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An interpretation of an Angular Momentum Density of Circularly Polarized Light Beams

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Abstract: Reasons are presented against considering an moment of momentum flux to be a spin flux. A spin tensor is proposed to describe spin of a photon in the frame of the classical electrodynamics.

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A circularly polarized light beam carries an angular momentum (AM) [1,2]. However, troubling questions exist: what is the distribution of this AM over the beam section, and what is the nature of the AM, orbital or spin?

A paraxial circularly polarized Laguerre-Gaussian beam [3], LG_p^l, in the cylindrical coordinates ρ, ϕ, z with the metric $dl^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2$, namely

$$\underset{\to}{E} = \exp\{i(l+1)\phi + i\omega(z-t)\}(\omega \underset{\to}{\rho} + i\omega\rho \underset{\to}{\phi} + i \underset{\to}{z}\,\partial_\rho)\, u_p^l(\rho, z), \quad \underset{\to}{B} = -i\underset{\to}{E},$$

$$u_p^l = \frac{C_p^l}{w(z)}\left[\left(\frac{\rho\sqrt{2}}{w}\right)^l L_p^l\left(\frac{2\rho^2}{w^2}\right)\right]\exp\left\{-\frac{\rho^2}{w^2} + \frac{i\rho^2}{w^2 z_R} - i(2p+l+1)\arctan\left(\frac{z}{z_R}\right)\right\} \;\ldots. \qquad .(1)$$

($\underset{\to}{\rho}, \underset{\to}{\phi}, \underset{\to}{z}$ are *covariant* coordinate vectors, $k = \omega$, $c = 1$) is an eigenfunction of the *orbital*, not spin, AM operator $-i\hbar\partial_\phi$ with the eigenvalue $\hbar(l+1)$. This means that both, the circular polarization and the spiral phase front related with l, carry only orbital AM, not spin, in the frame of the standard electrodynamics.

Now we consider an exact, not paraxial, solution of the Maxwell equations; the solution for the radiation of a rotating electric dipole [4-6] in the spherical coordinates r, θ, ϕ:

$$E^r = (2/r^3 - i2\omega/r^2)\sin\theta\,\exp[i\varphi + i\omega(r-t)]/4\pi, \qquad (2)$$

$$E^\theta = (-1/r^4 + i\omega/r^3 + \omega^2/r^2)\cos\theta\,\exp[i\varphi + i\omega(r-t)]/4\pi, \qquad (3)$$

$$E^\varphi = (-i/r^4 - \omega/r^3 + i\omega^2/r^2)\exp[i\varphi + i\omega(r-t)]/(4\pi\sin\theta), \qquad (4)$$

$$B_{r\theta} = (i\omega/r + \omega^2)\cos\theta\,\exp[i\varphi + i\omega(r-t)]/4\pi, \qquad (5)$$

$$B_{\varphi r} = (\omega/r - i\omega^2)\sin\theta\,\exp[i\varphi + i\omega(r-t)]/4\pi, \quad B_{\theta\varphi} = 0. \qquad (6)$$

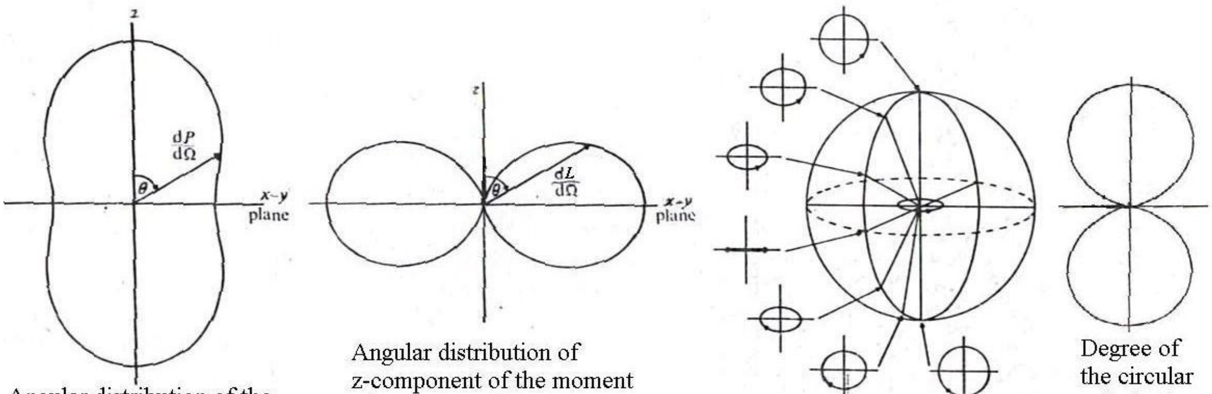
An angular distribution of the energy flux, $dP/d\Omega = <(\mathbf{E}\times\mathbf{B})_r r^2> = \omega^4(\cos^2\theta + 1)/(32\pi^2)$, and an angular distribution of z-component of the moment of momentum flux, i.e., of torque,

$$dL_z/dtd\Omega = d\tau_z/d\Omega = <[\mathbf{r}\times(\mathbf{E}\times\mathbf{B})]_z r^2> = \omega^3\sin^2\theta/(16\pi^2), \qquad (7)$$

are depicted. The total power and total torque are $P = \omega^4/6\pi$ and $\tau_z = \omega^3/6\pi$. We present also a distribution of the degree of circular polarization σ of the radiation [4], which approximately equals the ratio of lengths of the axes of the ellipse: $\sigma \cong \cos\theta$.

It is seen that AM (7) is emitted mainly into the equatorial part of space, situated near the $x-y$-plane where the polarization is elliptic or linear. Polar regions, situated near the z-axis, are scanty by AM (7), although they are intensively illuminated by the almost circularly polarized radiation. So, if we associate spin of an electromagnetic radiation with a circular polarization, we must recognize AM (7) is an *orbital* AM, not spin. Also note, fields (2) – (6) are eigenfunctions of the *orbital*, not spin, AM operator, $-i\hbar\partial_\phi$, with eigenvalue $\hbar$. This confirms the orbital nature of AM (7).

Thus we must recognise the standard electrodynamics cannot catch sight of spin of electromagnetic fields (1) – (6), and it is in need of an expansion.



Angular distribution of the energy flux. $dP/d\Omega \propto (\cos^2\theta+1)$

Angular distribution of z-component of the moment of momentum flux $dL_z/dtd\Omega \propto \sin^2\theta$

Polarization of the electric field seen by looking from different direction at a circular oscillator

Degree of the circular polarization $\sigma \cong \cos\theta$

The classical field theory points the way to the expansion. The Lagrange formalism gives two divergence-free tensors for free fields, energy-momentum and spin tensors [7]:

$$T^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \mathsf{L}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu}\mathsf{L}\ , \quad \Upsilon^{\lambda\mu\nu} = -2A^{[\lambda}\delta_\alpha^{\mu]}\frac{\partial \mathsf{L}}{\partial(\partial_\nu A_\alpha)}. \qquad (8)$$

Unfortunately, the standard Belinfante-Rosenfeld procedure [8,9] eliminates the spin tensor of electrodynamics [10,11]. So, we proposed an alternative procedure [12,13], which gives the Maxwell energy-momentum tensor and an elecrtrodynamics' spin tensor

$$\Upsilon^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}. \qquad (9)$$

Here A^λ and Π^λ are the magnetic and electric vector potentials which satisfy $\partial_\lambda A^\lambda = \partial_\lambda \Pi^\lambda = 0$, $2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}$, $2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}$, where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field; $e_{\mu\nu\alpha\beta}$ is the Levi-Civita antisymmetric tensor density. Using (9) yields an angular distribution of z-component of the spin flux in the rotating electric dipole radiation [5,6]:

$$dS_z/dtd\Omega = \omega^3\cos^2\theta/(16\pi^2)\,, \qquad (10)$$

and the total flux of z-component of the spin, $dS_z/dt = \omega^3/(12\pi)$, which is half of the total orbital angular momentum flux. However, the ratio of the spin flux density to the power density at $\theta = 0$ equals to $1/\omega$,

$$\left.\frac{\omega^3\cos^2\theta/(16\pi^2)}{\omega^4(\cos^2\theta+1)/(32\pi^2)}\right|_{\theta=0} = \frac{1}{\omega}, \qquad (11)$$

just as for a photon because the radiation is circularly polarized with plane phase front along z-axis:

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