The radius of the proton in the self-consistent model

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Based on the notion of strong gravitation, acting at the level of elementary particles, and on the equality of the magnetic moment of the proton and the limiting magnetic moment of the rotating non-uniformly charged ball, the radius of the proton is found, which conforms to the experimental data. At the same time the dependence is derived of distribution of the mass and charge density inside the proton. The ratio of the density in the center of the proton to the average density is found, which equals 1.57 .

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1. Introduction

Since the discovery of the proton in 1917 the question arose how to determine the radius of this elementary particle. There are many theoretical estimates of the radius of the proton. One of them is based on the fact that in the particles, when they are excited, standing electromagnetic waves emerge. The maximum energy of these standing waves does not exceed the rest energy in order to avoid the decay of particles. From this it can be derived that the de Broglie waves are electromagnetic oscillations, detectable in the laboratory frame in the interaction of moving particles. To describe these oscillations it is necessary to apply the Lorentz transformations to the standing waves inside the particles and to find their form in the laboratory reference frame [1], [2]. If we assume that the length of the standing wave is equal to $\lambda = 2R$, where R is the radius of the proton, then from the equality of the wave energy and the rest energy of the proton we obtain:

$$\nu = \frac{c}{\lambda} = \frac{c}{2R}, \qquad M_p c^2 = h\nu = \frac{hc}{2R}, \qquad R = \frac{h}{2M_p c} = 6.6 \cdot 10^{-16} \text{ m},$$

here ν is the oscillation frequency,

c is the speed of light,

M_p is the mass of the proton,

h is the Planck constant.

Another method assumes that the difference between the rest energy of the neutron and the proton is due to the electrical energy of the proton charge. In this case, it should be:

$$(M_n - M_p)c^2 = \frac{ke^2}{4\pi\varepsilon_0 R}, \tag{1}$$

where M_n is the mass of the neutron,

e is the elementary charge,

ε_0 is the vacuum permittivity.

In (1) for the case of the uniform distribution of the charge in the volume of the proton $k=0.6$, as a result the estimation of the proton radius gives the value of $R=6.68\cdot 10^{-16}$ m.

In [3] and [4], the radius of the proton was found from the condition that the limiting angular momentum of the strong gravitation field inside the proton is equal in magnitude to the spin of the proton. This leads to the following formula:

$$R=\frac{5\Gamma M_p}{21c^2}=6.7\cdot 10^{-16} \text{ m}. \qquad (2)$$

In (2) the strong gravitational constant Γ is used. According to [1], this constant is determined from the equation of electric force and the force from the strong gravitation field, acting in the hydrogen atom on the electron with the mass M_e, which is located in the ground state on the Bohr radius R_B:

$$\frac{e^2}{4\pi\varepsilon_0 R_B^2}=\frac{\Gamma M_p M_e}{R_B^2}, \qquad \Gamma=\frac{e^2}{4\pi\varepsilon_0 M_p M_e}=1.514\cdot 10^{29} \text{ m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}, \qquad (3)$$

In addition to the attractive forces from gravitation and the charges of the nucleus and the electron, in the hydrogen atom the electron substance in the form of the rotating disc is influenced by the repulsive forces acting away from the nucleus. One of these forces is the electric force of repulsion of the charged substance of the electron cloud from itself. In the rotating non-inertial reference frame in which an arbitrary part of the electron substance is at rest, there is also the force of inertia in the form of the centrifugal force, which depends on the velocity of rotation of this substance around the nucleus. In the first approximation, these forces are equal in magnitude, which leads to (3).

We shall remind that the idea of strong gravitation was introduced into science in the works of Abdus Salam and his colleagues [5], [6] as the alternative explanation of the strong interaction of the particles.

With the help of the strong gravitation constant (3) we can express the fine structure constant:

$$\alpha = \frac{\Gamma M_p M_e}{\hbar c} = \frac{1}{137.035999},$$

here $\hbar = \frac{h}{2\pi}$ is the Dirac constant.

Another estimate of the radius of the proton follows from the equality of the rest energy and the absolute value of the total energy, which, taking into account the virial theorem, is approximately equal to the half of the absolute value of the strong gravitation energy associated with the proton [1]:

$$M_p c^2 = \frac{k \Gamma M_p^2}{2R}. \qquad (4)$$

If we take $k = 0.6$ for the case of the uniform mass distribution, then from (4) it follows that $R = 8.4 \cdot 10^{-16}$ m.

All of the above estimates are based on the classical approach to the proton as to the material object of small size in the form of the ball with the radius R. It is assumed that the strong gravitation acts at the level of elementary particles in the same way as ordinary gravitation at the level of planets and stars.

In the Standard model of elementary particles and in quantum chromodynamics it is assumed that the nucleons and other hadrons consist of quarks, and baryons have three quarks, while mesons have two quarks. Instead of the strong gravitation, the action of gluon fields is assumed to hold the quarks in hadrons. Quarks are considered to be charged elementary particles, therefore as the radius of the proton the charge and magnetic mean square radii are considered. These radii are determined by the electric and magnetic interactions of the proton and can differ from each other.

The estimate of the mean square charge radius of the proton can be made with the help of the experiments on the scattering of charged particles on the proton target [7]. In such experiments the total cross sections of interaction of the particles σ are found. For the case of the protons scattering on nucleons with energies more than 10 GeV we can assume that $\sigma = \pi R^2$, and $\sigma = 3.8 \cdot 10^{-30}$ m^2. Hence we obtain $R = 7.8 \cdot 10^{-16}$ m.

2. The self-consistent model

Our aim will be to find a more exact value of the radius of the proton by using classical methods. In the calculations we shall use only the tabular data on the mass, charge and magnetic moment of the proton. The proton will be considered from the standpoint of the theory of infinite nesting of matter [8], in which the analogue of the proton at the level of stars is a magnetar or a charged neutron star with a very large magnetic and gravitational field. Similarly to the magnetar, the substance of the proton must be magnetized and held by a strong gravitation field.

To take into account the non-uniformity of the substance density inside the proton we shall use the formula in which the substance density changes linearly increasing to the center:

$$\rho = \rho_c (1 - Ar), \qquad (5)$$

where ρ_c is the central density,

r is the current radius,

$0 < A < \frac{1}{R}$ is the coefficient which should be determined.

To estimate the values A and the radius R we shall consider the integral for the proton mass in the spherical coordinates:

$$M_p = \int \rho_c (1 - Ar) r^2 dr \sin\theta \, d\theta \, d\varphi = \frac{4\pi R^3 \rho_c}{3} \left(1 - \frac{3AR}{4} \right). \qquad (6)$$

In (6) there are three unknown quantities, to obtain which two more equations are required. We shall assume the virial theorem to be valid and equate the rest energy of the proton to the half of the absolute value of the energy of the static field of strong gravitation:

$$M_p c^2 = -\frac{1}{2} \int_0^\infty u \, dV = \frac{1}{16\pi \Gamma} \int_0^\infty G^2 dV, \qquad (7)$$

where $u=-\frac{G^2}{8\pi\Gamma}$ is the energy density of the strong gravitation field according to [1], G is the gravitational acceleration.

In (7), the integration of the energy density of the field should be done both inside and outside of the proton. The value G inside the proton can be conveniently found by integrating the equation for the strong gravitation field $\nabla\cdot\mathbf{G}=-4\pi\Gamma\rho$, which is part of the equations of the Lorentz-invariant theory of gravitation [9]. After integrating over the spherical volume with the radius $r\le R$, and then using the Gauss theorem, that is making transition to integrating over the area of the indicated sphere inside this proton, in view of (6) we obtain:

$$\int\nabla\cdot\mathbf{G}_i\,dV=\oint\mathbf{G}_i\cdot\mathbf{n}\,dS=4\pi r^2G_i=-\int4\pi\Gamma\rho\,dV\ ,$$

$$\mathbf{G}_i=-\frac{4\pi\Gamma\rho_c\mathbf{r}}{3}\left(1-\frac{3Ar}{4}\right). \qquad (8)$$

Outside the proton the gravitational acceleration is equal to:

$$\mathbf{G}_o=-\frac{\Gamma M_p\mathbf{r}}{r^3}. \qquad (9)$$

Substituting (8) and (9) in (7), we obtain the relation:

$$M_pc^2=4\pi^2\Gamma\rho_c^2R^5\left(\frac{1}{45}-\frac{AR}{36}+\frac{A^2R^2}{112}\right)+\frac{\Gamma M_p^2}{4R}. \qquad (10)$$

In (10) we can eliminate the value ρ_c using (6), which give the dependence of A on R in the form of the quadratic equation:

$$A^2R^2\left(\frac{\Gamma M_p}{14}-\frac{Rc^2}{4}\right)+AR\left(-\frac{7\Gamma M_p}{36}+\frac{2Rc^2}{3}\right)+\frac{2\Gamma M_p}{15}-\frac{4Rc^2}{9}=0.$$

The analysis of this equation shows that it has the following solution:

$$AR = \frac{\frac{7\Gamma M_p}{36} - \frac{2Rc^2}{3} + \sqrt{\frac{\Gamma M_p Rc^2}{945} - \frac{13\Gamma^2 M_p^2}{45360}}}{\frac{\Gamma M_p}{7} - \frac{Rc^2}{2}}, \qquad (11)$$

on condition that when $0.3 < \frac{Rc^2}{\Gamma M_p} < \frac{13}{35} \approx 0.371$, then accordingly $0 < AR < 1$.

We shall now turn to the magnetic moment of the proton. As in [1], we assume that the magnetic moment of the proton is equal to the magnetic moment, which is formed due to the maximum rapid rotation of the charged substance of the proton. In spherical coordinates, the magnetic moment can be approximately calculated as the sum of the elementary magnetic moments of the separate rings with their radius $r\sin\theta$, which have the magnetic moment due to the current di flowing in them from the rotation of the charge:

$$P_m = \int dP_m = \int \pi r^2 \sin^2\theta\, di = \int \pi r^2 \sin^2\theta \frac{dq}{dt} =$$
$$= \int \pi r^2 \sin^2\theta\, \rho_{qc}(1 - Ar) r^2 dr \sin\theta\, d\theta \frac{d\varphi}{dt} = \frac{4\pi R^5 \omega_L \rho_{qc}}{15}\left(1 - \frac{5AR}{6}\right). \qquad (12)$$

The angular velocity $\omega_L = \frac{d\varphi}{dt}$ of the maximum rotation of the proton can be found from the condition of limiting rotation, with the equality of the centripetal force and the gravitation force at the equator: $\frac{\Gamma M_p}{R^2} = \omega_L^2 R$. Further we believe that for the charge density and the substance density the equation $\frac{\rho_{qc}}{\rho_c} = \frac{e}{M_p}$ holds, and we use (6). This gives the following:

$$P_m = \frac{4e\sqrt{\Gamma M_p R}\,(6 - 5AR)}{30(4 - 3AR)}. \qquad (13)$$

3. Conclusions

The relation (13) together with (11) allow us to find the radius of the proton $R = 8.73 \cdot 10^{-16}$ m, as well as the value $A = \frac{0.48}{R}$. From (6) we obtain then the central substance density $\rho_c = 9.4 \cdot 10^{17}$ kg/m^3, which exceeds the average density of the proton 1.57 times. The maximum angular velocity of rotation of the proton in view of (12) is equal to $\omega_L = 6.17 \cdot 10^{23}$ rad/s. At the same time, if the spin of the proton in the approximation of the uniform density of substance would be equal to the standard value for the spin of the fermion $L = 0.4 M_p R^2 \omega = \frac{\hbar}{2}$, then the angular velocity of rotation $\omega = 1.03 \cdot 10^{23}$ rad/s would correspond to this spin.

For comparison with the experimental data we shall point to the results of calculations of electron scattering from [10], where the mean square charge radius $R = 8.7 \cdot 10^{-16}$ m is obtained taking into account only the scattering on protons, $R = 8.71 \cdot 10^{-16}$ m – taking into account the data on the pion scattering, and $R = 8.8 \cdot 10^{-16}$ m – taking into account the data on the neutron scattering. In [11] the mean square charge radius $R = 8.4184 \cdot 10^{-16}$ m was found in the study of the coupled system of the proton and the negative muon. The charge radius $R = 8.77 \cdot 10^{-16}$ m and the magnetic radius $R = 7.77 \cdot 10^{-16}$ m of the proton are listed on the site of *Particle data group* [12]. In the database CODATA [13] the proton charge radius is equal to $R = 8.775 \cdot 10^{-16}$ m.

The value $R = 8.73 \cdot 10^{-16}$ m obtained in the framework of the self-consistent model is close to the experimental values of the radius of the proton, which confirms the possibility of applying the idea of strong gravitation to describe the strong interaction of elementary particles.

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