

An interstellar position fixing method

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Abstract

A method is outlined to enable a navigator or an automated system to fix a ship's position in charted interstellar space with the assistance of a three dimensional computer based stellar chart and star camera spectrometers capable of measuring angular separations between three sets of pair stars. The method offers another tool for the navigator to rely on if alternative position fixing methods are not available or if the navigator wishes to verify the validity of one's position given by other means.

1 Introduction

"To drop a pea at the end of every mile of a voyage on a limitless ocean to the nearest fixed star, would require a fleet of 10,000 ships of 600 tons burthen, each starting with a full cargo of peas."

John Herschel, c.1850.

Before looking into the interstellar case, it is first useful to look into the following known position fixing method used in nautical coastal navigation which uses Horizontal Sextant Angles (HSA) to obtain a position fix at sea [2] provided suitably charted landmarks can be identified. The angular separation between (for eg) lighthouses is measured by using the sextant horizontally to give us the angle. Let A, B and C shown in Figure 1 be lighthouses, the ship is located somewhere off the coast and the navigator needs to fix its position on the chart.

As an example, the HSA between A and B was measured to be 30° and the HSA between B and C to be 50° . Before going through the mathematical

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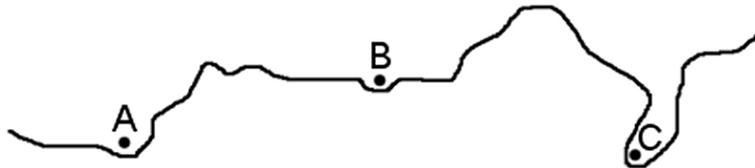


Figure 1: Three lighthouses A, B and C.

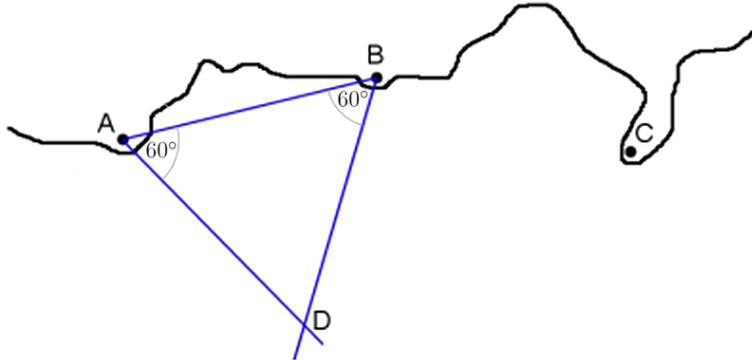


Figure 2: Constructing triangle ABD.

proof of the method, it is useful to describe first how the method is used. A line is drawn on the chart between A and B. Then draw a line from A which is $(90^\circ - HSA^\circ)$ from this line so in this case a line is drawn at $90^\circ - 30^\circ = 60^\circ$ from the line AB to seaward with the aid of a protractor. Similarly draw another line 60° from BA. These lines intersect at D as shown in Figure 2 forming the triangle ABD. If this was done correctly, the triangle ABD is an isosceles triangle as the angle $DAB = DBA = 60^\circ$ in this example and hence $AD = BD$. A drawing compass is now used, by placing the center of the compass at D and a full circle is drawn with a radius AD or DB which are equal, see Figure 3. The circle will intersect A and B. This is known as the ship's **position circle**. The ship could be anywhere to seaward on this blue circle and the navigator will measure a HSA between lighthouses A and B to be 30° (why this is the case will be shown later) however it is not known at this stage where on this circle the ship is located hence the name position circle.

This method is now repeated by measuring the HSA between lighthouses B and C. Ideally the ship is stopped or moving very slowly between these measurements. A HSA of 50° is measured between B and C and the second position circle is now plotted using the above method as shown in Figure 4. As before the ship must be anywhere to seaward on this green position circle for a HSA between B and C to be 50° . Since the ship must also be on the blue position circle as well, the only position where this qualifies is where both position circles intersect: the ship's position is now plotted on the chart. Notice there is another location where both position circles also intersect however this is at the lighthouse B and is on land so this can be ruled out. Any slight measurement errors in the first two HSAs (vessel rolling in heavy seas, instrument errors etc) using the sextant will affect the accuracy of the position fix and a third position circle needs to be plotted to confirm the ship's position. Ideally all three position circles should intersect at one spot however in practice sometimes (and often) they don't as shown in Figure 5. The aim of the position fix is to obtain the smallest "cocked hat". The ship's position is marked in the middle of the "hat" or closest to any dangers such as nearby reefs, rocks etc. A very large cocked hat indicates that one or more of the HSAs has a large error or a wrong charted landmark was used. However very accurate position fixes can be obtained with this method [3].

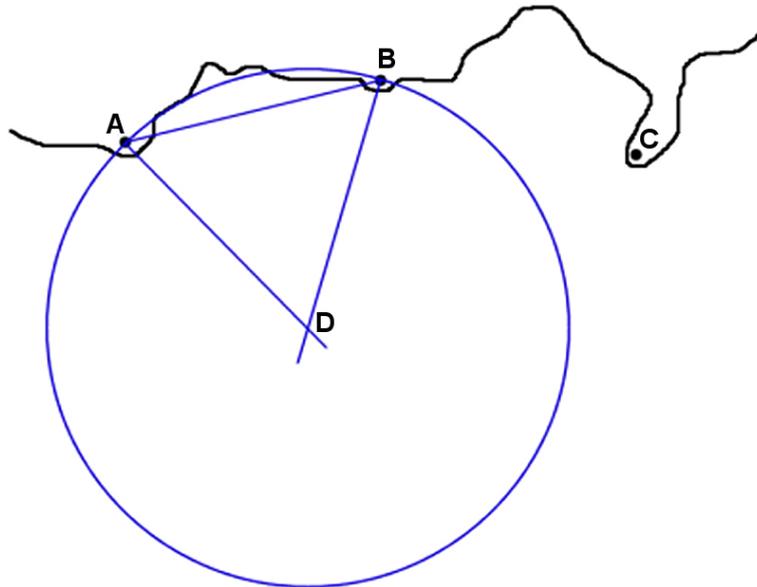


Figure 3: A position circle.

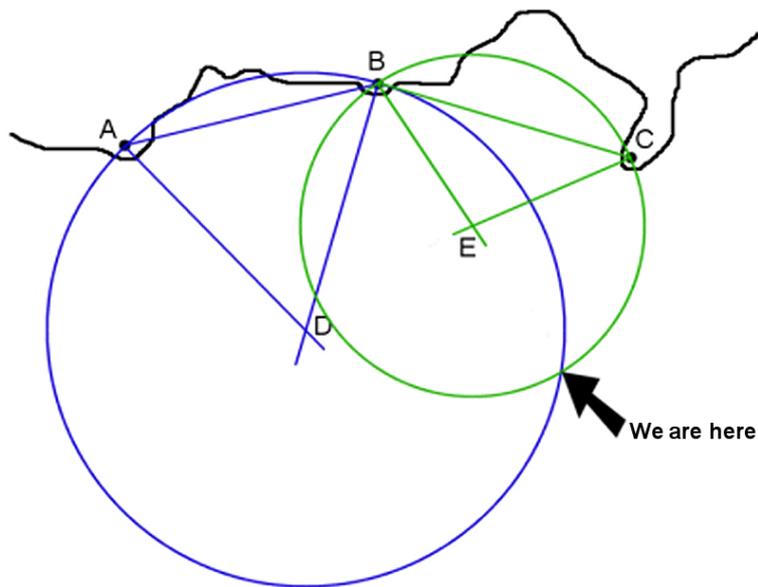


Figure 4: Two position circles showing the ship's position.

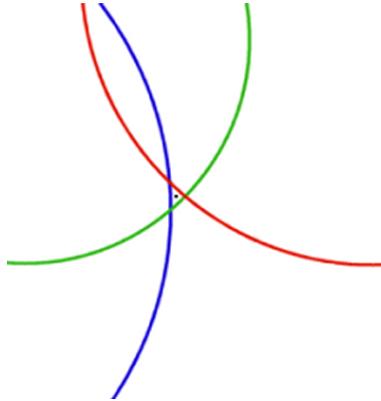


Figure 5: Three position circles showing a cocked hat.

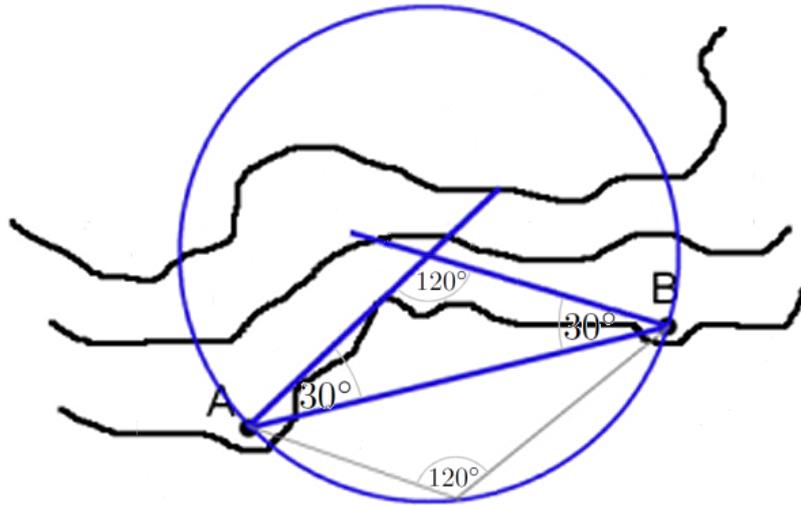


Figure 6: The case where HSA is greater than 90° .

In the example given above the measured HSAs were less than 90° and in this case the center of the position circles lie on the ship's side. What if the measured HSA was over 90° ? Consider a HSA between two lighthouses A and B to be 120° . In this case the center of the position circle lies on the opposite side of the line joining the lighthouses from the vessel and the base angles of the triangle becomes $120^\circ - 90^\circ = 30^\circ$ as shown in Figure 6. The ship could be anywhere on this position circle to seaward. Notice the arc where the position could be located is smaller now than the landward arc as opposed to the previous HSAs when they were less than 90° , the arcs that were part of the position circle to seaward were greater than the landward arcs (these major and minor arcs will be discussed further on).

The situation where the measured $HSA = 90^\circ$ is now considered as shown in Figure 7. As an example consider the ship is now located somewhere on the ocean at night and the only landmarks closeby are two small islands each with a

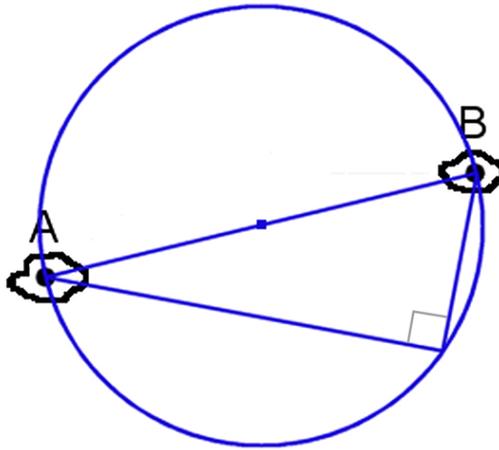


Figure 7: Position circle for $HSA = 90^\circ$.

lit lighthouse having their own unique light sequence. With the measured HSA between these lighthouse of 90° , the center of the position circle is now at the bisecting point of the line joining the lighthouses (again why this is so will be shown later), there is no isocetes triangle as with the previous examples because $90^\circ - 90^\circ = 0^\circ$ and the line AB is the diameter of the circle. Note that the ship could be anywhere on this position circle (except the islands) because in this case the navigator could also have measured a HSA of 90° if the ship was on the other side of the islands since there is water there as well.

The following three cases have been covered for measured:

- $HSA < 90^\circ$
- $HSA = 90^\circ$
- $HSA > 90^\circ$

and it has been shown how to construct a position circle on a two dimensional chart for each case.

2 Mathematical background

The mathematical proof of the above method it now shown and an explanation is given as to why it works. A number of theorems concerning circles apply here. An inscribed angle is an angle formed by two chords in a circle which have a common endpoint. A chord of a circle is a line joining any two points on the circumference. This common endpoint forms the vertex of the inscribed angle. The other two endpoints define what is called an intercepted arc on the circle. The intercepted arc might be thought of as the part of the circle which is “inside” the inscribed angle, see Figure 8.

A central angle is any angle where the vertex is located at the center of a circle. A central angle necessarily passes through two points on the circle, which in turn divide the circle into two arcs: a major arc and a minor arc. The minor

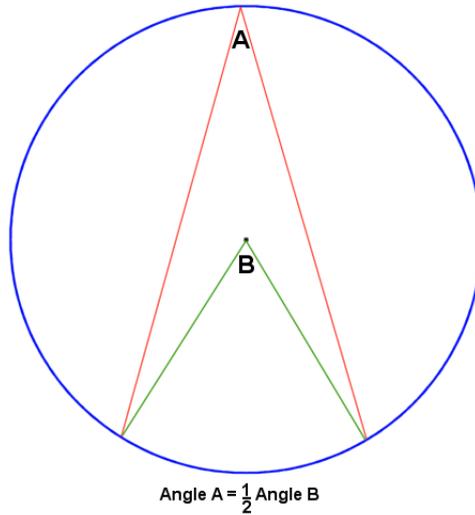


Figure 8: A is an *inscribed* angle, B is a *central* angle.

arc is the smaller of the two arcs, while the major arc is the bigger. The arc angle is defined to be the measure of the central angle which intercepts it.

Fixing one's position using the geometrical method described previously relies on a number of well know geometrical theorems. In the Appendix, proofs for the following are shown:

- *The measure of the intercepted arc (equal to its central angle) is exactly twice the measure of the inscribed angle.*
- *All angles subtended by a chord at the circumference are equal.*
- *If A, B and C are points on a circle where the line AC is a diameter of the circle, then the angle ABC is a right angle.*
- *Opposite angles in any quadrilateral inscribed in a circle are supplements of each other. (Their measures add up to 180°).*

From the above theorems it has been shown why the method is correct and why the ship needs to be on the position circle for a given measured HSA between two landmarks.

3 The Interstellar case

The navigator who sailed the oceans on Earth has now moved on to a new ship (a starship), venturing in interstellar space and has to fix the ship's position in three dimensions between a vast array of stars available in our galaxy. Previous investigations [9, 10, 11] have shown the options available in obtaining interstellar position fixes and the limitations associated with each of the different methods are outlined. To use the position fixing method outlined in this paper, a number of tools are required: a reasonably accurate computer based 3D stellar chart and star camera spectrometers which allows the navigator to identify

the stars measured and to measure angles between them. This could also be a computer controlled system which will do these tasks automatically for the crew.

A 3D stellar chart is required to fix the ship's position among the stars. How accurate does the chart have to be? Stars have radial and proper motions and hence the chart will have to be corrected regularly to keep track of the changing stellar positions (similarly to nautical charts which are updated regularly via Notices to Mariners). Our Sun is moving approximately 22 Km/s towards the constellation of Crater or Leo with respect to the average motion of our local stars and gas with typical star motions of the order of 10 Km/s relative to us (0.003 ly / 100 years) [4]. With current baselines for parallax data available (Earth's orbit), distances to stars can be relatively difficult to measure and a standard error of 1% is considered high accuracy however the distance accuracy of closer stars is better, with a distance to Proxima Centauri, closest star to our Sun, given at 4.243 ± 0.002 ly [5]. As probes are sent further out to the rims of the solar system and beyond thus extending the parallax baseline measurement, the standard error in distance measurements will be reduced [1]. Depending on the interstellar mission scenario and ship design, high accuracy of the order of ± 0.0001 ly may be required [4].

The coordinate system used for the chart also needs to be considered. One may use standard spherical coordinates (Right Ascension / Declination) based from Earth's vantage however for an interstellar navigator travelling between different star systems, one should preferably use Galactic rectilinear x, y, z coordinates to facilitate navigation purposes. Both coordinate systems can be readily converted to each other if required [12, 13]. Figure 9 is an example of a 3D representation showing all 32 stars within 14 lightyears (ly) from our Sun using Right Ascension / Declination coordinates [6]. As more ships venture further into the interstellar medium, these charts can be refined as more data is obtained.

Another problem to consider is the ship's velocity during the measurement of the angular separation of pair stars (HSAs). The faster the ship is travelling, the more pronounced will be stellar aberration which is an apparent position change of stars due to the finite speed of light c . If the ship is travelling at a relativistic velocity (V), stellar aberration must be calculated and is given as follows [8]:

$$\cos \theta_1 = \frac{\cos \theta + \frac{V}{c}}{1 + \frac{V}{c} \cos \theta}$$

where θ_1 is the apparent angle of a star from the velocity vector of the ship and θ is the apparent angle to the same star when the ship is at rest relative to the star. This also means if we know θ and we measure θ_1 , we can deduce the ship's velocity relative to the reference star. At $0.1c$, aberration causes an apparent shift of star positions to be shifted forward by 6° for those located on the sides and less for stars in front and behind. Brightness of stars is affected as well, for the same velocity, causing an increase in brightness by 20% for those in front and decreasing brightness by 20% for those behind [4, 15]. The most accurate measurements with this method will be obtained when the ship is stopped or travelling at a slow velocity of less than $0.01c$ with the propulsion drive off. This may not be an option due to the mission requirements or the

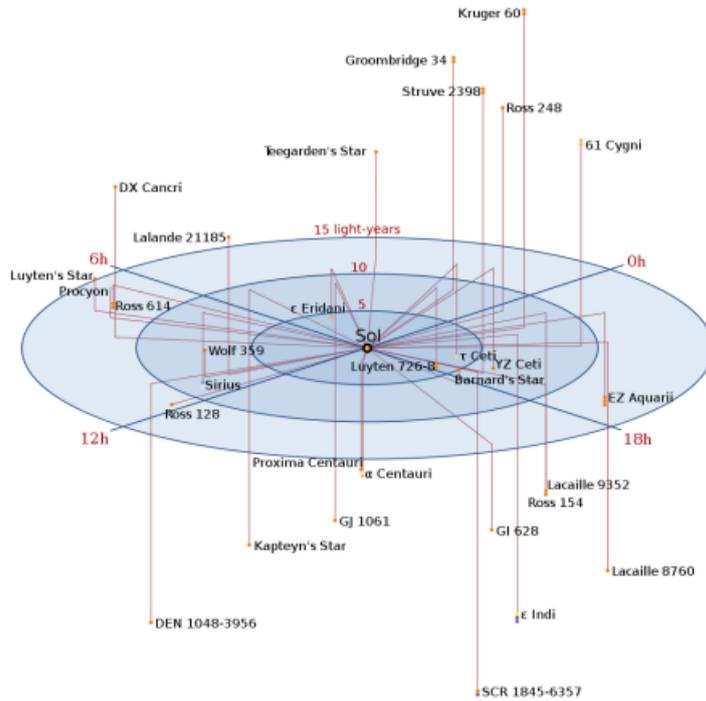


Figure 9: Map showing all 32 star systems within 14ly from our Sun.

energy limitations for the propulsion drive used by the ship [14], in this case relativistic mathematics will have to be employed to take into account the ship's velocity. Ship velocity data could also be obtained from measurements of the doppler shift in the stellar spectral lines and the ship should have an extensive database of stellar spectra when at rest for onboard comparison with at velocity measured spectra over a wider range than just the visible spectrum to account for the doppler shifts at high relativistic velocities [10]. Another method which could also provide accurate GPS-like interstellar position fixes involves using X-Ray pulsars (also known as XNav or X-Ray Navigation). Astronomers have mapped a substantial number of X-Ray pulsars whose pulsed emissions are as regular as atomic clocks. Since these sources are highly reliable and relatively fixed in position, phase measurements can be used to obtain a position fix potentially anywhere in the galaxy [7].

Ideally the navigator will choose closeby pairs of stars to do the position fix which means the navigator should have a rough idea where the ship is located in the galaxy. After two suitable stars are identified, the navigator measures the HSA between them and constructs a position circle in the 3D stellar chart. For the interstellar case, we don't know which plane to draw the position circle and our position can lie in any plane. Because of the extra third dimension, we need to draw in all position circles that satisfy the measured HSA between these two stars around the axis formed by the line drawn between these two stars as shown in Figure 10. The surface formed by the locus of all position circles rotated around the axis formed by the line drawn between these two stars yields

a torus and our position can be anywhere on the surface of this torus, this a **position torus**. This will need to be constructed with the aid of a computer in the 3D stellar chart. The process is now repeated and the navigator needs to pick two more pairs of stars to plot two more position torii on the chart to obtain a position fix. It was shown previously that for the ocean navigator's case, ideally all three position circles should intersect at one point however in practice they don't because of errors in measurements etc. Figure 11 shows an example of three position torii plots.

The geometrical center of the enclosed curved surface formed by all three position torii is the ship's position. The smaller the three dimensional "cocked hat", the more confident the navigator can be on the accuracy of the obtained position fix. Figure 12 is an example of the volume intersection of three position torii resulting in the cocked hat.

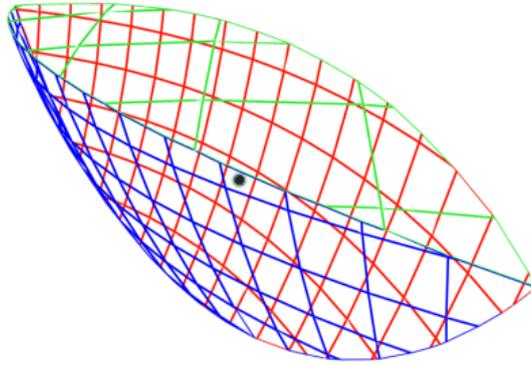


Figure 12: Three dimensional cocked hat showing the ship's position in the geometrical center of the obtained volume. Each coloured surface belongs to a unique position torus.

The ship's position is located in the geometrical center of such volume. If the 3D stellar chart is using galactic rectilinear x, y, z coordinates in lightyears, then the uncertainty of the position fix can be evaluated directly from the cocked hat dimensions. It will be noted that the construction method of the position torii or position spheres follows the rules described earlier for all three cases $HSA < 90^\circ$, $HSA = 90^\circ$ and $HSA > 90^\circ$ ie draw a position circle in any plane according to the HSA measured between the two stars and perform a full 360° rotation around the axis formed between the pair stars used to obtain either the position torii (for $HSA < 90^\circ$ or $HSA > 90^\circ$) or the position sphere (for $HSA = 90^\circ$) and after repeating the procedure for two more times, obtain the ship's position by obtaining the geometrical center of the volume enclosed by the cocked hat plotted in the 3D stellar chart. Any large errors in the position fix will be immediately visible as this would result in a large cocked hat. If the navigator is confident on the accuracy of the 3D stellar chart then more HSAs can be measured for replotting if required.

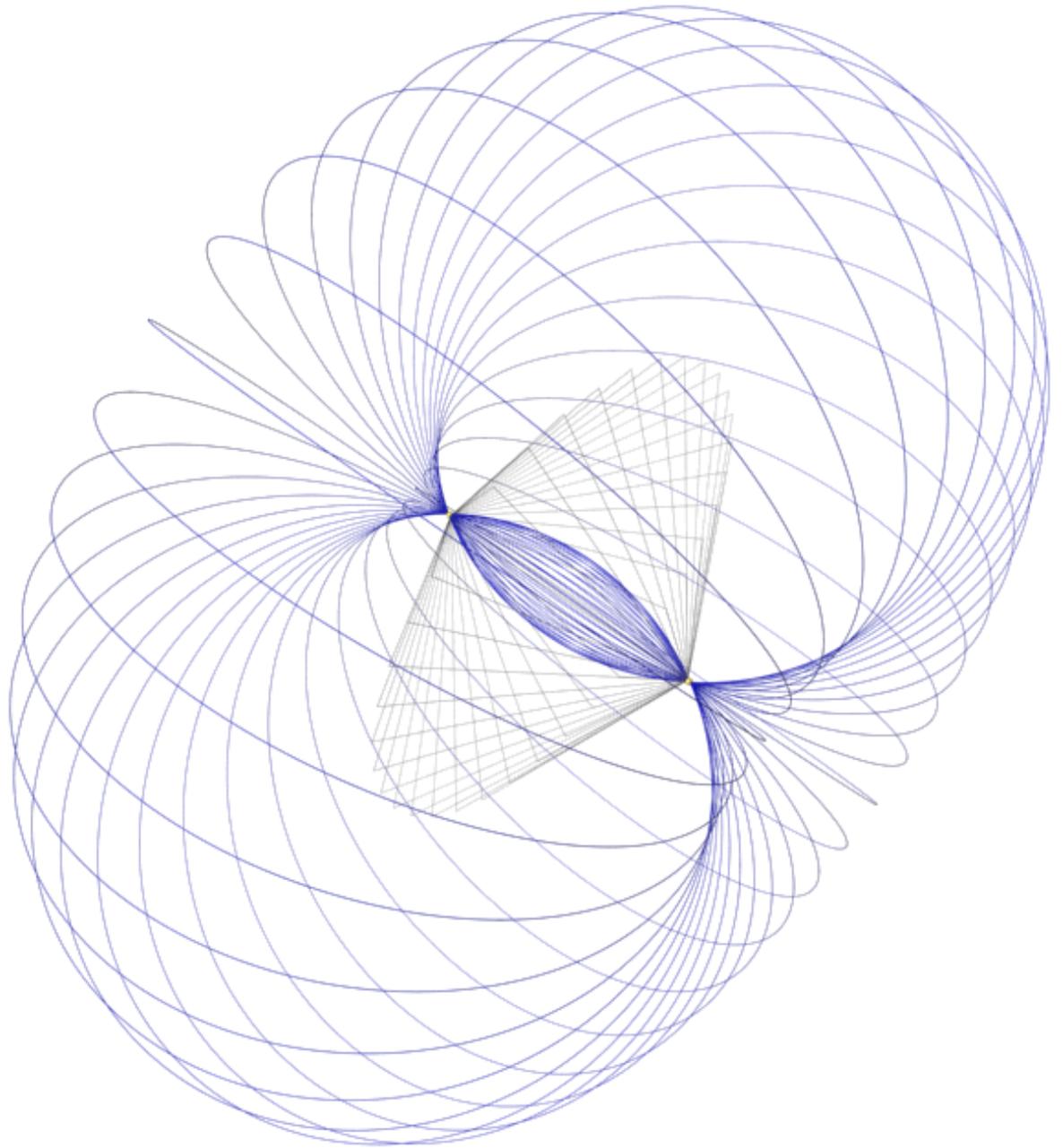


Figure 10: A position torus using two stars

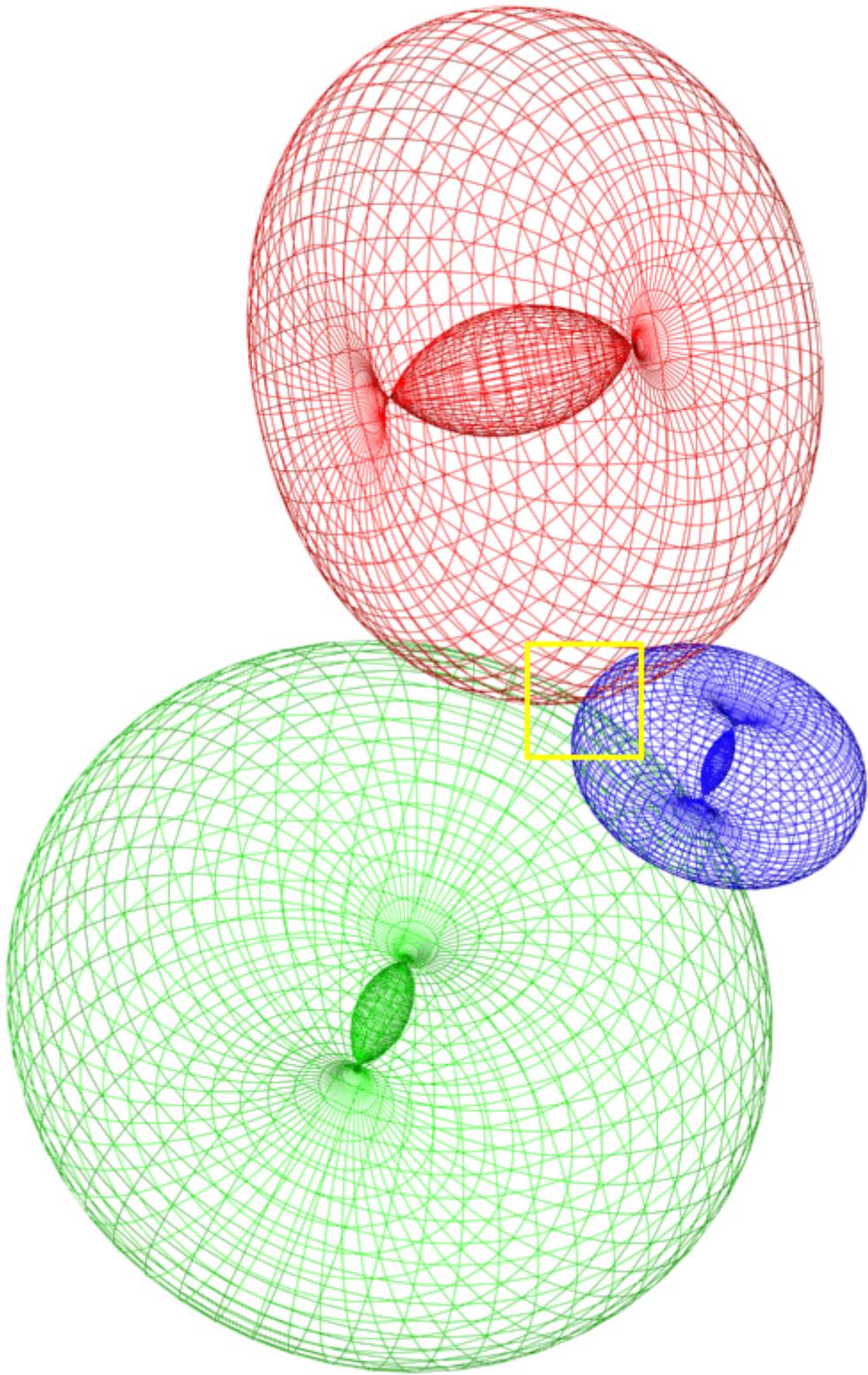


Figure 11: An interstellar position fix using three position torii (6 stars).

4 Conclusion

A method is shown to enable an interstellar navigator or automated system to obtain a position fix in charted interstellar space. The method shown can be used to complement other position fixing methods or be used to double check the validity of a position obtained by other means. The method relies on measurements of angular separations between stars alone which are easily measured.

5 Appendix

The following propositions together with their proofs are shown. These are relevant to the position fixing method described.

Proposition: *The measure of the intercepted arc (equal to its central angle) is exactly twice the measure of the inscribed angle.*

Consider line AB in Figure 13 to be a screen for a drive in movie theater and the professionals informed the owner on the optimum angle θ that the movie screen AB should present to the viewer. But only one customer can sit in the preferred spot V directly in front of the screen. The owner is interested in locating other points, U, from which the screen subtends the same angle θ . The answer is the circle passing through the three points A, B and V. *AUB is measured by the same arc as AVB, the angle at U is the same as the angle at V as only one circle can pass through three points not in the same straight line.* This is also the HSA that was measured between the lighthouses.

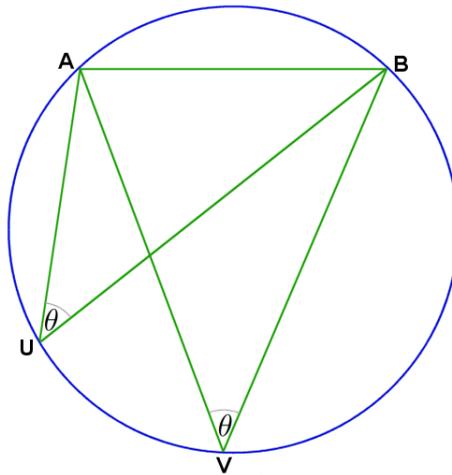


Figure 13: Optimum viewing angles.

Proof:

Let O be the center of a circle as shown in Figure 14. Choose two points on the circle, and call them V and A. Draw line VO and extended past O so that it intersects the circle at point B which is diametrically opposite the point V. Draw an angle whose vertex is point V and whose sides pass through points A

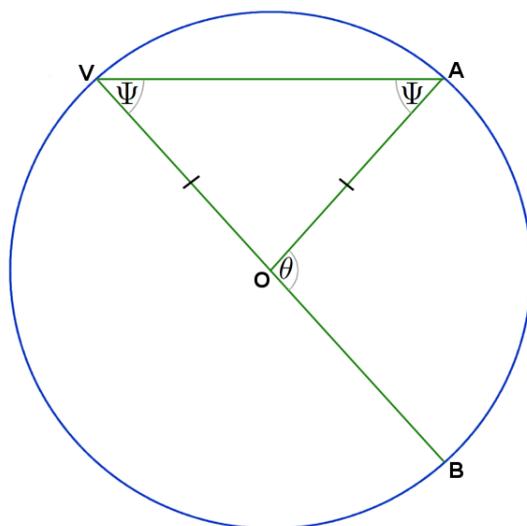


Figure 14: Central and inscribed angles.

and B. $\angle BOA$ is a central angle; call it θ . Draw line OA . Lines OV and OA are both radii of the circle, so they have equal lengths. Therefore triangle $\triangle VOA$ is isosceles, so $\angle BVA$ (the inscribed angle) and $\angle VAO$ are equal; let each of them be denoted as Ψ .

Angles BOA and AOV are supplementary. They add up to 180° , since line VB passing through O is a straight line. Therefore $\angle AOV$ measures $180^\circ - \theta$.

It is known that the three angles of a triangle add up to 180° , and the three angles of a triangle VOA are $180^\circ - \theta$, Ψ and Ψ . Therefore

$$2\psi + 180^\circ - \theta = 180^\circ.$$

Subtract 180° from both sides,

$$2\psi = \theta,$$

where θ is the central angle subtending arc AB and ψ is the inscribed angle subtending arc AB . The inscribed angle is only defined for points on the major arc (the longest path around the circle between the two given points) hence the inscribed angle is undefined in the shorter (minor) arc. Further below another theorem will be shown to find the angle in the minor arc.

Consider now an inscribed angle with the center of the circle in their interior, Figure 15.

Given a circle whose center is point O , choose three points V , C and D on the circle. Draw lines VC and VD : $\angle DVC$ is an inscribed angle. Now draw line VO and extend it past point O so that it intersects the circle at point E . $\angle DVC$ subtends arc DC on the circle.

Suppose this arc includes point E within it. Point E is diametrically opposite to point V . Angles DVE and EVC are also inscribed angles, but both of these angles have one side which passes through the center of the circle, hence the above theorem can be applied to them. Therefore:

$$\angle DVC = \angle DVE + \angle EVC.$$

then let

$$\psi_0 = \angle DVC,$$

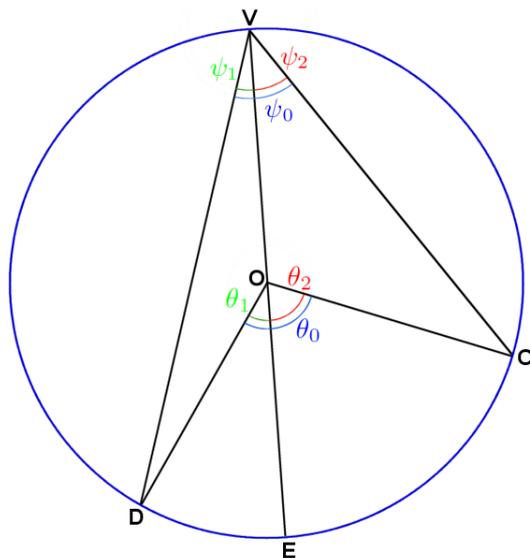


Figure 15: Inscribed angles.

$$\psi_1 = \angle DVE,$$

$$\psi_2 = \angle EVC,$$

so that

$$\psi_0 = \psi_1 + \psi_2. \text{ (Eq.1)}$$

Draw lines OC and OD. $\angle DOC$ is a central angle, but so are angles DOE and EOC, and

$$\angle DOC = \angle DOE + \angle EOC.$$

Let

$$\theta_0 = \angle DOC,$$

$$\theta_1 = \angle DOE,$$

$$\theta_2 = \angle EOC,$$

so that

$$\theta_0 = \theta_1 + \theta_2 \text{ (Eq. 2)}$$

From the above theorem it was shown that $\theta_1 = 2\psi_1$ and $\theta_2 = 2\psi_2$. Combining these results with Eq. 2 yields

$$\theta_0 = 2\psi_1 + 2\psi_2$$

therefore, by Eq. 1,

$$\theta_0 = 2\psi_0.$$

This is known as the *Central Angle Theorem*: the measure of the inscribed angle is always half the measure of the central angle. The inscribed angle is the HSA that was measured with the sextant.

Proposition: *If A, B and C are points on a circle where the line AC is a diameter of the circle, then the angle ABC is a right angle.*

Proof:

The following facts are used:

- The sum of the angles in a triangle is equal to two right angles (180°).

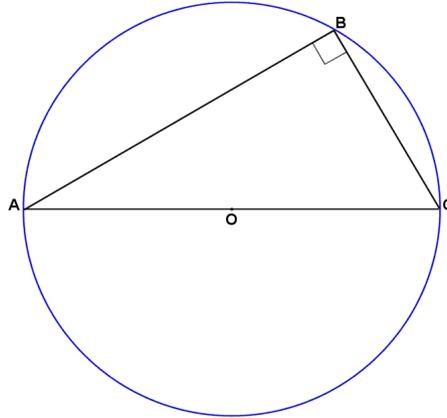


Figure 16: Angle ABC is a right angle

- The base angles of an isosceles triangle are equal.

Let O be the center of the circle, Figure 17. Since $OA = OB = OC$, $\triangle OAB$ and $\triangle OBC$ are isosceles triangles, and by equality of the base angles of an isosceles triangle, $\angle OBC = \angle OCB$ and $\angle BAO = \angle ABO$. Let $\alpha = \angle BAO$ and $\beta = \angle OBC$. The three internal angles of $\triangle ABC$ are α , $\alpha + \beta$ and β . Since the sum of the angles of a triangle is equal to two right angles, then

$$\alpha + (\alpha + \beta) + \beta = 180^\circ$$

then

$$2\alpha + 2\beta = 180^\circ$$

or simply

$$\alpha + \beta = 90^\circ$$

This is known as *Thales' Theorem*: the diameter of a circle always subtends a right angle to any point on the circle. This applies to the situation where a HSA of 90° was measured.

Proposition: Opposite angles in any quadrilateral inscribed in a circle are supplements of each other. (Their measures add up to 180°).

Proof:

From Figure 18, the main result needed is that an inscribed angle has half the measure of the intercepted arc. Here, the intercepted arc for $\angle A$ is the Arc(BCD) and for $\angle C$ is the Arc(DAB).

$$\text{Arc}(BCD) = 2\angle A \text{ and } \text{Arc}(DAB) = 2\angle C$$

$$\text{Arc}(BCD) + \text{Arc}(DAB) = 360^\circ$$

$$2\angle A + 2\angle C = 2(\angle A + \angle C) = 360^\circ$$

$$\angle A + \angle C = 180^\circ.$$

Hence opposite angles in any quadrilateral inscribed in a circle add up to 180° .

Now consider the situation where the measured HSA was 120° and see why the navigator needs to draw the triangle opposite to the ship's position, refer

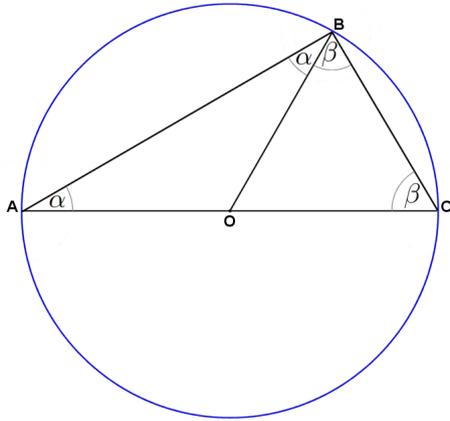


Figure 17: Angles $\alpha + \beta = 90^\circ$.

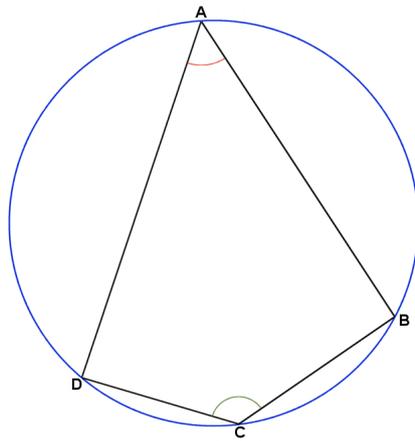


Figure 18: Opposite angles in a quadrilateral inscribed in a circle.

to Figure 19. Recall the method was started off by drawing the base of the triangle with angles of $120^\circ - 90^\circ = 30^\circ$. Following from the inscribed angles theorem, if the ship was anywhere on the major arc (landward) the navigator would measure a HSA of 60° . However the ship cannot be on that side as the measured $HSA = 120^\circ$. From the above theorem it was found that opposite angles in any quadrilateral inscribed in a circle add up to 180° . This is only true if the ship is on the minor arc since $60^\circ + 120^\circ = 180^\circ$.

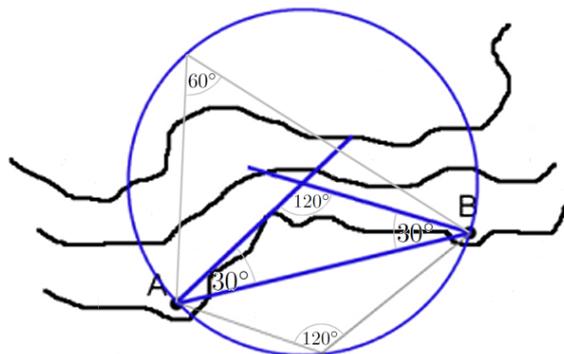


Figure 19: HSA over 90° .

References

- [1] Paul Gilster, “Centauri Dreams”, p204, Copernicus Books, 2004.
- [2] Captain Dick Gandy, “Australian Boating Manual”, p467, Ocean Publications, 2009.
- [3] Gerald B. Mills, “Analysis of Random Errors in Horizontal Sextant Angles”, Master’s thesis, Naval Postgraduate School Monterey CA, Sep. 1980.
- [4] John H. Mauldin, “Prospects for Interstellar Travel”, p167, American Astronautical Society, 1992.
- [5] Benedict, G. Fritz et al., "Interferometric Astrometry of Proxima Centauri and Barnard’s Star Using Hubble Space Telescope Fine Guidance Sensor 3: Detection Limits for Substellar Companions". The Astronomical Journal **118** (2): 1086–1100. <http://arxiv.org/abs/astro-ph/9905318>
- [6] Accessed online 23/06/2011 from Wikipedia: http://en.wikipedia.org/wiki/List_of_nearest_stars
- [7] Suneel I. Sheikh, Paul S. Ray et al., “Spacecraft Navigation Using X-Ray Pulsars”, Journal Of Guidance, Control, and Dynamics Vol. **29**, No. 1, January–February 2006. <http://pdf.aiaa.org/jaPreview/JGCD/2006/PVJA13331.pdf>
- [8] Eugene Mallove, Gregory Matloff, “The Starflight Handbook”, p179, Wiley Science Editions, 1989.

- [9] Robert Cornog, “Selected problems in Interstellar Navigation”, 16th annual meeting of the Institute of Navigation, 23-25th June 1960.
<http://www.scribd.com/doc/186932/Selected-Problems-in-Interstellar-Navigation>
- [10] James R. Wertz, “Interstellar Navigation”, Spaceflight 14, June 1972.
<http://www.scribd.com/doc/3535954/Interstellar-Navigation>
- [11] David G. Hoag, Walter Wrigley, “Navigation and Guidance in Interstellar Space”, Acta Astronautica Volume 2, Issues 5-6, June 1975.
<http://www.sciencedirect.com/science/article/pii/009457657590065X>
- [12] Robin M. Green, “Spherical Astronomy”, p40, Cambridge University Press, 1985.
- [13] Accessed online 25/06/2011 from NASA HEASARC online coordinate converter
<http://heasarc.gsfc.nasa.gov/cgi-bin/Tools/convcoord/convcoord.pl>
- [14] Marc G. Millis and Eric W. Davis, “Frontiers of Propulsion Science”, Chapter 2: Limits of Interstellar Flight Technology, Volume 227 Progress in Astronautics and Aeronautics, American Institute of Aeronautics and Astronautics, 2009.
- [15] Accessed online 26/06/2011 from the Physics FAQ: What would a relativistic interstellar traveller see?
<http://math.ucr.edu/home/baez/physics/Relativity/SR/Spaceship/spaceship.html>