The spherical inner solution's non-existence of the general relativity theory and the gravity field equation

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ABSTRACT

In the general relativity theory, using Einstein's gravity field equation and acceleration in the spherical skin's condition, prove that the spherical inner solution don't exist .

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I.Introduction

This paper is that it prove non-existence of the spherical coordinate system's inner solution in the general relativity theory's field equation that it has spherical skin's condition

The general relativity theory's field equation is written completely.

$$R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}R=-\frac{8\pi G}{c^2}T_{\mu\upsilon} \quad (1)$$

Eq (1) multiply $g^{\mu\upsilon}$ and does contraction,

$$g^{\mu\upsilon}R_{\mu\upsilon}-\frac{1}{2}g^{\mu\upsilon}g_{\mu\upsilon}R$$

$$=-R=-\frac{8\pi G}{c^2}T^{\lambda}{}_{\lambda} \quad (2)$$

Therefore, Eq (1) is

$$R_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}\frac{8\pi G}{c^2}T^{\lambda}{}_{\lambda}=-\frac{8\pi G}{c^2}T_{\mu\upsilon}$$

$$R_{\mu\upsilon}=-\frac{8\pi G}{c^2}(T_{\mu\upsilon}-\frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) \quad (3)$$

In this time, if the spherical coordinate system's inner solution is, energy-momentum tensor $T_{\mu\upsilon}$ is by Reference [2] (Tolman-Oppenheimer-Volkoff equation,TOV equation),

$$ds^2=-c^2d\tau^2=g_{00}c^2dt^2+g_{rr}dr^2+g_{\theta\theta}d\theta^2+g_{\phi\phi}d\phi^2$$

Stress-energy tensor Schwarzchild field hydrostatic density

$$T_{00}=-g_{00}\rho(r)>0$$

Stress-energy tensor Schwarzchild field hydrostatic pressure

$$T_{rr}(r)=-p(r)g_{rr}/c^2<0$$

otherwise $T_{\mu\upsilon}=0$

$$T^{\lambda}{}_{\lambda}=g^{\mu\lambda}T_{\mu\lambda}=g^{00}T_{00}+g^{rr}T_{rr}=-\rho(r)-p(r)/c^2 \quad (4)$$

$\rho(r)$ is the mass density , $p(r)$ is the pressure

The spherical coordinate system's inner solution's invariant time is

$$d\tau^2=A(t,r)dt^2-\frac{1}{c^2}[B(t,r)dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2] \quad (5)$$

Using Eq(5)'s metric tensor, save the Riemannian-curvature tensor, and does contraction,save Ricci-tensor, Eq (5) is

$$R_{tt}=-\frac{A''}{2B}+\frac{A'B'}{4B^2}-\frac{A'}{Br}+\frac{A'^2}{4AB}+\frac{\ddot{B}}{2B}-\frac{\dot{B}^2}{4B^2}-\frac{\dot{A}\dot{B}}{4AB}$$

$$=-\frac{8\pi G}{c^2}[T_{00}-\frac{1}{2}g_{00}g^{\mu\lambda}T_{\mu\lambda}]$$

$$=-\frac{8\pi G}{c^2}[T_{00}-\frac{1}{2}g_{00}\{g^{00}T_{00}+g^{rr}T_{rr}\}]$$

$$=-\frac{8\pi G}{c^2}[-g_{00}\rho(r)-\frac{1}{2}g_{00}\{-\rho(r)-p(r)/c^2\}]$$

$$=-\frac{8\pi G}{c^2}[-\frac{1}{2}g_{00}\rho(r)+\frac{1}{2}g_{00}p(r)/c^2]$$

$$=\frac{4\pi G}{c^2}A[-\rho(r)+p(r)/c^2] \qquad (6)$$

$$R_{rr}=\frac{A''}{2A}-\frac{A'^2}{4A^2}-\frac{A'B'}{4AB}-\frac{B'}{Br}-\frac{\ddot{B}}{2A}+\frac{\dot{A}\dot{B}}{4A^2}+\frac{\dot{B}^2}{4AB}$$

$$=-\frac{8\pi G}{c^2}[T_{rr}-\frac{1}{2}g_{rr}\{g^{00}T_{00}+g^{rr}T_{rr}\}]$$

$$=-\frac{8\pi G}{c^2}[-p(r)g_{rr}/c^2-\frac{1}{2}g_{rr}\{-\rho(r)-p(r)/c^2\}]$$

$$=-\frac{8\pi G}{c^2}[-\frac{1}{2}p(r)g_{rr}/c^2+\frac{1}{2}g_{rr}\rho(r)]$$

$$=\frac{4\pi G}{c^2}B[p(r)/c^2-\rho(r)] \qquad (7)$$

$$R_{\theta\theta}=-1+\frac{1}{B}-\frac{rB'}{2B^2}+\frac{rA'}{2AB}$$

$$=-\frac{8\pi G}{c^2}[T_{\theta\theta}-\frac{1}{2}g_{\theta\theta}\{g^{00}T_{00}+g^{rr}T_{rr}\}]$$

$$=-\frac{8\pi G}{c^2}[-\frac{1}{2}g_{\theta\theta}\{-\rho(r)-p(r)/c^2\}]$$

$$=\frac{4\pi G}{c^2}r^2[-p(r)/c^2-\rho(r)] \qquad (8)$$

$$R_{\phi\phi}=R_{\theta\theta}\sin^2\theta=\frac{4\pi G}{c^2}r^2\sin^2\theta[-p(r)/c^2-\rho(r)] \qquad (9)$$

$$R_{tr}=-\frac{\dot{B}}{Br}=0 \; (10) \qquad R_{t\theta}=R_{t\phi}=R_{r\theta}=R_{r\phi}=R_{\theta\phi}=0 \; (11)$$

II.Additional chapter-I

In this time, in the general relativity theory's the spherical coordinate system's inner solution, metric tensor and the general relativistic gravity acceleration has to coincide that is in the general relativity theory's the spherical coordinate system's vacuum solution in the spherical skin $r = R$.

$$A|_{r=R} = 1 - \frac{2GM}{Rc^2} \quad (12), \quad B|_{r=R} = \frac{1}{1 - \frac{2GM}{Rc^2}} \quad (13)$$

The vacuum's gravity acceleration has to coincide the inner solution's gravity acceleration in the spherical skin $r = R$.

$$\frac{d^2r}{d\tau^2}|_{r=R} = [-\Gamma^r{}_{00}(\frac{cdt}{d\tau})^2 - \Gamma^r{}_{rr}(\frac{dr}{d\tau})^2 - \Gamma^r{}_{\theta\theta}(\frac{rd\theta}{d\tau})^2 - \Gamma^r{}_{\phi\phi}(\frac{r\sin\theta d\phi}{d\tau})^2]|_{r=R}$$

$$= [-(1 - \frac{2GM}{rc^2})\frac{GM}{r^2c^2}(\frac{cdt}{d\tau})^2 + \frac{1}{(1 - \frac{2GM}{rc^2})}\frac{GM}{r^2c^2}(\frac{dr}{d\tau})^2 + r(1 - \frac{2GM}{rc^2})(\frac{rd\theta}{d\tau})^2$$

$$+ r\sin^2\theta(1 - \frac{2GM}{rc^2})(\frac{r\sin\theta d\phi}{d\tau})^2]|_{r=R}$$

$$= [-(1 - \frac{2GM}{Rc^2})\frac{GM}{R^2c^2}(\frac{cdt}{d\tau})^2|_{r=R} + \frac{1}{(1 - \frac{2GM}{Rc^2})}\frac{GM}{R^2c^2}(\frac{dr}{d\tau})^2|_{r=R}$$

$$+ R(1 - \frac{2GM}{Rc^2})(\frac{Rd\theta}{d\tau})^2|_{r=R} + R\sin^2\theta(1 - \frac{2GM}{Rc^2})(\frac{R\sin\theta d\phi}{d\tau})^2|_{r=R}]$$

$$= -\frac{1}{2}\frac{A'}{B}|_{r=R}(\frac{cdt}{d\tau})^2|_{r=R} - \frac{1}{2}\frac{B'}{B}|_{r=R}(\frac{dr}{d\tau})^2|_{r=R} + \frac{1}{B}|_{r=R} \cdot R(\frac{Rd\theta}{d\tau})^2|_{r=R}$$

$$+ \frac{1}{B}|_{r=R} \cdot R\sin^2\theta(\frac{R\sin\theta d\phi}{d\tau})^2|_{r=R} \quad (14)$$

Therefore, In the spherical skin $r = R$

$$\frac{1}{B}|_{r=R} = 1 - \frac{2GM}{Rc^2}, \quad \frac{B'}{B}|_{r=R} = B'|_{r=R} \cdot \frac{1}{B}|_{r=R} = -\frac{1}{1 - \frac{2GM}{Rc^2}}\frac{2GM}{R^2c^2} \quad (15)$$

Therefore

$$B'|_{r=R} = -\frac{1}{(1 - \frac{2GM}{Rc^2})^2}\frac{2GM}{R^2c^2} \quad (16)$$

Therefore, In the spherical skin $r = R$

$$\frac{1}{B}|_{r=R} = 1 - \frac{2GM}{Rc^2}, \quad \frac{A'}{B}|_{r=R} = A'|_{r=R} \cdot \frac{1}{B}|_{r=R} = (1 - \frac{2GM}{Rc^2})\frac{2GM}{R^2c^2} \quad (17)$$

Therefore

$$A'|_{r=R} = \frac{2GM}{R^2c^2} \quad (18)$$

By Eq(12) and Eq (13)

$$A|_{r=R} = 1 - \frac{2GM}{Rc^2}, \quad B|_{r=R} = \frac{1}{1 - \frac{2GM}{Rc^2}} \quad (19)$$

III.Additional chapter-II

In this time, Eq (8) is

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = \frac{4\pi G}{c^2} r^2[-p(r)/c^2 - \rho(r)] \quad (20)$$

In Eq (20),both term is treated in the spherical skin $r = R$'s situation, Eq (20)'s left term is

$$(-1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB})|_{r=R}$$

$$= -1 + 1 - \frac{2GM}{Rc^2} + \frac{1}{2}R(1 - \frac{2GM}{Rc^2})^2 \cdot \frac{1}{(1 - \frac{2GM}{Rc^2})^2}\frac{2GM}{R^2c^2} + \frac{1}{2}R\frac{2GM}{R^2c^2} = 0 \quad (21)$$

In the spherical skin $r = R$, the pressure $p(R)$ has to zero because sphere's radius invariant . In the spherical skin $r = R$, Eq (20)'s right term is

$$\frac{4\pi G}{c^2}[r^2\{-p(r)/c^2 - \rho(r)\}]|_{r=R}$$

$$= \frac{4\pi G}{c^2} \cdot -R^2\rho(R) < 0, \quad p(R) = 0 \quad (22)$$

In this time, the mass density $\rho(R) > 0$

In the spherical skin $r = R$, Eq (20)'s right term don't coincide Eq (20)'s left term.

IV.Additional chapter-III

By Eq (10)

$$\dot{B} = 0 \quad (23)$$

And use this result, by Eq (6) and Eq (7)

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{A'}{ABr} - \frac{B'}{B^2r} = -\frac{8\pi G}{c^2}[\{-\rho(r) + p(r)/c^2\} + \{p(r)/c^2 - \rho(r)\}]$$

$$= -\frac{8\pi G}{c^2}[-2\rho(r) + 2p(r)/c^2] \quad (24)$$

In this time, in Eq (24), in the spherical skin $r = R$'s situation, the Eq (24)'s left term is

$$(-\frac{A'}{ABr} - \frac{B'}{B^2r})|_{r=R} = -\frac{2GM}{R^2c^2}\frac{1}{R} - (1 - \frac{2GM}{Rc^2})^2 \cdot -\frac{1}{(1 - \frac{2GM}{Rc^2})^2}\frac{2GM}{R^2c^2}\frac{1}{R} = 0 \quad (25)$$

In the spherical skin $r = R$'s situation, the fomula (24)'s right term is

$$-\frac{8\pi G}{c^2}[-2\rho(r)+2p(r)/c^2]|_{r=R}$$

$$=\frac{16\pi G}{c^2}\rho(R)>0\,,\quad p(R)=0 \quad (26)$$

In the spherical skin $r = R$, Eq (24)'s right term don't coincide Eq (24)'s left term.

IV.Conclusion

According to Eq (22),Eq(26), the spherical coordinate system's inner solution don't exist.

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