. Applications of Euclidian Snyder geometry to the foundations of space time physics

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Abstract This thought experiment supposition will be raised, as a way to start investigations as to being able to choose either LQG, or string theory, as an initial space time template for emergent gravity. The author intends to explore the applications of deformed Euclidian space to questions as of the role of either string theory and/or LQG as to what degree the fundamental constants of nature are preserved between different cosmological cycles, and also the degree that gravity is an emergent field which is either partly/ largely classical, with extreme non linearity, or a far more quantum phenomenon.

Introduction

Recent papers in LQG which the author was exposed to in the 12 Marcel Grossman conference, presented that a big bounce replaced the singularity conditions Hawkings, Ellis, and others use. In particular, Marco Valerio Batistini, in a PRD article (2009) uses Snyder geometry to find a common basis in which to make a limiting approximation as to how to either derive either brane world, or LQG conditions for cosmological evolution. The heart of what Batistini works with is a deformed Euclidian Snyder space, when we use the $\hbar = c = 1$ units, obtaining then $[q, p] = i \cdot \sqrt{1-\alpha \cdot p^2} \Leftrightarrow \Delta q \Delta p \geq \frac{1}{2} \cdot \left| \left\langle \sqrt{1-\alpha \cdot p^2} \right\rangle \right|$. The LQG condition is $\alpha > 0$, and Brane worlds have, instead $\alpha < 0$. As Batistini indicated, in PRD, 2009, it is possible to obtain a string theory limit of $\Delta q \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p \right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$. We will use this result explicitly in the document as to differentiating between criteria as to information transfer from a prior to a present universe, as a way to distinguish, how to determine if minimum spatial uncertainty requirements for space time can distinguish between LQG, and brane world scenarios.

What is at stake can be parsed as follows.

How much information is in an individual Graviton? And how can one analyze normalized GW density in terms of gravitons?

Consider the following: i.e. we will put a first principle introduction. As to what can be said about gravitational wave density and its detection? It is useful to note that normalized energy density of gravitational waves, as given by Maggiore (2008)

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}} \right] \cdot \left(\frac{f}{1kHz} \right)^4 \quad (1)$$

Where n_f is a frequency based count of gravitons per unit cell of phase space. In terms of early universe nucleation, the choice of n_f may also depend upon interaction of gravitons with neutrinos. The supposition is, that eventually, Eq. (1) above could be actually modified with a change of

$$n_f \propto n_f[graviton] + n_f[neutrinos] \quad (2)$$

And also a weighted average of neutrino-graviton coupled frequency $\langle f \rangle$, so that for detectors

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f[graviton] + n_f[neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4 \quad (3)$$

The supposition to be investigated will be what if Eq (3) were true, how to actually measure it, and some consequences, especially if Fuller and Kishimoto in PRL, 2009 is legitimate. Among other things, the author is convinced that the spread out of the neutrino, as outlined by Fuller and Kishimoto , may be one of the factors leading to the graviton having, in later times a small mass, perhaps on the order of $m_{graviton} \propto 10^{-65}$ grams. The consequences of such a small rest mass are in figure 1

Consequences, which the author believes should be investigated

As brought up in Beckwith (2009), there is a signatory effect of gravitons which may have macro consequences, i.e. that of Gravitons contributing to the re acceleration of the Universe. In a re do of Alves et al. (2009) treatment of the Jerk calculation , i.e. re acceleration for the universe one billion years ago, Beckwith obtained for a brane world treatment of the Friedman equation leading to the following behavior Assume X is red shift, Z. q(X) is De - Celeration . Here we have a graph of de celeration parameter due to small $m_{graviton} \propto 10^{-65}$ grams, with q(Z) defined as below

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{4}$$

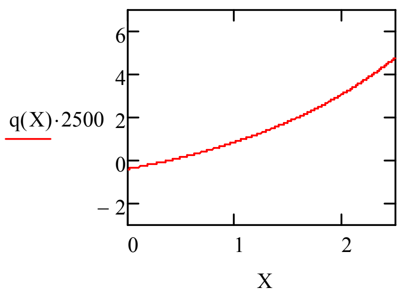

Figure 1: Re duplication of basic results of Alves, et al. (2009), using their parameter values, with an additional term of C for 'Dark flow' added, and corresponding to one KK additional dimensions.

The treatment of the jerk calculation follows what Beckwith (2009) did for a brane world plot and analysis of the jerk, q(Z), with Z set = X in the calculation above. This assumes that a small mass exists for the graviton, and that this is for a brane world treatment of the Friedman equation, along the lines of writing the density of a brane world having

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G\hbar^2}\right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right) \tag{5}$$

The above Eq (5) is assuming use of the following inequality, for a change in the HUP

$$\Delta q \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \tag{5a}$$

The supposition that the author is entertaining, is that the mass of the graviton is partly due to the stretching alluded to by Fuller and Kishimoto (2009), a supposition the author is investigating for a slight modification of a joint KK tower of gravitons as given by Maartens (2005) for DM which the author believes is promising. I.e. what if the following actually occurred? Assume that the stretching of neutrinos would lead to the KK tower of gravitons is, for when $\alpha < 0$, and higher dimensions are being used, of having:

$$m_n(Graviton) = \frac{n}{L} + 10^{-65} \text{ grams,} \tag{6}$$

As well as having the following way of calculating the JERK, i.e. If the following modification of the HUP is set, $\Delta q \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$, with the LQG condition is $\alpha > 0$, and Brane worlds have, instead, $\alpha < 0$. When $\alpha < 0$, we effectively have higher dimensional gravity, and a representation of gravitons in KK space. This leads to the following treatment of the JERK calculation: when Brane worlds imply $\alpha < 0$

What if a brane world, and KK tower for representing Gravitons were used in the friedman equation? What happens to the JERK calculation?

As can be related to, if we wish to look at string theory versions of the FRW equation , in Friedman-Roberson – Walker metric space, we can do the following de composition , with different limiting values of the mass, and other expressions, e.g. as a function of an existing cosmological constant

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{6a}$$

As well as

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{\left(\rho_{Total} + 3p_{Total}\right)}{6M_{Planck}^2} + \frac{\Lambda}{3} \tag{6b}$$

Not only this, if looking at the brane theory Friedman equations as presented by / for Randall Sundrum theory, it would be prudent working with

$$\dot{a}^2 = \left[\left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{\rho^2}{36M_{Planck}^2}\right)a^2 - \kappa + \frac{C}{a^2}\right] \tag{6c}$$

For the purpose of Randal Sundrum brane worlds, 6.c is what will be differentiated with respect to $d/d\tau$, and then terms from (6b) will be used, and put into a derivable equation which will be for a RS brane world version of $q = -\frac{\ddot{a}a}{\dot{a}^2}$. Several different versions of what q should be will be offered as far as what the time dependence of terms in 6c actually is. Note that Roy Maartens has written as of 2004 that KK modes (graviton) satisfy a 4 Dimensional Klein – Gordon equation, with an effective 4 dim mass, $m_n(Graviton) = \frac{n}{L}$, with $m_0(Graviton) = 0$, and L as the stated 'dimensional value' of higher dimensions. The value $m_0(Graviton) \sim` 10^{-65} - 10^{-60}$ gram in value picked is very small, but ALMOST zero. Grossing has shown how the Schrodinger and Klein Gordon equations can be derived from classical Lagrangians, i.e. using a version of the relativistic Hamilton-Jacobi- Bohm equation, with a wave functional $\psi \sim \exp(-iS/\hbar)$, with S the action, so as to obtain working values of for a tier of purported masses of a graviton from the equation , for 4 D of $\left[g^{\alpha\beta}\partial_\alpha\partial_\beta \xrightarrow[FLAT-SPACE]{} \nabla^2 - \partial_\tau^2\right]$, and $\left[\nabla^2 - \partial_\tau^2\right]\cdot\psi_n = m_n^2(graviton)\cdot\psi_n$ If one is adding , instead the small mass of $m_n(Graviton) = \frac{n}{L} + 10^{-65}$ grams, with $m_0(Graviton) \approx 10^{-65}$ grams,

Creating an analysis of how graviton mass, assuming branes, can influence expansion of the universe

Following development of (6c) as mentioned above, with inputs from Friedman eqns. To do this, the following normalizations will be used, i.e. $\hbar = c = 1$, so then

$$q = A1 + A2 + A3 + A4 \tag{6d}$$

Where

$$A1 = \frac{C}{a^3} \cdot \left[1 \Big/ \sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)} \right] \tag{6e}$$

$$A2 = -\left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \Big/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right] \tag{6f}$$

$$A3 = -\frac{1}{2} \cdot \left[\frac{(d\rho/d\tau)}{3M_4^2} + \frac{(d\Lambda_4/d\tau)}{3} + \frac{1}{18} \cdot \frac{\rho \cdot (d\rho/d\tau)}{M_P^2} \right] \Big/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{3/2} \tag{6g}$$

$$A4 = \frac{\kappa}{a^3} \cdot \left[\frac{(da/d\tau)}{3} \right] \Big/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{3/2} \tag{6h}$$

Furthermore, if we are using density according to whether or not 4 dimensional graviton mass is used, then

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 - \left[\frac{m_g c^6}{8\pi G \hbar^2} \right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2} \right) \tag{6i}$$

So, then one can look at $d\rho/d\tau$ obtaining

$$d\rho/d\tau = -\left(\frac{\dot{a}}{a} \right) \cdot \left[3 \cdot \rho_0 \cdot \left(\frac{a_9}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g c^6}{8\pi G \hbar^2} \right) \right] \tag{6j}$$

Here, use, $\left(\frac{\dot{a}}{a} \right) = \sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)}$, and assume eqn. (6i) covers ρ, then

If $\hbar \equiv c \equiv 1$,

$$d\rho/d\tau = -\sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)} \cdot \left[3 \cdot \rho_0 \cdot \left(\frac{a_9}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \tag{6k}$$

Now, if, to first order, $d\Lambda_4/d\tau \sim 0$ and, also, we neglect Λ_4 as of being not a major contributor

$$d\rho/d\tau \cong -\sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right)} \cdot \left[3 \cdot \rho_9 \cdot \left(\frac{a_0}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \tag{6l}$$

$$A3 \cong \frac{1}{2} \left(\cdot \left[\frac{1}{3M_4^2} + \frac{1}{18} \cdot \frac{\rho \cdot}{M_P^6} \right] \Big/ \left[\frac{C}{a^4} - \frac{\kappa}{a^2} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{1/2} \right) \cdot \left[3 \cdot \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \cdot \tag{6m}$$

Also, then, set the curvature equal to zero. i.e. $\kappa = 0$. So then $A4 = 0$, and

$$A3 \cong \frac{1}{2} \left(\cdot \left[\frac{1}{3M_4^2} + \frac{1}{18} \cdot \frac{\rho \cdot}{M_P^6} \right] \Big/ \left[\frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_P^6} \right) \right]^{1/2} \right) \cdot \left[3 \cdot \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 + 4 \cdot \left(\frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left(\frac{m_g}{8\pi G} \right) \right] \cdot \tag{6n}$$

Then

$$A2 \cong -\left(\frac{\rho}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right) \Bigg/ \left[\frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\right] \qquad (6o)$$

$$A1 \cong \frac{C}{a^3}\cdot\left[1\Bigg/\sqrt{\frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)}\right] \qquad (6p)$$

Pick, here, $\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g}{8\pi G}\right]\cdot\left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right)$, after $\hbar = c = 1$, and also set

$$\Phi(\rho, a, C) = \frac{C}{a^4} + \left(\frac{\rho}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right) \qquad (6q)$$

$$A3 \cong \frac{1}{2}\left(\cdot\left[\frac{1}{3M_4^2} + \frac{1}{18}\cdot\frac{\rho\cdot}{M_P^6}\right]\Bigg/[\Phi(\rho,a,C)]^{1/2}\right)\cdot\left[3\cdot\rho_0\cdot\left(\frac{a_0}{a}\right)^3 + 4\cdot\left(\frac{a^4}{14} + \frac{a^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right] \qquad (6r)$$

$$A2 \cong -\left(\frac{\rho}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho^2}{M_P^6}\right)\Bigg/[\Phi(\rho,a,C)] \qquad (6s)$$

$$A1 \cong \frac{C}{a^3}\cdot\left[1\Big/\sqrt{\Phi(\rho,a,C)}\right] \qquad (6t)$$

For what it is worth, the above can have the shift to red shift put in by the following substitution. I.e. use $1 + z = a_0 / a$. Assume also that C is the dark radiation term which in the brane version of the Friedman equation scales as a^{-4} and has no relationship to the speed of light. a_0 Is the value of the scale factor in the present era, when red shift z =0, and $a \equiv a(\tau)$ in the past era, where τ is an interval of time after the onset of the big bang. $(a_0 / a)^3 = (1+z)^3$, and $a \equiv a_0 / (1+z)$, Then

$$\rho(z) \equiv \rho_0\cdot(1+z)^3 - \left[\frac{m_g}{8\pi G}\right]\cdot\left(\frac{a_0^4}{14\cdot(1+z)^4} + \frac{2a_0^2}{5\cdot(1+z)^2} - \frac{1}{2}\right) \qquad (6u)$$

$$A1(z) \cong \frac{C\cdot(1+z)^3}{a_0^3}\cdot\left[1\Big/\sqrt{\Phi(\rho(z), a_0/(1+z), C)}\right] \qquad (6v)$$

$$A2(z) \cong -\left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho(z)^2}{M_P^6}\right)\Bigg/[\Phi(\rho(z), a_0/(1+z), C)] \qquad (6w)$$

$$A3(z) \cong \frac{1}{2}\left(\cdot\left[\frac{1}{3M_4^2} + \frac{1}{18}\cdot\frac{\rho(z)\cdot}{M_P^6}\right]\Bigg/[\Phi(\rho(z), a_0/(1+z), C)]^{1/2}\right)\cdot$$
$$\left[3\cdot\rho_0\cdot(1+z)^3 + 4\cdot\left(\frac{a_0^4/(1+z)^4}{14} + \frac{a_0^2/(1+z)^2}{5}\right)\cdot\left(\frac{m_g}{8\pi G}\right)\right] \qquad (6x)$$

$$\Phi(\rho(z), a_0/(1+z)), C) = \frac{C\cdot(1+z)^4}{a_0^4} + \left(\frac{\rho(z)}{3M_4^2} + \frac{1}{36}\cdot\frac{\rho(z)^2}{M_P^6}\right) \qquad (6y)$$

So, for $4 < z \leq 0$, i.e. not for the range, say $z \sim 1100$ 380 thousand years after the big bang, it would be possible to model, here

$$q(z) = A1(z) + A2(z) + A3(z) \qquad (6z)$$

Easy to see though, that to first order, $q(z) = A1(z) + A2(z) + A3(z)$ would be enormous when $z \sim 1100$, and also that for Z =0, $q(0) = A1(0) + A2(0) + A3(0) > 0$.Negative values for eqn. (6z) appear probable at about $z \sim 1.5$, when eqn. (6a1) would dominate, leading to $q(z \sim 1.5))$ with a negative expression/ value . The positive value conditions rely upon, the C dark radiation term,

Final result. The JERK calculation can be done, for the braneworld case, and KK gravitons. However, it also is a major problem as to explain exactly what may have contributed to the graviton having a slight mass which contravenes the correspondence principle. We will get to this in the last part of this article.

Now what can one expect with LQG condition with respect to the HUP, with $\alpha > 0$?

What happens, is that most of the complexity drops out, and above all, the following

When using the LQG condition $\alpha > 0$, in Snyder geometry modified HUP

The claim is that almost all the complexity is removed, and what is left is a set of equations similar to the tried and true $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3}$. To get an idea of what happens with LQG versions of the Friedman equation, one can look at Taveras's (2008) treatment of the Friedman equations, and he obtains, to first order, if ρ is a scalar field DENSITY.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3} \cdot \rho \qquad (7a)$$

As well as

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{2 \cdot \kappa}{3} \cdot \rho \qquad (7b)$$

The interpretation of ρ as a scalar field DENSITY, and if one does as Aves et al did, i.e work with flat space, with k=0, in $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3}$, as well as $\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G\hbar^2}\right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right)$

The sticking point in all of this is to interpret the role of ρ. In the purported LQG version brought up by Taveras's (2008) article, the $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3} \cdot \rho$ may be re written to be, as follows, i.e. if conjugate momentum is in many cases, 'almost' , or actually a constant, then to good effect

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv \frac{\kappa}{6} \cdot \frac{p_\phi^2}{a^6} \qquad (7c)$$

This is assuming that the conjugate dimension, in this case has connectivity with a quantum connection specified via an effective scalar field, ϕ, obeying the relationship

$$\dot{\phi} = -\frac{\hbar}{i} \cdot \frac{\partial}{\partial \cdot p_\phi} \qquad (7d)$$

For what it is worth, it is appropriate to consider, to first order that Alves et al's program can probably be carried out, especially if Eq (7d) is, true, but this is a matter of subjective interpretation of Eq (7d) above. The main point being though that there should be an interpretation as to what the graviton actually is, which is in common with regards to both the LQG condition $\alpha > 0$, and the Brane world case, when $\alpha < 0$

What is in common, with both models as far as 4 dimensional representations of the Graviton. For both $\alpha < 0$ and $\alpha > 0$

Two hypothesis, to consider. First is that there is an interaction between neutrinos, and gravitons. Bashinsky [6] (2005), gave details in his article about an alleged modification of density fluctuations via neutrino-graviton interactions. A far more radical hypothesis, is that there are a few 'stretched neutrinos', to contend with Bashinsky, which may span many light years, and these stretched neutrinos may affect gravitons, i.e. as implied by Bashinsky may lend credence to George Fuller and Chad Kishimoto's PRL supposition that as the "universe expanded, the most massive of these states slowed down in the relic neutrinos, stretching them across the universe". If there is a coupling between gravitons, and neutrinos, as speculated by Bashinsky, the author believes the result will bring into question the correspondence principle which is usually used to require gravitons to be spin 2, with zero mass. This will in the latter part of the manuscript.

Statement as to stretching of the neutrino, and its probable effect upon graviton wave lengths

Assume that with stretching of the neutrino, and graviton neutrino coupling with zeroth order value of $m_0(Graviton) \approx 10^{-65}$ grams as a consequence **of at least a few** of the neutrino-gravitons obeying density fluctuation modified , according to Bashinsky [6] $\left[1 - 5 \cdot (\rho_{neutrino}/\rho) + \vartheta([\rho_{neutrino}/\rho]^2)\right]$ according to Bashinsky [6] (2005), as well as having equivalent neutrino-graviton wave lengths becomes, instead the same order of magnitude as the matter wave values of neutrinos, with, initially

$$
\begin{aligned}
& m_{graviton}\Big|_{RELATIVISTIC} < 4.4 \times 10^{-22} h^{-1} eV / c^2 \\
& \Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} meters
\end{aligned}
\tag{7c}
$$

A few select gravitons, coupled to almost infinite wave length stretched neutrinos would lead to Eq(7) , if they were sufficiently large, as of

$$
\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 10^4 meters \text{ or larger} \tag{8}
$$

The correspondence principle, and t'Hooft's supposition of 'Deterministic QM' as applied to gravitons

What to look into? The author frankly is suggesting that the stretch out of the graviton implied by Eq (8) above may be a sign that the correspondence principle, used by string theorists and others, as a way to insist that the graviton be of zero mass, may have to be amended. After presenting why the author states this, the author will suggest a mechanism for replacement of the correspondence principle, which the author believes is consistent with t'Hoofts deterministic quantum mechanics. The final part to this will be in making a suggestion as to what 'information' a particle like the graviton may carry

What can be stated about the 'Correspondence principle' and its connections as to gravitons? Rothman and Boughn wrote out a well considered article (2006) arguing that it is unrealistic given current detector technology to envision gravitons ever being measured. The author will summarize Rothman and Bohns findings with a statement as to what he views as a weak point in their presentation which may be amendable to investigations, and to from there to lay out as to how and why the graviton may carry physical information. Finally, upon doing this, the author will look into what a graviton 'construction' with a tiny mass may entail as to instanton-anti instantons, and its relationship to t'Hoofs deterministic quantum mechanics. To recap what they are suggesting, it is useful to note the formula 2.1 which will be reproduced here, as , when $\tilde{n}$ is the purported numerical density of 'detector - particles', σ is the detector cross area, and $\tilde{\lambda}$ is the mean 'distance' a graviton would have to travel, i.e. look at

$$\tilde{n} \cdot \sigma \cdot \tilde{\lambda} \geq 1 \tag{9}$$

The author does not quarrel with the basic physics of Eq (9) above. Assume though that, for an instant, that the cross sectional area for a graviton would have to be larger 'than the diameter of Jupiter. $\tilde{n}$ is given by Rothman and Bohn, to be $\tilde{n} \equiv M_{\det} / \lfloor m_{proton} \cdot V_{\det} \rfloor$. I.e. this is for a detector with gravitons interacting with some version of hydrogen, with $M_{\det}$ the 'mass' of the detector, and with $V_{\det}$ the purported volume of the detector. Also, m_{proton} is the mass of protons in the detector which the gravitons may interact with . If one wishes to get a detector, then if one is using such technology, then the figures for the volume $V_{\det}$ being Jupiter sized do look very reasonable.

Rothman and Bohn go further, re writing Eq (9) as implying the following for a numerical total of gravitons detected during the life time of an experiment as, when $L_{graviton-production}$ is the luminosity of graviton production, R as the purported distance the graviton would travel, while setting up the right hand side with $\frac{A_{\det} \cdot \tau_B}{\in_{graviton}} \equiv$ (detector cross sectional area* time of process for the graviton source to be operating) / graviton energy . Also, $\tau_B \leq \frac{M_{graviton-generating-source}}{L}$. Here $M_{graviton-generating-source}$ is the relative mass of the graviton producing source, and L the luminosity of the source. The bounds for τ_B effectively get thrown through the window, if the graviton production 'site' is relic early universe gravitons, instead of what is cited, i.e. for non zero graviton energies, $\in_{graviton}$

$$N_{graviton=\exp-lifetime} \equiv \left[\frac{L_{graviton-production}}{4\pi R^2}\right] \cdot \left[\frac{A_{\det} \cdot \tau_B}{\in_{graviton}}\right] \tag{9a}$$

Rothman and Bohn give a very coherent argument that for neutron stars, black holes and the like that Eq(9a) has an upper bound of $N_{graviton=\exp-lifetime} \approx 10^{-5}$. The author, in lieu of what may be stated as to relic gravitons categorically states that the total source luminosity L versus luminosity of graviton production process of the source $L_{graviton-production}$ may be very different from the ratio values given by Rothman, and Bohn, of $L_{graviton-production} / L = f_{graviton} \sim .01 - .02$. If the $f_{graviton}$ is over ten times larger, plus the life time $\tau_B \leq \frac{M_{graviton-generating-source}}{L}$ >> life time of graviton production from black holes with a larger time due to having a value of $M_{graviton-generating-source} >> 10^{15}$ grams , with 10^{15} grams ~ mass of a black hole, then $N_{graviton=\exp-lifetime} \approx 10^{-5}$ may be way too small. Furthermore, if the stretched neutrino hypothesis, with coupling to the graviton occurs, then , assuming that there is at a minimum $\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 10^4\, meters$, instead of $\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8}\, meters$, one even with a non giant planet sized detector would see an effective $N_{graviton=\exp-lifetime} >> N_{Rothman-calculated-graviton-\exp er-lifetime} \approx 10^{-5}$, perhaps as high as nearly unity. And this primarily due to re calibration of the different input coefficients. This is, however, using very old GW/ Graviton detector technology. It will lead up to the author questioning the standard correspondence principle used to characterize gravitons, and to mention an alternative as to having Gravitons with spin 2, but perhaps masses slightly larger than zero. Eventually , this will lead to considering the correspondence principle, as well as t'Hoofts 'deterministic' quantum mechanics as a way to consider the nature of gravitons.

Can relic GW be observed?

The main problem in these assumptions is in the specification that one is looking into relic GW. While this is a huge problem, there does exist an argument, which the author has borrowed from Dr. Fangyu Li, which implies that relic GW, and by implicit assumption, gravitons are not ruled out. From Dr. Li, personal notes. The assumptions so being as stated that with careful calibration, there is a way to obtain measurable relic GW, and also, possibly, graviton measurements.

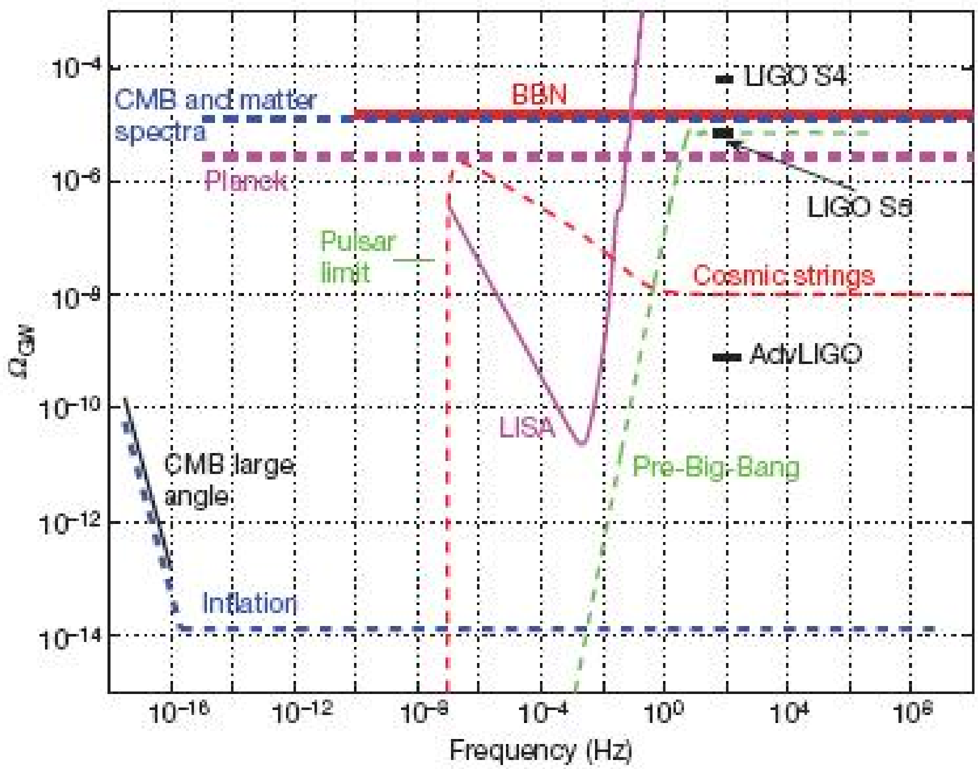

Figure2. This figure is from B.P. Abbott, et.al., Nature 460, 991 (2009). The figure shows the relation between Ω_g and frequency. The curve of the pre-big-bang models shows that Ω_g of the relic GWs is almost constant $\sim 6.9\times10^{-6}$ from 10^{-1}Hz to 10^{10}Hz. Ω_g of the cosmic string models is about 10^{-8} in the region 1Hz to 10^{10}Hz, its peak value region is about 10^{-7}-10^{-6}Hz.

According to more accepted estimation, the upper limit of Ω_g on relic GWs should be smaller than 10^{-5}, while very recent date analysis [B.P. Abbott et al, Nature 460, 990-993 (2009)] shows, the upper limit of Ω_g should be 6.9×10^{-6}. By using such parameters, Dr. Li estimates the spectrum $h(\nu_g,\tau)$ and the r.m.s. amplitude h_{rms}. The relation between Ω_g and the spectrum $h(\nu_g,\tau)$ is often expressed as (L. P. Grishchuk, Lect. Notes Phys. 562, 167 (2001))

$$\Omega_g \approx \frac{\pi^2}{3}\left(\frac{\nu}{\nu_H}\right)^2 h^2(\nu,\tau), \tag{9b}$$

so

$$h(\nu,\tau) \approx \frac{\sqrt{3\Omega_g}}{\pi}\frac{\nu_H}{\nu}, \tag{9c}$$

Where $\nu_H = H_0 \quad 2\times10^{-18}\,Hz$, it is the present value of the Hubble frequency. From Esq. (9b) and (9c), we have

(a) If $\nu = 10GHz, \qquad h = 10^{-30}$, then $\Omega_g = 8.3\times10^{-5}$, (9d)

If $\nu = 10GHz, \qquad h = 10^{-31}$, then $\Omega_g = 8.3\times10^{-7} < \Omega_{g\max}$, (9e)

If $\nu = 10GHz, \qquad \Omega_g = \Omega_{g\max} = 6.9\times10^{-6}$, then $h = 2.9\times10^{-31}$ (9f)

(b) If $\nu = 5GHz, \qquad h = 10^{-30}$

Then $\quad \Omega_g = 2.1\times10^{-5}$ (9g)

If $\nu = 5GHz, \qquad$ h=10^{-31} then $\Omega_g = 2.1\times10^{-7} < \Omega_{g\max}$ (9h)

If $\nu = 5GHz, \qquad \Omega_g = \Omega_{g\max} = 6.9\times10^{-5}$, then $h = 5.7\times10^{-31}$ (9i)

Such values of $\nu = 5GHz, \qquad \Omega_g = \Omega_{g\max} = 6.9\times10^{-5}$, would be essential, in being able to ascertain the possibility of detection of GW from relic conditions, whereas Ω_g being referenced here, is with regards to $\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f)\cdot\Omega_{gw}(f)$. Furthermore, $h_0^2\Omega_{gw}(f) \cong \frac{3.6}{2}\cdot\left[\frac{n_f[graviton]+n_f[neutrino]}{10^{37}}\right]\cdot\left(\frac{\langle f\rangle}{1kHz}\right)^4$ if one is looking at a very narrow range of frequencies, that to first approximation do a comparison between what is brought up between an integral representation of Ω_g and $h_0^2\Omega_{gw}(f)$. Note also that Dr. Li suggests, as an optimal upper frequency to investigate $\nu_g = 2.9GHz$ (see below, suggestion 1-3), $\Delta\nu$=3KHz, then

$$h \approx \frac{\sqrt{3\Omega_g}}{\pi}\frac{\nu_H}{\nu_g} \approx 1.0\times10^{-30}, \tag{9j}$$

and

$$h_{rms} = \sqrt{\langle h^2\rangle} \approx h\left[\frac{\Delta\nu}{\nu_g}\right]^{\frac{1}{2}} \approx 1.02\times10^{-33} \tag{9k}$$

These are upper values of the spectrum, and should be considered as preliminary. Needed in this mix of calculations would be a way to try to ascertain a set of input values into the numerical count of $h_0^2\Omega_{gw}(f) \cong \frac{3.6}{2}\cdot\left[\frac{n_f[graviton]+n_f[neutrino]}{10^{37}}\right]\cdot\left(\frac{\langle f\rangle}{1kHz}\right)^4$. If there is roughly a 1-1 correspondence between graivitons, and neutrios (highly unlikely) , then $h_0^2\Omega_{gw}(f) \sim 3.6\cdot\left[\frac{n_f[graviton]}{10^{37}}\right]\cdot\left(\frac{\langle f\rangle}{1kHz}\right)^4$.

The counting of the number of gravitons, per cell space, should also be done in terms of considering what Buoanno wrote, for Les Houches, namely that if one looks at BBN, that the following bound should be considered . Here, Buoanno is using $f > f_* = 4.4 \times 10^{-9}\, Hz$, and a reference from Kosowoky, Mack, and Kahniashhvili (2002) as well as Jenet et al (2006)

$$h_0^2 \Omega_{gw}(f) \le 4.8 \times 10^{-9} \cdot (f / f_*)^2 \qquad (9.1)$$

Using this upper bound, if one insist upon assuming , as Buoanno (2006) does, that the frequency today depends upon the relation

$$f \equiv f_* \cdot [a_* / a_0] \qquad (9.m)$$

The problem in this is that the ratio $[a_* / a_0]$ << 1, assuming that a_0 is "todays" scale factor. In fact, using this estimate, Buoanno comes up with a peak frequency value for relic/ early universe values of the electro weak era generated GW/ graviton production of

$$f_{Peak} \cong 10^{-8} \cdot [\beta / H_*] \cdot [T_* / 16GeV] \cdot [g_* / 100]^{1/6}\, Hz \qquad (9.0)$$

By conventional cosmological theory, limits of g_* are on the upper limit of 100-120, at most, according to Kolb and Turner (1991). $T_* \sim 10^2\, GeV$ is specified for nucleation of a bubble, as a generator of GW. Early universe models with $g_* \sim 1000$ or so are not in the realm of observational science, YET, as was told the author by Hector De La Vega at the Colmo, Italy astro particle physics school , July 2009. Furthermore, the range of accessible frequencies as given by Eq (9.o) is in sync with regards to $h_0^2 \Omega_{gw}(f) \sim 10^{-10}$ for peak frequencies with values as of 10 mHz. The net affect of such thinking is to rule that all relic GW are inaccessible. If one looks at Figure 2, $\Omega_{GW} > 10^{-6}$ for frequencies as high as up to 10^6 Hertz, but this runs up against what was declared by Turner and Wilzenk (1990) that inflation will terminate with observable frequencies in the range of 100 or so Hertz. The problem is though, that after several years of LIGO, no one has observed such a GW signal. Either from the early universe, from Black Holes, or any other source, yet. About the only way one may be able to observe a signal , for GW and/or gravutons may be to consider how to obtain a numerical count of gravitons and/or neutrinos for $h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f [graviton] + n_f [neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4$. And this leads to the question of how to account for a possible mass/ information content to the graviton

Can the graviton have a small mass? Issue as of embedding the laws of QM w.r.t. gravitons within a non linear theory.

Recently, an alternative to usual space time Gravitation theories was proposed , HoYYava gravity, and has been obtaining reviews in the Perimeter Institute, among other places. Robert Brandenberger in (2009) also modeled this new theory in terms of the early universe, with the claim that there was a matter bounce instead of standard inflation. This theory, ironically depends upon a chaotic inflationary potential $V(\phi) = (1/2) \cdot m^2 \phi^2$ for its pre bounce conditions, and uses 'dark radiation' for obtaining a 'bounce', and Shinji Mukohyama (2009) has presented what he calls "scale-invariant, super-horizon curvature perturbations" . Both Mukohyama, and Brandeberger accept scale free 'perturbations' so long as the contraction phase does use 'quantum vacuum fluctuations', and the author is waiting to see if HoYYava gravity develops or is provided with a mechanism to transfer energy to the standard model of comology predictions as to the radiation and matter eras. By way of contrast what the author will attempt to do is to with gravitons is far more modest, i.e. referencing the construction of a graviton in terms of instanton- anti instantons, and asking if a composition of a graviton as such an 'object' as a composition of such kink- anti kinks can be tied in with 'tHoofts "deterministic quantum mechanics"

Beginning the analysis, the author will review, briefly what he did with CDW in $1+\varepsilon^{+}$ dimensions , and then reference the chances for doing the same for 4 dimensions for gravitons,. Finally, closing with a description if the graviton can carry information, and what this says about graviton mass.

Brief review of S-S' in CDW, and its relevance to higher dimensional 'objects'

Seen below is a representation of CDW and instantons

The author is briefly presenting his density wave instanton- anti instanton construction for CDW, which has classical analogies, and then making a reference to such constructions in instanton type models in cosmology. As presented in Beckwith's PhD dissertation, kink- anti kink models have a classical analogy with

$$\phi_{\pm}(z,\tau) = 4\cdot\arctan\left(\exp\left\{\pm\frac{z+\beta\cdot\tau}{\sqrt{1-\beta^2}}\right\}\right) \tag{9.p}$$

Which is a solution to

$$\frac{\partial^2\phi(z,\tau)}{\partial\tau^2} - \frac{\partial^2\phi(z,\tau)}{\partial z^2} + \sin\phi(z,\tau) = 0 \tag{9.q}$$

A tunneling Hamiltonian version of such solutions had the following formalism, namely a Gaussian wave functional with

$$\Psi_{i,f}\ [\phi(\mathbf{x})]\Big|_{\phi=\phi_{ci,cf}} = c_{i,f}\cdot\exp\left\{-\int d\mathbf{x}\ \alpha\left[\phi_{Ci,f}(\mathbf{x}) - \phi_0(\mathbf{x})\right]^2\right\}, \tag{9.r}$$

Furthermore, this allowed us to derive, as mentioned in another publication a stunning confirmation of the fit between the false vacuum hypothesis and data obtained for current – applied electrical field values graphs (I-E) curves of experiments initiated in the mid 1980s by Dr. John Miller, et al. (1986) which lead to the modulus of the tunneling Hamiltonian being proportional to a current, with E_T a threshold pinning field

$$I \propto \widetilde{C}_1\cdot\left[\cosh\left[\sqrt{\frac{2\cdot E}{E_T\cdot c_V}} - \sqrt{\frac{E_T\cdot c_V}{E}}\right]\right]\cdot\exp\left(-\frac{E_T\cdot c_V}{E}\right) \tag{9.s}$$

The phase as put in eqn. (9.r) was such that it had the following graphical representation, and it is indicative of what instanton physics can be used for, i.e. this is not a substitute for a well thought out treatment of instantons which will be connected with appropriate metrics in GR. Figure 5 in particular , is a template as to how the author will model a pop up effect of a S-S' pair, in a quantum mode, using S and S' pairs.

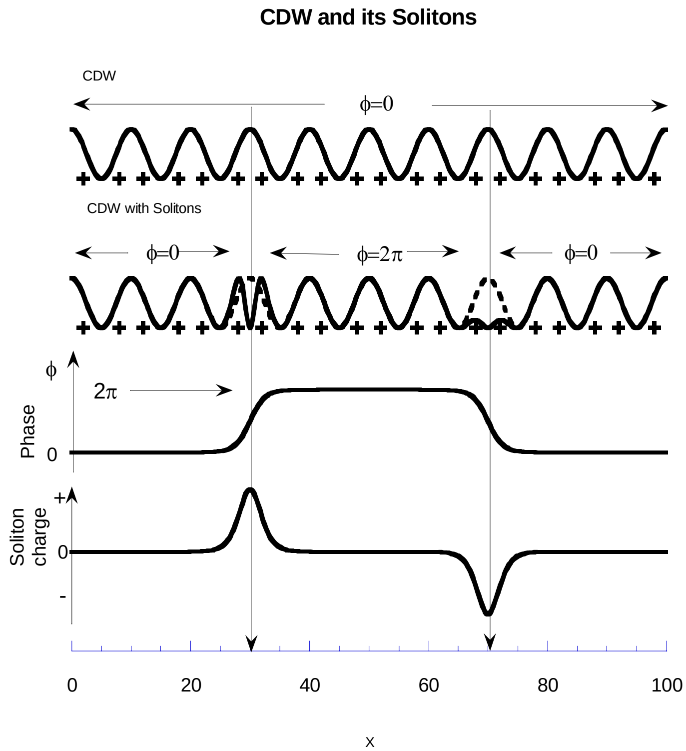

Figure 3 : Typical results of density wave physics instanton-anti instanton pairs. As from Beckwith(2001), and Beckwith (2006)

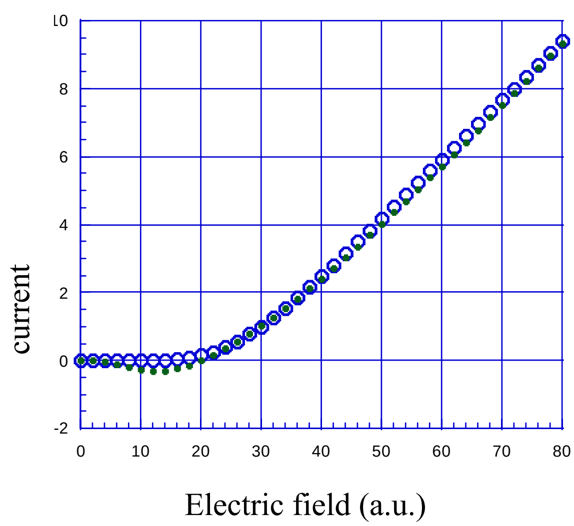

Figure 4 : Results of applying Eq (9.s) as opposed to $I \propto G_P \cdot (E - E_T) \cdot \exp\left(-\frac{E_T}{E}\right)$ if E > E_T , and setting I = 0 if $E \le E_T$. In figure 4, the blue dots represent Eq. (9.s) whereas the black dots represent uniformly applying the non zero plot for electric fields as given by the Zenier plot approximation.

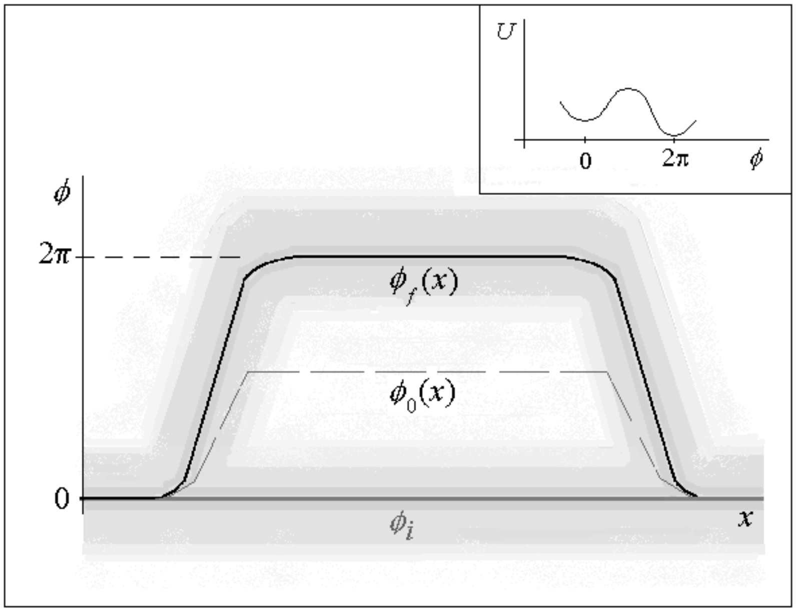

Figure 5. The pop up effects of an intanton- anti instanton in Euclidian space. Taken from Beckwith(2001)

In order to connect with GR, one needs to have a higher dimensional analog of $\phi_{\pm}(z,\tau) = 4 \cdot \arctan\left(\exp\left\{ \pm \frac{z + \beta \cdot \tau}{\sqrt{1-\beta^2}} \right\} \right)$ which is consistent with regards to space time metrics, a topic which will be presented in brief, in the next section.

Brief introduction to instantons in GR, which are consistent with respect to space time metrics

The best, physically consistent models of GR admissible solitons appears to be given by Belunski, and Verdaguer, 2001, in work which ties in the instanton formulation for gravitation to specific metrics in space time physics. In addition, the author will reference done by Givannini, 2006, which gives a kink- anti kink construction, which the author says is similar to what the author was doing with CDW, in order to obtain a model of the graviton. How this graviton, as a kink- anti kink construction fits with QM, and the usual comments as to a correspondence mass zero values for the graviton will be brought up with t'Hoofs version of a Deterministic QM , i.e. a highly non linear structure embeds quantum physics, w.r.t. the graviton.

Belunski, and Verdaguer, 2001, gave an example of how to match conditions of the instanton with space time metrics, and Givannini has another example of a kink- anti kink construction involving instantons which will be commented upon.

The author also has a paper which claims that instantons initially travel at low velocity, and which only reach speeds up to nearly light speed in nearly infinite distance travel. Aside from the CDW example, the author is convinced that the only way to avoid such conundrums is to have a kink- anti kink construction for the graviton.. The basic idea is how to generalize figure 5, which was in the authors PhD dissertation, in 2001.

Another argument as to how information can be attached to the graviton will be the closing part of this discussion., based upon a presentation which the author made in Chongquing University, November 2009.

Belunski, and Verdaguer, 2001, give an example of how to generalize an instanton from the metric g, with

$g \equiv diag\{t \cdot \exp(\phi), t \cdot \exp(-\phi)\}$ when put into the Einstein equations leads to obtaining a two part solution, which is further generalized on their page 198 to read, as

$$\phi \equiv d \cdot \ln t + \sum_{k=1}^{s} h_k \ln(\mu_k / t) \tag{9.s}$$

The 2nd part of this equation roughly corresponds to $\phi_+(z,\tau) = 4 \cdot \arctan\left(\exp\left\{\frac{z + \beta \cdot \tau}{\sqrt{1-\beta^2}}\right\}\right)$. Further work by Belunski, and Verdaguer, 2001 yields instanton- anti instanton solutions which are elaborations of Eq (9.s) above, which is in the case of instantons applied to cosmology can be justified by the warning given by J. Ibanez, and E. Verdaguer (1985) that instantons by themselves travel at speeds very much smaller at the speed of light, in cosmology and reach peak velocities only much later on, at 'infinite; distance from a source. To put it mildly, that is not going to work. Aside from other considerations, the warning by J. Ibanez, and E. Verdaguer (1985) is one of the reasons why the author is seeking higher dimensional versions of Figure 5 above, as a pop up version of when instantons can come into space time.

More on that later. It is important now to reference what was presented by Givannini, 2006 , namely from a least action version of the Einstein – Hilbert action for 'quadratic' theories of gravity involving Euler-Gauss-Bonnet, a scalar field which has the form of, when w in this case roughly corresponds to a time variable. Then.his equation 6 corresponds to

$$\phi = \tilde{v} + \arctan\left((bw)^{\nu}\right) \tag{9.t}$$

Givannini's (2006) manuscript also has a representation of Eq (9.t) as a kink, and makes references to an anti kink solution, in his figure 1. Furthermore the over lap between Eq. (9.t) and $\phi_+(z,\tau) = 4 \cdot \arctan\left(\exp\left\{\frac{z + \beta \cdot \tau}{\sqrt{1-\beta^2}}\right\}\right)$ is in its own way very obvious. If the two equations are similar, and if $\arctan\left((bw)^{\nu}\right)$ overlaps in behavior with $\sum_{k=1}^{s} h_k \ln(\mu_k / t)$ in certain limits, as far as the formation of an instanton, the problem is amendable to analysis. Furthermore, is considering what role a kink-anti kink model of an instanton would arise from. If a graviton is a kink-anti kink combination, arising from, in part a 5 dimensional line element

$$dS^2 = a(w) \cdot \left[\eta_{uv} dx^u dx^v - dw^2\right] \tag{9.u}$$

Then how the graviton may be nucleated in this space is important, and involves the transfer of information. How that information will be embedded and transferred to an instanton- anti instanton configuration will be the next topic of discussion of this manuscript . Before doing this, the geometry of where the instanton- anti instanton pair arises, in the beginning of inflation needs to be addressed.

Dropping in of 'information' to form an instanton- anti instanton pair, and avoiding the cosmological singularity via the 5th dimension?

As the author brought up in Chongquing, there is NO reliable way to reconcile the formation of an instanton-anti instanton pair, and to avoid having an instanton as an example disrupted by a cosmological singularity. What the author proposed, as a graphical example was to consider what if there was, in higher dimensions than just four dimensions, a transfer of region of space for when an instanton – anti instanton could pop up

This lead to the author writing up in Chonquing the region about the singularity definable via a ring of space – time about the origin, but not over lapping it, with a time dimension defined via

$$\Delta t \equiv 10^{\beta} \cdot t_{Planck} \tag{9.v}$$

The exact uncertainty principle, in five dimensions is open to discussion, but the author envisioned, as an example, a five dimensional version of $\Delta E \Delta t \geq \hbar$. IF one takes the tiny mass specified via the $m_{graviton} \propto 10^{-65}$ grams , and make energy equivalent to mass, then the small mass , times the speed of light, squared, in the case of instanton-anti instanton (kink – anti kink) would be the S-S' pair for the instanton nucleated about the cosmic singularity

The classical treatment of this problem would be in assuming that the transfer of information from a prior universe, to our own went through a 5th dimension, with the cosmic singularity, a 4th dimensional artifact. I.e. that the information was dropped via a 5th dimensional conduit to a 4th dimensional space time, in order to form a small mass for the graviton, i.e. $m_{graviton} \propto 10^{-65}$ grams, with, say a top value for the graviton mass, after acceleration being $m_{graviton} \propto 10^{-61}$ grams, I.e. abrupt acceleration making the graviton mass **at least** 10^{4} times heavier than initially. To understand why the author is investigating such a supposition, a brief review of typical field theories involving 'massive' gravitons and the limit $m_{graviton} \to 0$ will be presented, with a description of why these effects may lead to semi classical approximations.

Massive Graviton field theories, and the limit $m_{graviton} \to 0$

As given by M. Maggiore (2008), the massless equation of the Graviton evolution equation takes the form

$$\partial_{\mu} \partial^{\varpi} h_{\mu\nu} = \sqrt{32\pi G} \cdot \left(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T_{\mu}^{\mu} \right) \qquad (9.w)$$

When $m_{graviton} \neq 0$, the above becomes

$$\left(\partial_{\mu} \partial^{\varpi} - m_{graviton} \right) \cdot h_{\mu\nu} = \left[\sqrt{32\pi G} + \delta^{+} \right] \cdot \left(T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T_{\mu}^{\mu} + \frac{\partial_{\mu} \partial_{\nu} T_{\mu}^{\mu}}{3 m_{graviton}} \right) \qquad (9.x)$$

The mis match between these two equations, when $m_{graviton} \to 0$, is largely due to $m_{graviton} h_{\mu}^{\mu} \neq 0$ as $m_{graviton} \to 0$, which is in turn due to setting $m_{graviton} \cdot h_{\mu}^{\mu} = -\left[\sqrt{32\pi G} + \delta^{+} \right] \cdot T_{\mu}^{\mu}$. The miss match between these two expressions is one of several reasons why the author is looking at what happens for semi classical models for when $m_{graviton} \neq 0, m_{graviton} \sim 10^{-65}$ grams , noting that in QM, a spin 2 $m_{graviton} \neq 0$ has five degrees of freedom, whereas the $m_{graviton} \to 0$ gram case has two helicity states, only**.** Note that string theory treats gravitons as 'excitations' of a closed string , as given by Keifer , with a term added to a space time metric, $\bar{g}_{uv}$, such that $g_{uv} \equiv \bar{g}_{uv} + \sqrt{32\pi G} f_{\mu\nu}$ with $f_{\mu\nu}$ a linkage to coherent states of gravitons. This is partly in relation to the Venziano (1993) expression of $\Delta x \geq \frac{\hbar}{\Delta p} + \frac{l_s^2}{\hbar} \Delta p$, where $G \sim g^2 l_s^2$. Kieffer gives a correction due to quantum gravity in page 179 of the order of $\left(\frac{m}{M_{Planck}} \right)^2$ If the mass, $m_{graviton} \sim 10^{-65}$ g**,** then this is going to be hard to measure as an individual 'particle'**.** But, if $m_{graviton} \sim 10^{-65}$ g exists, as a macro effect, it may well pay a role as indicated by **Fig 1** above.

So, what about representing a graviton as a kink- anti kink ? How does this fit in with t'Hooft's deterministic QM?

T'Hooft used, in 2006 an equivalence class argument as an embedding space for simple harmonic oscillators, as given in his Figure 2, on page 8 of his 2006 article. It is also noteworthy to consider that in 2002, t'Hooft also wrote in his introduction, that "Beneath Quantum Mechanics, there may be a deterministic theory with (local) information loss. This may lead to a sufficiently complex vacuum state,". The author submits, that a kink-anti kink formulation of the graviton, when sufficiently refined, may , indeed create such a vacuum state, as a generalization of Fig 5 of this manuscript. In addition, the embedding equivalence class structure may be a consequence of a family of

$$\Psi_{i,f}\left[\phi(\mathbf{x})\right]\Big|_{\phi\equiv\phi_{ci,cf}} = c_{i,f}\cdot\exp\left\{-\int d\mathbf{x}\;\alpha\left[\phi_{Ci,f}(\mathbf{x})-\phi_0(\mathbf{x})\right]^2\right\},$$ solutions to a graviton state, if one is

taking the $\phi(x)$ as a kink-anti kink combination. I.e. looking at a history plot of equivalent solutions to the graviton problem, in a 5 dimensional space. The point being that the above 'functional', if one takes the tack of equivalence classes of solutions may, with work be part of a deterministic embedding space for the vacuum space of space time embedding the graviton. The author is trying to re formulate the above solution in terms of different values of $\phi_0(x)$ in a wave functional representation of a graviton, and trying to look for equivalence class embedding structures. This would mean as an example, a considerable refinement of the metric in 5 dimensions, given above, $dS^2 = a(w)\cdot\left[\eta_{uv}dx^u dx^v - dw^2\right]$

While doing this, the author is also asserting that the closeness of this fit, would , if worked out in detail perhaps give an explanation of the graviton mass problem. i.e. in looking at why $m_{graviton}\sim 10^{-65}$ exists.

The closeness of $m_{graviton}\sim 10^{-65}$ to a zero mass should not be seen as a failure of quantum physics, but a success story, whereas the author asserts that the hard work of establishing equivalence classes as part of a procedure to embed gravitons in space time will require generalizing t'Hoofts equations 4.3 and 4.4 of his 2006 manuscript to the wave functional the author asserts may be of use , namely looking at

$$\Psi_{i,f}\left[\phi(\mathbf{x})\right]\Big|_{\phi\equiv\phi_{ci,cf}} = c_{i,f}\cdot\exp\left\{-\int d\mathbf{x}\;\alpha\left[\phi_{Ci,f}(\mathbf{x})-\phi_0(\mathbf{x})\right]^2\right\},$$ in terms of a solution similar to the

equivalence class t'Hoot is working with harmonic oscillators showing up in his 2006 manuscripts figure 2. Having said that, it is time to look at if the graviton can actually carry information and what such information would imply for the cosmological constants.

How much information needs to be maintained to preserve the cosmological constants? From cosmological cycle to cycle?

No clear answer really emerges, YET. It is useful to note, that de La Peña in 1997 proposed an order-of-magnitude estimate to derive a relation between Planck's constant (as a measure of the strength of the field fluctuations) and cosmological constants. If , as an example, the fine structure constant has input parameter variance, as was explored by Livio, et al (1998), with an explanation of why fine structure constant has $\Delta\tilde{\alpha}/\tilde{\alpha}\leq 10^{-5}-10^{-6}$ when traveling from red shift values Z ~ 1.5 to the present era, and there is, as an example, from QED a proportional argument that $\tilde{\alpha}\equiv e^2/\hbar\cdot c$, with , in CGS units

$$\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc} \qquad (9y)$$

With a now commonly accepted version of $\dot{\tilde{\alpha}}/\tilde{\alpha} \le (-1.6 \pm 2.3) \times 10^{-17}\ year$. The supposition which the author will be investigating, as an example, will be if the energy needed to overcome the electrostatic repulsion between two electrons when the distance between them is reduced from infinity to some finite *d*, and (ii) the energy of a single photon of wavelength λ = 2π*d* has limiting grid values as to, in earlier conditions of cosmological expansion where the limits $\Delta q \ge \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$ could be investigated, and at least given limiting values.. This is where the LQG condition is $\alpha > 0$, and Brane worlds have, instead $\alpha < 0$. The author is fully aware of the inappropriateness of extrapolating eqn. (9w) before Z ~ 1100, and is, instead, looking for an equivalent statement as to what $\tilde{\alpha} \equiv e^2 / \hbar \cdot c$ would be at the onset of the big bang. Furthermore, the planck length, as given by $l_P \equiv \sqrt{\hbar G / c^3}$ would be, if followed through, a ay to make linkage between minimum length $\Delta q \ge \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$, and ways to obtain $\tilde{\alpha} \equiv e^2 / \hbar \cdot c$. If minimum uncertainty could be argued so as to look at

$$\Delta q \equiv 10^{\beta} \cdot l_P \sim \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \qquad (10)$$

Which was advanced by G. Veneziano , (1993), i.e. $10^{\beta} \cdot l_P \equiv l_{string}$ as a minimum length, it may be a way as to link choices of how much information could be stored in $\Delta q \equiv 10^{\beta} \cdot l_P$, with values of both the value $\tilde{\alpha} \equiv e^2 / \hbar \cdot c$, and $l_P \equiv \sqrt{\hbar G / c^3}$. The author is looking as to different algorithms of how to pack 'information' into minimum quantum lengths, $\Delta q \equiv 10^{\beta} \cdot l_P$, with the supposition that the momentum variance Δp could come from prior universe inputs into the present cosmos.

1st Conclusion, one needs a reliable information packing algorithm!

The author is working on it. Specifically one of the main hurtles is in finding linkage between information, as one can conceive of it, and entropy. If such a parameterization can be found, and analyzed, then Seth Lloyds short hand for entropy can then possibly be utilized. Namely as given by Lloyd (2002)

$$I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} = \left[\rho \cdot c^5 \cdot t^4 / \hbar\right]^{3/4} \qquad (11)$$

The author's supposition is that eqn (3) is basic, but that there could be a variance of inputs into eqn. (3) as far as inputs into the Planck's constant, $\hbar$ based upon arguments present at and after eqn (10)

Once resolution of the above ambiguities is finalized, one way or another, choices of inputs into eqn (2) and eqn. (3) will commence, with ways of trying to find how to select between the following. : the LQG condition is $\alpha > 0$, and Brane worlds have, instead $\alpha < 0$

If as an example, one is viewing gravitons according to the idea refined by Beckwith from Y.J. Ng, 2008, that a counting algorithm for entropy is de rigor according to **Appendix I**, then if say the total number of gravitons in inflation is of the order of $n \sim 10^{20}$ gravitons $\approx 10^{20}$ entropy counts, then Eq (11) above implies up to $\approx 10^{27}$ operations. If so, then there is at least a 1-1 relationship between an operation, and a bit of information, then a graviton has at least one 'bit' of information. The operation being considered is of the same form as a 2nd order phase transition.

What the author thinks, is that tentatively, higher dimensional versions of gravity perhaps need to be investigated, which may allow for such a counting algorithm. Either refinements as to determinisitic kink-anti kinks $\approx 10^{20}$ in number during inflation, according to a combination of **Appendix I ,** and the arguments given in page 17 of this document, or similar developments.

2nd Conclusion : Sensitivity limits as to detectors need to be revisited.

This document is in itself not only a HFGW document. The author though would like to re examine the question of HFGW, and to consider some of the proposals given to the author to obtain a range of GW and perhaps GRAVITONS.

Note that the initial standing question posed in the beginning was if there was a mass to the graviton. The stretch out of a graviton wave, perhaps greater than the size of the solar system gives, according to Maggiore (2008) an upper limit of a graviton mass, of $\lambda_{graviton} > 300 \cdot h_0 kpc \Leftrightarrow m_{graviton} < 2 \times 10^{-29} h_0^{-1} eV$. I .e a massively stretched graviton wave, ultra low frequency, may lead to a low mass limit. I.e. though more careful limits have narrowed the upper limit to about $10^{-20} h_0^{-1} eV$. Needless to state though for reasons given on page 17 of this document, the author finds the usual field theory treatments of graviton mass to be very difficult to maintain from a purely quantum field theoretic treatment.

Note, that ultra low frequency arguments and bounds to the graviton mass converged to the supposition of a kink- anti kink argument in the spirit of Giovannini's (2006) Classical and quantum gravity letter. The author sees no way to entertain a graviton mass without looking at a stretch out of a graviton to huge distances and then a permissible upper bound to the mass which is tiny.

This lead to the author entertaining a fifth dimensional conduit as to 'information' being exchanged from a prior universe, to our present universe. Having said that though, the material in appendix 1 argues in favor of perhaps a large number of gravitons having higher frequencies. The two items are not out of sync with one another. A counting algorithm, partly based upon the spirit of Appendix I with commensurate information attached to a graviton may be a way to give a minimum amount of information from a prior universe to our present universe put in Eq (9y). Note that in $\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc}$ that most of the information probably will be packed in the wave length given as λ above, and that the amount of information packed into this wave length λ may be amendable to how much information is packed into subsequent gravitons given in appendix I, below. I.e. what the author thinks is that what would be important would be, as an example for the fine structure constant, to seed a certain amount of information for its value via wave length values from nucleated kink-anti kink gravitons nucleated in a region of space more than Planck time after the big bang.

The author wishes to thank Professor Rainer Weiss, of MIT, in ADM 50, in November 7th (2009) for explaining the implications of a formula for HFGW of at least 1000 Hertz for GW which is a start in the right direction i.e. a strain value of, if L is the Interferometer length, and N is the number of quanta / second at a beam splitter, and τ is the integration time.

$$h \sim \frac{\lambda}{Lb\sqrt{N\tau}} \qquad (12)$$

For LIGO systems, and their derivatives, the usual statistics and technologies of present lasers as bench marked by available steady laser in puts given by Eq (12) appear to limit $h \sim 10^{-23}$. The problem is that as Weiss explained to the author, one of the most active, and perhaps guaranteed to obtain GW sources involves the interaction of super massive black holes in the center of colliding galaxies, which would need $h \sim 10^{-25}$ to obtain verifiable data. Going significantly below $h \sim 10^{-23}$ involves an argument as given as follows: The following question was posed by a reviewer of a document given to Dr. Fangyu Li, and the author has copied his response on page 10 of this document

Quote:

"*The most serious is that a background strain* $h \sim 10^{-30}$ *at 10GHz corresponds to a* Ω_g *(total)* $\sim 10^{-3}$ *which violates the baryon nuclei-synthesis epoch limit for either GWs or EMWs.* Ω_g *(Total) needs to be smaller than* 10^{-5} *otherwise the cosmological Helium/hydrogen abundance in the universe would be strongly affected......"*

The answer, which the author copied from Dr. Li, i.e. from page ten of this document that If $\nu = 10GHz, \quad h = 10^{-31}$, then $\Omega_g = 8.3 \times 10^{-7} < \Omega_{g\max}$, is an answer to this supposition .

Obviously, if Professor Lis supposition is correct, then a barrier to GW detection could be over come and that at least colliding super massive black holes in the center of merging galaxies can be investigated as a source of GW. Professor WEISS.

Doing so may mean that higher dimensional / semi classical models for graviton physics may have to be investigated. This leads to the authors tentative endorsement of , in the snyder geometry at least to first principle in looking at suitably modified Brane theory physics, with a possible opening to a modification of the above, if the instanton-anti instanton treatment of gravitons is verified..

Frankly though to fully maximize use of $h \sim \frac{\lambda}{Lb\sqrt{N\tau}}$ may be up to ten years off, i.e further advanced laser development, as cited by Dr. Weiss. .

Further Research questions to look into

If Eq(8) is true for a few select neutrinos and gravitons, then the author believes that it is reasonable to assume that as up to a billion years ago, $m_{graviton} \propto 10^{-65}$ grams. If so then the derivation of Figure 1 above is plausible. The problem the author is investigating is what is the consequence of Eq(8) for Eq(3). The author believes this problem is resolvable, and may imply a linkage between DE and DM in ways richer than what is done by the Chapygin gas models which are now currently a curiosity, Note that the proof of perhaps a kink- anti kink model as a bound for graviton mass is, initially a low frequency phenomenon

Appendix I : Entropy generation via Ng's Infinite Quantum Statistics

This discussion is motivated to show a purely string theory approach and to see if its predictions may over lap with semi classical WDM (semi classical) treatments of cosmology.. The contention being advanced is that if there is an over lap between these two methods that it may aid in obtaining experimentally falsifiable data sets for GW from relic conditions.

We wish to understand the linkage between dark matter and gravitons. How relic gravitational waves relate to relic gravitons"? To consider just that, we look at the "size" of the nucleation space, V for dark matter, DM. V for nucleation is HUGE. Graviton space V for nucleation is tiny, well inside inflation. Therefore, the log factor drops OUT of entropy S if V chosen properly for both eqn 1 and eqn 2. Ng's result begins with a modification of the entropy/ partition function Ng used the following approximation of temperature and its variation with respect to a spatial parameter, starting with temperature $T \approx R_H^{-1}$ (R_H can be thought of as a representation of the region of space where we take statistics of the particles in question). Furthermore, assume that the volume of space to be analyzed is of the form $V \approx R_H^3$ and look at a preliminary numerical factor we shall call $N \sim (R_H / l_P)^2$, where the denominator is Planck's length (on the order of 10^{-35} centimeters). We also specify a "wavelength" parameter $\lambda \approx T^{-1}$. So the value of $\lambda \approx T^{-1}$ and of R_H are approximately the same order of magnitude. Now this is how Jack Ng changes conventional statistics: he outlines how to get $S \approx N$, which with additional arguments we refine to be $S \approx < n >$ (where <n> is graviton density). Begin with a partition function

$$Z_N \sim \left(\frac{1}{N!}\right) \cdot \left(\frac{V}{\lambda^3}\right)^N \tag{0.1}$$

This, according to Ng, leads to entropy of the limiting value of, if $S = (\log[Z_N])$

$$S \approx N \cdot \left(\log\left[V / N\lambda^3\right] + 5/2\right) \xrightarrow[Ng-\inf inite-Quantum-Statistics]{} N \cdot \left(\log\left[V / \lambda^3\right] + 5/2\right) \approx N \tag{0.2}$$

But $V \approx R_H^3 \approx \lambda^3$, so unless N in eqn (0.2) above is about 1, S (entropy) would be < 0, which is a contradiction. Now this is where Jack Ng introduces removing the N! term in eqn (1) above , i.e., inside the Log expression we remove the expression of N in eqn. (0.2) above. The modification of Ng's entropy expression is in the region of space time for which the general temperature dependent entropy Kolb and Turner expression breaks down. In particular, the evaluation of entropy we do via the modified Ng argument above is in regions of space time where g before re heat is an unknown, unmeasurable number of degrees of freedom The Kolb and Turner entropy expression (1991(has a temperature T related entropy density which leads to that we are able to state total entropy as the entropy density time's space time volume V_4 with $g_{re-heat} \approx 1000$, according to De Vega, while dropping to $g_{electro-weakt} \approx 100$ in the electro weak era. This value of the space time degrees of freedom, according to de Vega has reached a low of $g_{today} \approx 2-3$ today. We assert that eqn (0.2) above occurs in a region of space time before $g_{re-heat} \approx 1000$, so after re heating eqn (0.2) no longer holds, and we instead can look at

$$S_{total} \equiv s_{Density} \cdot V_4 = \frac{2\pi^2}{45} \cdot g_{\bullet} \cdot T^3 \cdot V_4 \tag{0.3}$$

where $T < 10^{32} K$. We can compare eqn (0.1) and (0.2), as how they stack up with Glinka's (2007) quantum gas, if we

identify $\Omega = \dfrac{1}{2|u|^2 - 1}$ as a partition function (with u part of a Bogoliubov transformation) due to a graviton-quintessence gas, to get information theory based entropy

$$S \equiv \ln \Omega \tag{0.4}$$

Such a linkage would open up the possibility that the density of primordial gravitational waves could be examined, and linked to modeling gravity as an effective theory. The details of linking what is done with (0.2) and bridging it to (0.3) await additional theoretical development , and are probably conceptually understandable if the following is used to link the two regimes. I.e. we can use the number of space time operations used to create (0.2), via Seth Lloyds

$$I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} = \left[\rho \cdot c^5 \cdot t^4 / \hbar\right]^{3/4} \tag{0.5}$$

Essentially, what will be done is to use eqn.(0.5) to show linkage between a largely thermally based production of entropy, as implied by (0.3) and a particle counting algorithm, as given by eqn.(0.2). This due to the problems inherent in making connections between a particle count generation of entropy, and thermal contributions. I.e. two different processes are involved.

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