R_{\theta\phi} = \mathbf{Ric} \neq 0$), heat radiation gravitation energy in that region of space-time, which is weak curvature, in an extended gravitational field.

When there is the **absence of matter** in the vacuum of space-time, the source of gravitation geodesic arc length ($G_{Space\,\theta\phi} = 0$), is equal to zero. Also, the Stress Energy-Momentum ($T_{Energy\,\theta\phi} = 0$), and the Matter/Mass ($\mathrm{m}_{\mathrm{Net}}$) at the source is equal to zero; and the Riemann Tensor ($R_{\theta\phi} \neq 0$).

1.17

$$G_{Space\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{\mathrm{c}_{\mathrm{Light}}^4}{\mathrm{G}} \right)} \right) = \left[R_{\theta\phi} - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\mathrm{Einstein}} \right) \right] \rightarrow m$$

In this **Dynamic Vacuum Solution** ($\Lambda_{\mathrm{Einstein}} \neq 0$), and where the source of gravitation geodesic arc length ($G_{Space\,\theta\phi} = 0$), is equal to zero, the ***Ricci Tensor*** ($R_{\theta\phi} = \mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\mathrm{Einstein}} \right)$) Dark Energy Gravitation term, is equal to the Riemann Tensor "Geodesic" Arc Length ($R_{\theta\phi} \neq 0$), and weak curvature, in an extended gravitational field.

1.18

$$R_{\theta\phi} = \mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\mathrm{Einstein}} \right) \rightarrow m$$

In the above **absence of matter** condition and equation, means that there is an absence of mass/matter in a region of space-time, where the Matter & Energy "Geodesic" Arc Length, Einstein Tensor term ($G_{Space\,\theta\phi} = 0$) is equal to zero.

Likewise in the above **absence of matter** condition the non-zero Riemann Tensor ($R_{\theta\phi} \neq 0$) term is equal to Ricci Tensor ($R_{\theta\phi} = \mathbf{Ric} \neq 0$), Dark Energy Expansion Gravitation Energy in that region of space-time.

In these above conditions the Principle of Equivalence, Special Relativity, General Relativity, and the constancy of the speed of light ($\mathrm{c}_{\mathrm{Light}}$) of photon propagation, are all valid!

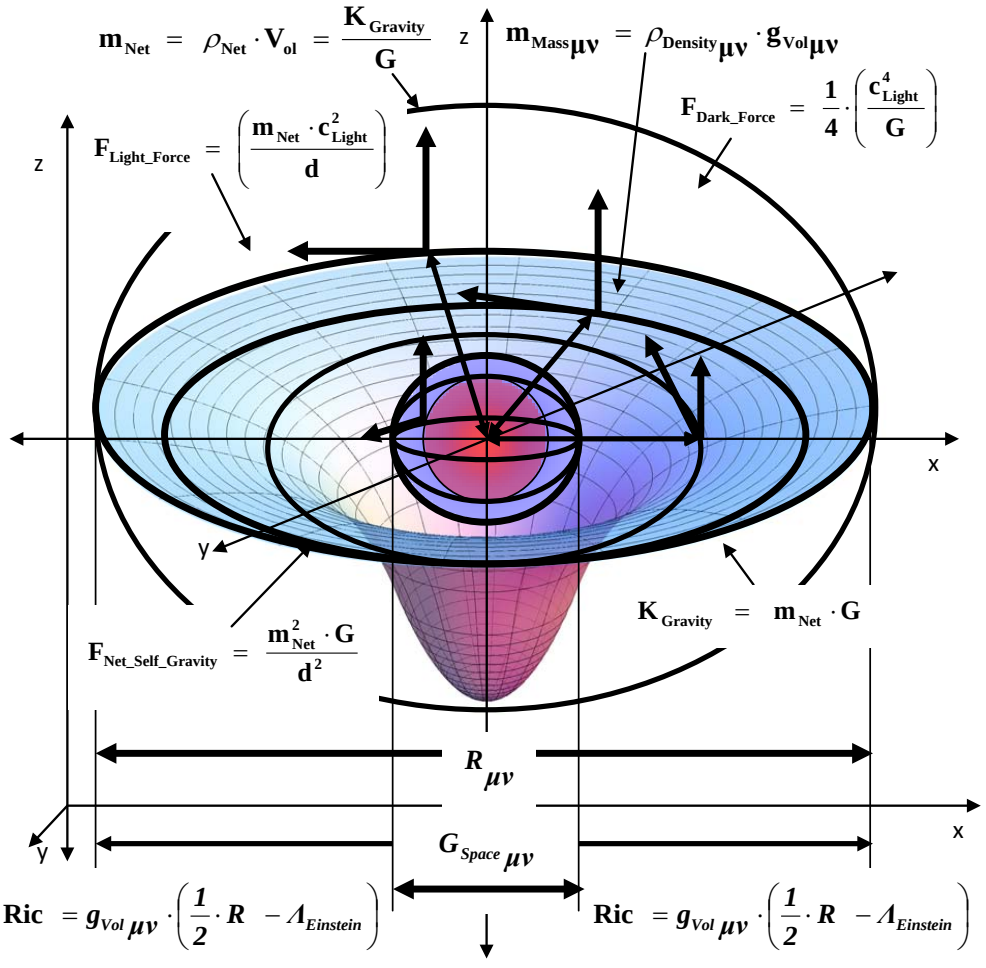
In concluding this section, there are four (4) conditions for the presence or absence of Mass/Matter in a localized region of space-time, given by the following conditions:

1. **Absence of Matter/Mass** – $(G_{Space\,\theta\phi} = 0)$ & $(R_{\theta\phi} = \mathbf{Ric} \neq 0)$

2. **Presence of Matter/Mass** – $(R_{\theta\phi} = 0)$ & $(G_{Space\,\theta\phi} = \mathbf{Ric} \neq 0)$

3. **Absence of Matter/Mass** – $(R_{\theta\phi} = G_{Space\,\theta\phi} = 0)$ & $(\mathbf{Ric} = 0)$

4. **Presence of Matter/Mass** – $(R_{\theta\phi} = G_{Space\,\theta\phi})$ & $(\mathbf{Ric} = 0)$



1.5. Mathematical Conditions for the Absence and Presence of Matter in Regions of Space-time – Part #5

In the second and final paragraphs of section #1, Crothers [1] writes:

> "It is therefore *impossible* for the Principle of Equivalence and Special Relativity to manifest in the spacetime of $R_{\mu\nu} = 0$ since $R_{\mu\nu} = 0$ contains no matter by mathematical construction. Therefore $R_{\mu\nu} = 0$ violates the physical principles required by Einstein for his gravitational field and is therefore inadmissible, having no physical meaning."

The arguments made by Crothers about the Einstein Field Equation (EFE) for Gravitation above are summarized in the following:

1. The EFE "*makes the assumption that **in the absence of matter** ($R_{\mu\nu} = 0$)*"

2. The above EFE, Riemann Tensor *($R_{\mu\nu} = 0$)* condition, ***violates*** the Principle of Equivalence, "*based on the assumption that **in empty space** and **in the absence of matter***"; is an inadmissible condition that disproves Einstein and GR; and this condition has no physical meaning.

Now let's check to see if the above is true. The **Principle of Equivalence** makes the claims that the isotropy and the constancy of the speed of light (c_{Light}), remains constant and isotropic, for electromagnetic "heat radiation" propagating photons, in both "Inertial Frames" and "Gravitational Field" frames of reference.

And, likewise the **Principle of Equivalence** makes the following proportionally claims about the total quantity of matter/mass (m_{Net}) of a gravitational field system body, which is directly proportional to the total amount of "uniform acceleration" ($a_{Acceleration} = g_{Gravity} = \frac{m_{Net} \cdot G}{r^2}$), in a localized frame of reference. The frame of reference is "Inertial" for the considerations of Special Relativity (SR), and "Gravitational" for the considerations of General Relativity (GR). The essence of the "equivalence principle" is that it makes the claims, that wherever there is mass or matter there is a gravitational field.

$$a_{Acceleration} = g_{Gravity} \propto m_{Net}$$

I argue that the statement made by Crothers that "It is therefore *impossible* for the Principle of Equivalence and Special Relativity to manifest in the spacetime", is not true.

Condition #4 - Einstein Field Equation (EFE) – ($G_{Space\,\theta\phi} = 0$) – ($R_{\theta\phi} \neq 0$)

The true EFE condition where there is the **absence of Matter/Mass**, is given by the Einstein Tensor ($G_{Space\,\theta\phi} = 0$) being equal to zero, and in that region of the vacuum of space-time, the Stress Energy-Momentum ($T_{Energy\,\theta\phi} = 0$) is also equal to zero.

In this true EFE condition ($G_{Space\,\theta\phi} = 0$), where there is the **absence of Matter/Mass**, the Riemann Tensor "Geodesic" Arc Length ($R_{\theta\phi} \neq 0$), extended gravitational field, is a non-zero term.

In this **Dynamic Vacuum Solution** ($\Lambda_{\text{Einstein}} \neq 0$), where there is the **absence of Matter/Mass** and where the source of gravitation ($G_{Space\,\theta\phi} = 0$), is equal to zero, the ***Ricci Tensor*** ($R_{\theta\phi} = \mathbf{Ric}$) Dark Energy Gravitation term, is equal to the Riemann Tensor "Geodesic" Arc Length ($R_{\theta\phi} \neq 0$), and weak curvature, in an extended gravitational field.

1.19

$$R_{\theta\phi} = \mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}} \right) \quad \rightarrow m$$

In this **Static Vacuum Solution** ($\Lambda_{\text{Einstein}} = 0$), where there is the **absence of Matter/Mass** and where the source of gravitation ($G_{Space\,\theta\phi} = 0$), is equal to zero, the ***Ricci Tensor*** ($R_{\theta\phi} = \mathbf{Ric}$) Electromagnetic Heat Radiation Gravitation term, is equal to the Riemann Tensor ($R_{\theta\phi} \neq 0$), "Geodesic" Arc Length and weak curvature, in an extended gravitational field.

1.20

$$R_{\theta\phi} = \mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \quad \rightarrow m$$

In the above **absence of matter** Einstein Tensor ($G_{Space\,\theta\phi} = 0$) is equal to zero condition, there still is gravitational field energy present in the form of non-zero Ricci Tensor ($R_{\theta\phi} = \mathbf{Ric} \neq 0$), made of pure "Electromagnetic Heat Radiation Gravitation" or "Dark Expansion Gravitation Energy" in that "empty space" region of space-time; where there is the absence matter.

Therefore in the absence of matter, there still can remain "Electromagnetic Heat Radiation Gravitation" or "Dark Expansion Gravitation Energy" in that localized "empty space" gravitational field region of the vacuum of space-time; where there is the absence matter.

Furthermore, in the above **absence of matter** condition ($G_{Space\,\theta\phi} = 0$), the non-zero Riemann Tensor ($R_{\theta\phi} \neq 0$) term is equal to Ricci "Expansion Space" Tensor ($R_{\theta\phi} = \mathbf{Ric} \neq 0$), and weak curvature, in an extended gravitational field; made of "Electromagnetic Heat Radiation Gravitation" or "Dark Expansion Gravitation Energy" in that localized "empty space" gravitational field region of the vacuum of space-time.

In this condition the Principle of Equivalence, Special Relativity, General Relativity, and the constancy of the speed of light (c_{Light}) of photon propagation, are all valid!

Condition #5 - Einstein Field Equation (EFE) – ($G_{Space\,\theta\phi} = 0$) – ($R_{\theta\phi} = 0$)

Condition, where there is the **absence of Mass**, ($R_{\theta\phi} = G_{Space\,\theta\phi} = 0$), there is a physical condition that describes "Electromagnetic Heat Radiation Gravitation" energy and "Vacuum Energy" that is localized in a gravitational field region of space-time; given by Ricci Tensor ($\mathbf{Ric} = 0$) equal to zero.

In this **Dynamic Vacuum Solution** ($\Lambda_{Einstein} \neq 0$), where there is the **absence of Matter/Mass** and where the source of gravitation ($G_{Space\,\theta\phi} = 0$), is equal to zero, and the Riemann Tensor "Geodesic" Arc Length ($R_{\theta\phi} = 0$), is equal to zero, the ***Ricci Tensor*** ($\mathbf{Ric} = 0$) the Dark Energy Expansion Gravitation term is also equal to zero.

1.21

$$\mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{Einstein} \right) = 0 \quad \rightarrow m$$

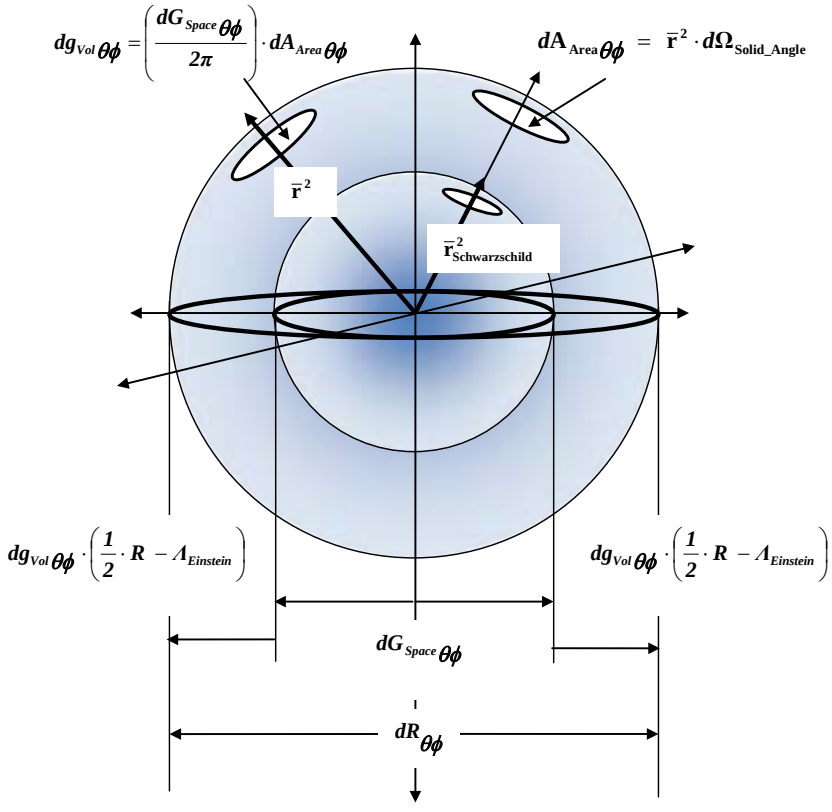
The above EFE condition ($R_{\theta\phi} = G_{Space}\theta\phi = 0$), along with the **Dynamic Vacuum Solution** condition, ($\Lambda_{\text{Einstein}} = \frac{1}{2}\cdot R_{Heat} \neq 0$), means that there is the **absence of matter** or no mass or matter energy present, but only "Electromagnetic Heat Radiation Gravitation" energy ($\frac{1}{2}\cdot R_{Heat}$) equal to the "Vacuum Energy" ($\Lambda_{\text{Einstein}}$), is present in that localized "empty space" gravitational field region of the vacuum of space-time

1.22

$$\frac{1}{2}\cdot R_{Heat} = \Lambda_{\text{Einstein}} \rightarrow {}^{1}\!/_{m^2}$$

In this condition the Principle of Equivalence, Special Relativity, General Relativity, and the constancy of the speed of light (c_{Light}) of photon propagation, are all still valid!

The Einstein Field Equations (EFEs) do have physical meaning, of mass and energy, in the absence or the presence of mass and energy, localized in a gravitational field in the vacuum of space-time; this process is called gravitation.



Therefore, there are the four (4) conditions for discussing the **presence or the absence of matter**, in any region of a gravitational field, described by the **General Relativity - Einstein Field Equations (EFEs) for Gravitation**; and in summary:

1. (**Matter & Energy *With no* Extended Gravity Field**) $\rightarrow (R_{\theta\phi} = 0)$

$$G_{Space\,\theta\phi} = \mathbf{Ric} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \rightarrow m$$

2. (**Matter & Energy *With* Extended Gravity Field**) $\rightarrow (R_{\theta\phi} = G_{Space\,\theta\phi})$

$$G_{Space\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = R_{\theta\phi} \rightarrow m$$

3. (**Absence of Matter *With no* Extended Gravity Field**)$\rightarrow (R_{\theta\phi} = G_{Space\,\theta\phi} = 0)$

*Note – The Dynamic Vacuum Solution is required for this condition, $(\Lambda_{Einstein} \neq 0)$

$$\mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{Einstein} \right) = 0 \rightarrow m$$

4. (**Absence of Matter *With* Extended Gravity Field**) $\rightarrow (G_{Space\,\theta\phi} = 0)$

$$R_{\theta\phi} = \mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{Einstein} \right) \rightarrow m$$

The Static Vacuum Solution condition $(\Lambda_{Einstein} = 0)$, will only apply to conditions (1, 2, & 4), of the EFE.

But the Dynamic Vacuum Solution $(\Lambda_{Einstein} \neq 0)$ will apply to all conditions (1, 2, 3, & 4) of the EFE. Therefore, I totally disagree with Crothers's conclusion about the interpreted results of Einstein and General Relativity.

1.6. Mathematical Conditions for the Black Hole Event Horizon in Regions of Space-time

In the final paragraphs of section #1, Crothers [1] writes:

> Therefore the solution for $R_{\mu\nu} = 0$ (the so-called 'Schwarzschild solution' actually due to David Hilbert) also violates the physical principles of General Relativity and so also has no physical meaning. But it is from Hilbert's solution to $R_{\mu\nu} = 0$ that the black hole was spawned. Indeed, we note that,
>
> "*Black holes were first discovered as purely mathematical solutions of Einstein's field equations. This solution, the Schwarzschild black hole, is a nonlinear solution of the Einstein equations of General Relativity. It contains no matter, and exists forever in an asymptotically flat space-time*." [7]
>
> Thus the black hole violates the physical principles of General Relativity and so it too has no physical meaning. In other words, General Relativity does not predict the black hole. The black hole thus fails to have any theoretical justification by means of General Relativity."

In this final paragraph of section #1 of Crothers's paper he argues that Black Holes, contain no matter, and have no physical meaning in General Relativity.

I totally disagree with this statement, because not only are Black Holes predicted by the mathematics of General Relativity, but we are currently directly observing Black Holes using various astronomical measuring experiments and techniques.

In the following description, I will show directly in the mathematics, of the General Relativity (GR) Einstein Field Equation (EFE), where the solution discovered by Karl Schwarzschild (1915) of the Black Hole Event Horizon radius. It was Schwarzschild and not Hilbert that discovered the Black Hole solution that is inherent in the EFE.

The Einstein Tensor source of matter gravitation ($G_{Space\,\theta\phi}$) "Geodesic" Arc Length, which is measured at the surface of the Black Hole Event Horizon; and this spatial distance known as the Black Hole Schwarzschild radius ($r_{Schwarzschild}$), is directly proportional to the Net Inertial Mass (m_{Net}), of a gravitational field.

The Black Hole Schwarzschild radius ($r_{Schwarzschild}$), is the description of the source of gravitation, mass, and the gravitational field, and is the smallest, infinitesimal, and most minimum distance, and maximum curvature ($G_{Space}\theta\phi$), for the gravitational field; whose strength, acceleration, and energy is proportional to the Net Inertial Mass (m_{Net}), and the Stress Energy-Momentum ($T_{Energy}\theta\phi$) of a gravitational field.

Likewise, the Einstein Tensor source of matter gravitation ($G_{Space}\theta\phi$) "Geodesic" Arc Length, at the Black Hole Event Horizon is inversely proportional to the square of the maximum orbiting tangential velocity ($\frac{1}{v^2_{Gravity}}$) of the gravitational field potential, at the, surface of the Black Hole Event Horizon.

Located at the "event horizon" or ground state potential of the Black Hole, there is the fastest or maximum value, of the square of the orbiting gravitational field, tangential velocity ($v^2_{Gravity} = \frac{c^2_{Light}}{2} = \left(\frac{m_{Net} \cdot G}{r_{Schwarzschild}}\right)$), which is equal to the square of the "Speed of Light" velocity (c^2_{Light}) divided by two (2).

For any isolated Net Inertial Mass (m_{Net}) system body, and gravitational field, the magnitude or size of the Einstein Tensor ($\overline{G}_{Space}\theta\phi$) Geodesic gets larger, the slower the orbital gravitational squared velocity ($v^2_{Gravity} = \left(\frac{m_{Net} \cdot G}{r}\right)$), and the larger the semi-major radius (r), and the further away the semi-major radius (r) is from the "Black Hole" Schwarzschild radius.

Likewise, the Einstein Tensor source of matter gravitation ($G_{Space}\theta\phi$) "Geodesic", gets larger, as the gravity field radius ($r = \left(\frac{m_{Net} \cdot G}{v^2_{Gravity}}\right)$) of the sphere gets larger; and the slower the orbital squared tangential velocity ($v^2_{Gravity}$) of the gravitational field potential; and the lower the gravitational energy of that potential.

The Source of Gravitation Geodesic Arc Length ($G_{Space\,\theta\phi}$) Einstein Tensor is a function matter/mass (m_{Net}), Stress Energy-Momentum ($T_{Energy\,\theta\phi}$), and distance/space (r), which is measured relative to the center of the system.

1.23

$$G_{Space\,\theta\phi} = r \cdot \overline{\Omega}_{Map\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = \left[R_{\theta\phi} - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \right] \rightarrow m$$

The Source of Gravitation Geodesic Arc Length ($G_{Space\,\theta\phi}$) Einstein Tensor gets larger or smaller inversely proportional to the square of the speed of the orbital gravitational velocity ($\frac{1}{v_{Gravity}^2}$).

The square of the orbiting gravitational field speed or velocity ($v_{Gravity}^2$) and the energy potentials of the gravitational field, depend on the amount of the Net Inertial Mass (m_{Net}) of the system, described by the following limits.

$$\left(\begin{matrix} \textbf{\textit{Maximum Distance Curvature}} \\ \textbf{\textit{Slow Speed}} \end{matrix} \right) \leq v_{Gravity}^2 \leq \left(\begin{matrix} \textbf{\textit{Minimum Distance Curvature (Black Hole)}} \\ \textbf{\textit{Finite Speed}} \end{matrix} \right)$$

$$(0) \leq v_{Gravity}^2 = \left(\frac{m_{Net} \cdot G}{r} \right) \leq \left(\frac{c_{Light}^2}{2} \right)$$

The Source of Gravitation Geodesic Arc Length ($G_{Space\,\theta\phi}$) Einstein Tensor gets larger or smaller in direct proportional to the radial distance (r) in a gravitational field; determined by a fixed amount of the Net Inertial Mass (m_{Net}) of the system, described by the following limits.

$$\left(\begin{matrix} \textbf{\textit{Maximum Distance Curvature}} \\ \textbf{\textit{Slow Speed}} \end{matrix} \right) \leq r \leq \left(\begin{matrix} \textbf{\textit{Minimum Distance Curvature (Black Hole)}} \\ \textbf{\textit{Finite Speed}} \end{matrix} \right)$$

$$(\infty) \geq r = \left(\frac{m_{Net} \cdot G}{v_{Gravity}^2} \right) \geq \left(r_{Schwarzschild} = 2 \cdot \left(\frac{m_{Net} \cdot G}{c_{Light}^2} \right) \right)$$

The Source of Gravitation Geodesic Arc Length ($G_{Space}\theta\phi$) function of the inverse square of the orbiting gravitational field speed or velocity ($\frac{1}{v^2_{Gravity}}$)

1.24

$$G_{Space}\theta\phi = \left(\frac{m_{Net}\cdot G}{v^2_{Gravity}}\right)\cdot\Omega_{Map\,\theta\phi} = 2\pi\cdot\left(\frac{T_{Energy}\theta\phi}{\frac{1}{4}\cdot\left(\frac{c^4_{Light}}{G}\right)}\right) = \left[R_{\theta\phi} - g_{Vol}\theta\phi\cdot\left(\frac{1}{2}\cdot R_{Heat}\right)\right]$$

Therefore the above description of the mathematics of General Relativity (GR), proves that the Source of Gravitation Geodesic Arc Length ($G_{Space}\theta\phi$) which is curvature around the source of the gravitation is directly proportional to the Net Inertial Mass or Matter (m_{Net}); proportional to the distance/ space (r); proportional to the inverse square of the orbiting gravitational field speed or velocity ($\frac{1}{v^2_{Gravity}}$); and proportional to the Map/Patch/Manifold Curvature ($\Omega_{Map\,\theta\phi}$) of space-time parameter.

$$G_{Space}\theta\phi \propto \Omega_{Map\,\theta\phi} \propto m_{Net} \propto \frac{1}{v^2_{Gravity}} \propto r$$

The Map/Patch/Manifold Curvature ($\Omega_{Map\,\theta\phi} = \sqrt{\theta^2_{Latitude} + \phi^2_{Longitude}\cdot\sin^2(\theta_{Latitude})}$)

Thus, the Black Hole Event Horizon represents maximum curvature of space-time, and the most minimum distance of the gravitational field, given by the Schwarzschild radius ($r_{Schwarzschild}$). Likewise, the Black Hole Event Horizon represents the maximum, "Square of the Orbiting Velocity" ($v^2_{Gravity} = \frac{c^2_{Light}}{2}$) of the gravity field; equal to the square of the speed of light divide by two (2).

Black Hole Schwarzschild radius ($r = r_{Schwarzschild}$) (Scalar)

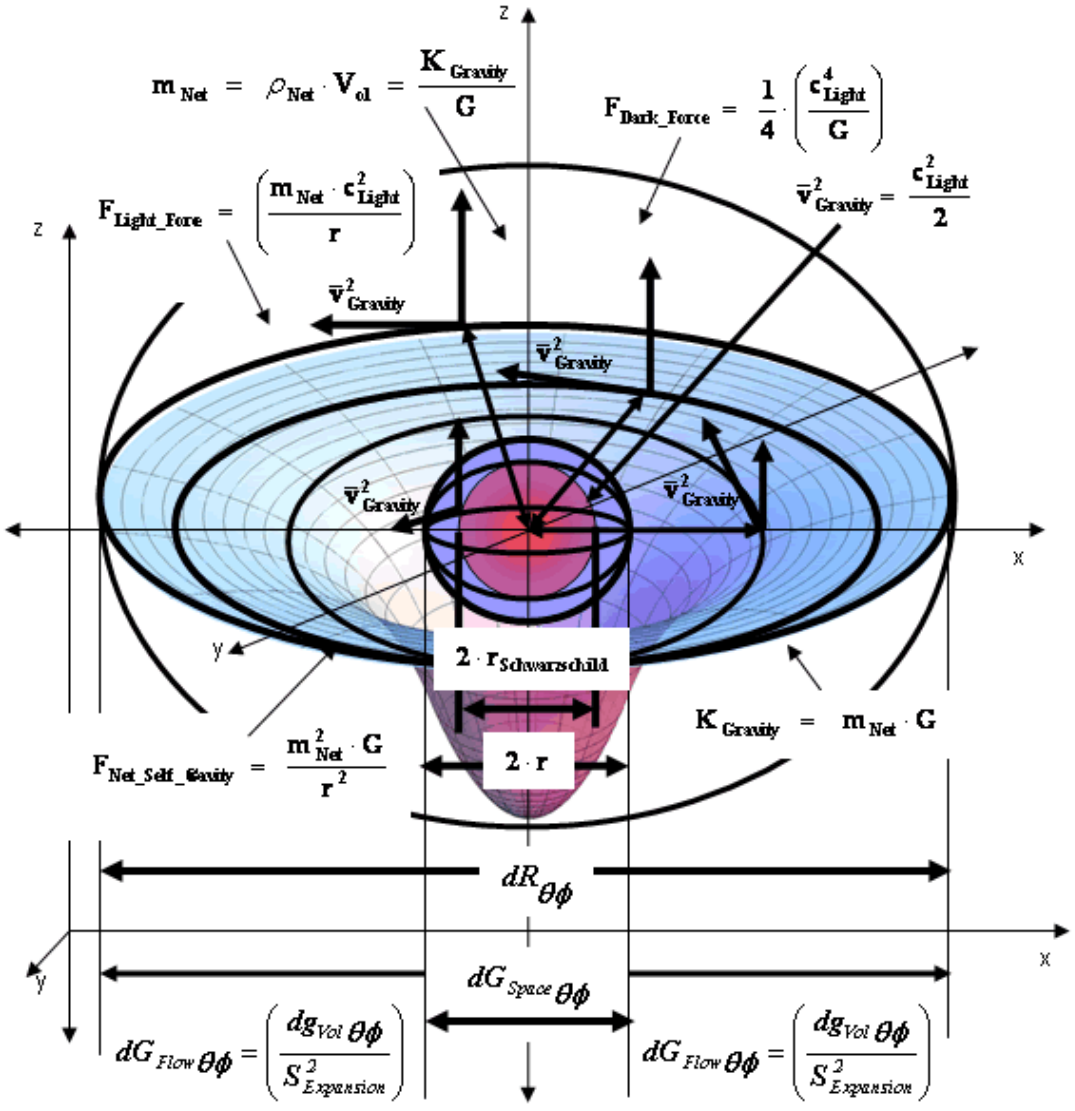
1.25

$$r = r_{Schwarzschild} = 2\cdot\left(\frac{m_{Net}\cdot G}{c^2_{Light}}\right) = \left(\frac{G_{Space}\theta\phi}{\Omega_{Map\,\theta\phi}}\right) \rightarrow m$$

Gravitational Field Potential — Maximum — Square of the Orbiting Speed or Velocity ($v^2_{Gravity} = \frac{c^2_{Light}}{2}$) of a Black Hole Gravitational Field Potential

1.26

$$v^2_{Gravity} = \frac{c^2_{Light}}{2} = \left(\frac{m_{Net} \cdot G}{r_{Schwarzschild}} \right) \rightarrow m^2/s^2$$



Therefore, in the **presence of matter** ($G_{Space\,\theta\phi} \neq 0$) condition, and when considering the ***Black Hole Event Horizon*** solution of the Einstein Field Equation (EFE) for Gravitation; there are the various conditions of the EFE in summary below:

1. (**Black Hole - Matter & Energy *With no* Extended Gravity Field Present**)

$$(R_{\theta\phi} = 0) \quad \& \quad (G_{Space\,\theta\phi} \neq 0)$$

Static Vacuum Solution – $(\Lambda_{\text{Einstein}} = 0)$

$$G_{Space\,\theta\phi} = r_{\text{Schwarzschild}} \cdot \Omega_{Map\,\theta\phi} = 2 \cdot \left(\frac{m_{\text{Net}} \cdot G}{c^2_{\text{Light}}} \right) \cdot \overline{\Omega}_{Map\,\theta\phi}$$

$$G_{Space\,\theta\phi} = \mathbf{Ric} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c^4_{\text{Light}}}{G} \right)} \right) = -\, g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \rightarrow m$$

2. (**Black Hole - Matter & Energy *With no* Extended Gravity Field Present**)

$$(R_{\theta\phi} = 0) \quad \& \quad (G_{Space\,\theta\phi} \neq 0)$$

Dynamic Vacuum Solution – $(\Lambda_{\text{Einstein}} \neq 0)$

$$G_{Space\,\theta\phi} = r_{\text{Schwarzschild}} \cdot \Omega_{Map\,\theta\phi} = 2 \cdot \left(\frac{m_{\text{Net}} \cdot G}{c^2_{\text{Light}}} \right) \cdot \Omega_{Map\,\theta\phi}$$

$$G_{Space\,\theta\phi} = \mathbf{Ric} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c^4_{\text{Light}}}{G} \right)} \right) = -\, g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{\text{Einstein}} \right) \rightarrow m$$

3. (**Black Hole - Matter & Energy *With* Extended Gravity Field Present**)

$$(R_{\theta\phi} = G_{Space\,\theta\phi}) \quad \& \quad (G_{Space\,\theta\phi} \neq 0)$$

Static or Dynamic Vacuum Solution

$$G_{Space\,\theta\phi} = r_{Schwarzschild} \cdot \Omega_{Map\,\theta\phi} = 2 \cdot \left(\frac{m_{Net} \cdot G}{c^2_{Light}} \right) \cdot \Omega_{Map\,\theta\phi}$$

$$G_{Space\,\theta\phi} = r_{Schwarzschild} \cdot \overline{\Omega}_{Map\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c^4_{Light}}{G} \right)} \right) = R_{\theta\phi} \rightarrow m$$

In the condition $(R_{\theta\phi} = G_{Space\,\theta\phi})$

The Ricci "Expansion Space" Tensor - $(\mathbf{Ric} = 0)$

The **Static Vacuum Solution** is ***"invalid"***: $\left(\frac{1}{2} \cdot R_{Heat}\right) = 0$ & $(\Lambda_{Einstein} = 0)$

The **Dynamic Vacuum Solution** in this condition, $(\Lambda_{Einstein} = \frac{1}{2} \cdot R_{Heat} \neq 0)$

Therefore the above description of the mathematics of General Relativity (GR), proves that the Source of Gravitation Geodesic Arc Length ($G_{Space\,\theta\phi}$) which is curvature around the source of the gravitation is directly proportional to the Net Inertial Mass or Matter (m_{Net}); and likewise directly proportional to the maximum curvature of space-time given by the Black Hole Event Horizon, Schwarzschild Semi-Manor Radius ($r_{Schwarzschild}$) of a gravitational field.

$$G_{Space\,\theta\phi} \propto m_{Net} \propto r_{Schwarzschild}$$

Therefore these three **presences of matter** conditions of the Einstein Field Equation (EFE) above all reveal where the Schwarzschild Black Hole Event Horizon distance/space solution, is described by the mathematics and physical descriptions of the EFE.

2. Conclusion

Therefore, there are the four (4) conditions for discussing the **presence or the absence of matter**, in any region of a gravitational field, described by the **General Relativity - Einstein Field Equations (EFEs) for Gravitation**; and in summary:

1. (**Matter & Energy *With no* Extended Gravity Field**) $\rightarrow (R_{\theta\phi} = 0)$

$$G_{Space\,\theta\phi} = \mathbf{Ric} = \mathrm{r} \cdot \Omega_{Map\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right) \rightarrow m$$

2. (**Matter & Energy *With* Extended Gravity Field**) $\rightarrow (R_{\theta\phi} = G_{Space\,\theta\phi})$
$\rightarrow (\mathbf{Ric} = 0)$

$$G_{Space\,\theta\phi} = \mathrm{r} \cdot \Omega_{Map\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = R_{\theta\phi} \rightarrow m$$

3. (**Absence of Matter *With no* Extended Gravity Field**)$\rightarrow (R_{\theta\phi} = G_{Space\,\theta\phi} = 0)$

*Note – The Dynamic Vacuum Solution is required for this condition, $(\Lambda_{Einstein} \neq 0)$

$$\mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{Einstein} \right) = 0 \rightarrow m$$

4. (**Absence of Matter *With* Extended Gravity Field**) $\rightarrow (G_{Space\,\theta\phi} = 0)$

$$R_{\theta\phi} = \mathbf{Ric} = g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{Einstein} \right) \rightarrow m$$

Therefore, in the **presence of matter**, and when considering the **Black Hole Event Horizon** solution of the Einstein Field Equation (EFE) for Gravitation; and considering the condition ($R_{\theta\phi} = 0$) and the condition ($R_{\theta\phi} = G_{Space\,\theta\phi}$) in summary:

1. (**Black Hole - Matter & Energy *With no* Extended Gravity Field**) →

$$(R_{\theta\phi} = 0) \quad \& \quad (G_{Space\,\theta\phi} = \mathbf{Ric} \neq 0)$$

Static Vacuum Solution

$$G_{Space\,\theta\phi} = r_{Schwarzschild} \cdot \Omega_{Map\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} \right)$$

2. (**Black Hole - Matter & Energy *With no* Extended Gravity Field**) →

$$(R_{\theta\phi} = 0) \quad \& \quad (G_{Space\,\theta\phi} = \mathbf{Ric} \neq 0)$$

Dynamic Vacuum Solution

$$G_{Space\,\theta\phi} = r_{Schwarzschild} \cdot \Omega_{Map\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = - g_{Vol\,\theta\phi} \cdot \left(\frac{1}{2} \cdot R_{Heat} - \Lambda_{Einstein} \right)$$

3. (**Black Hole - Matter & Energy *With* Extended Gravity Field**) →

$$(R_{\theta\phi} = G_{Space\,\theta\phi}) \quad \& \quad (G_{Space\,\theta\phi} \neq 0) \quad \& \quad (\mathbf{Ric} = 0)$$

$$G_{Space\,\theta\phi} = r_{Schwarzschild} \cdot \Omega_{Map\,\theta\phi} = 2\pi \cdot \left(\frac{T_{Energy\,\theta\phi}}{\frac{1}{4} \cdot \left(\frac{c_{Light}^4}{G} \right)} \right) = R_{\theta\phi}$$

The four (4) conditions for the presence or absence of Mass/Matter in a localized region of space-time are given by the following conditions:

1. **Absence of Matter/Mass** – $(G_{Space\,\theta\phi} = 0)$ & $(R_{\theta\phi} = \mathbf{Ric} \neq 0)$

2. **Presence of Matter/Mass** – $(R_{\theta\phi} = 0)$ & $(G_{Space\,\theta\phi} = \mathbf{Ric} \neq 0)$

3. **Absence of Matter/Mass** – $(R_{\theta\phi} = G_{Space\,\theta\phi} = 0)$ & $(\mathbf{Ric} = 0)$

4. **Presence of Matter/Mass** – $(R_{\theta\phi} = G_{Space\,\theta\phi})$ & $(\mathbf{Ric} = 0)$

This also concludes ***and disproves all claims*** made by Steven Crothers, Section #1 in the paper "Proof of the Invalidity of the Black Hole and Einstein's Field Equations".

I like the questions raised by Steven Crothers that are addressed and written in this work. They represents valid questions about General Relativity and the Einstein Field Equations (EFEs), which definitely require a clear explanation, to remove all ambiguity, about the claims, made by the theory.

In my opinion, the questions raised by Steven Crothers, and which are addressed in this work are tantamount to understanding the EFE conclusions, and claims made, regarding the presence or absence of matter in a localized gravitational field region of the vacuum of space-time - "empty" space.

Crothers's paper can be found at the link below:
http://gsjournal.net/Science-Journals/Research%20Papers-Relativity%20Theory/Download/4326

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