Towards an Einsteinian Quantum Theory

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Abstract

A theory of quantum mechanics in terms of a quantized space-time shows that Einstein was correct in his debate with Bohr. The conflict of the axioms of quantum field theory and the axioms of general relativity may resolved by modifying both and equating quantum field theory with harmonic analysis on the complex space-time $QAdS = U(3,2)/U(3,1) \times U(1)$. This is consistent with the geometry of particle interactions introduced in Love [113, 114].

> The real goal of my research has always been the simplification and unification of the system of theoretical physics. I attained this goal satisfactorily for macroscopic phenomena, but not for the phenomena of quanta and atomic structure. I believe that, despite considerable success, the modern quantum theory is also still far from a satisfactory solution of the latter group of problems.
>
> —Albert Einstein[55]

1 Introduction

Niels Bohr [21] recollected parts of his famous debate with Einstein on the foundations of quantum mechanics:

> Einstein's own views at that time are presented in an article "Physics and Reality," published in 1936 in the Journal of the Franklin Institute. Starting from a most illuminating exposition of the gradual development of the fundamental principles in the theories of classical physics and their relation to the problem of physical reality, Einstein here argues that the quantum-mechanical description is to be considered merely as a means of accounting for the average behaviour of a large number of atomic systems and his attitude to the belief that it should offer an exhaustive description of the individual phenomena is expressed in the following words: "To believe this is logically possible without contradiction; but it is so very contrary to my scientific instinct that I cannot forego the search for a more complete conception."
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> Even if such an attitude might seem well-balanced in itself, it nevertheless implies a rejection of the whole argumentation exposed in the preceding, aiming to show that, in quantum mechanics, we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena, but with a recognition that such an analysis is *in principle* excluded.

One of the early masters of quantum theory, Paul Dirac [41], saw serious problems in quantum theory, and he clearly perceived the direction physics should go:

Physicists have been very clever in finding ways of turning a blind eye to terms they prefer not to see in an equation. They may go on to get useful results, but this procedure is of course very far from the way in which Einstein thought that nature should work.

It seems clear that the present quantum mechanics is not in its final form. Some further changes will be needed just about as drastic as the changes made in passing from Bohr's orbit theory to quantum mechanics. Some day a new quantum mechanics, a relativistic one will be discovered, in which we will not have these infinities occurring at all. It might very well be that the new quantum mechanics will have determinism in the way that Einstein wanted. This determinism will be introduced only at the expense of abandoning some other preconceptions that physicists now hold. So, under these conditions I think it is very likely, or at any rate quite possible that in the long run Einstein will turn out to be correct, even though for the time being physicists have to accept the Bohr probability interpretation, especially if they have examinations in front of them.

Elsewhere Dirac [40] affirmed his agreement with Einstein:

There are great difficulties... in connection with the present quantum mechanics. It is the best that one can do up till now. But, one should not suppose that it will survive indefinitely into the future. And I think that it is quite likely that at some future time we may get an improved quantum mechanics in which there will be a return to determinism and which will, therefore, justify the Einstein point of view.

Micahel Atiyah [7] asks:

...do we need to look for new foundations? I confess that I myself remain an Einsteinian and would be happy to see quantum mechanics replaced by something deeper. This remains, as in Einstein's day, a minority opinion but one shared for example by Roger Penrose.

Although it is not acknowledged, the currently popular theory of strings goes against the philosophy of Bohr both in looking for something deeper than quantum mechanics and in modeling matter as other than a point particle. The probabilistic interpretation requires point particles.

My goal in this book is to construct "a more detailed analysis of atomic phenomena" and in so doing show that Einstein's instinct was correct.

2 QM Brings Us No Enlightenment

In 1926, Erwin Schrödinger published his now famous wave equation of the description of a "particle" moving in the potential created by another particle. There were some problems with this wave equation, not the least of which was the interpretation of the solutions. The Schrödinger equation describes a particle in terms of "waves", the ψ-function. But if the object is a classical "point particle," what is it that is waving? The wave function is an extended object in contradistinction to the point particle. How are the two related? Schrödinger [160] suggested

> ...that we should try to connect the function ψ with some *vibration* process in the atom, which would more nearly approach reality than the electronic orbits, the real existence of which is being very much questioned today...It is hardly necessary to emphasize how much more congenial it would be to imagine that at a quantum transition the energy changes over from one form of vibration to another than to think of a jumping electron (pp. 9-11).

Is the electron a particle or a wave? Schrödinger wanted to eliminate the concept of "particle" from quantum theory and deal only with waves:

> Now in the cases treated, the ψ potential energy arises from the interaction of particles, of which perhaps one at least, may be regarded in wave mechanics also as forming a point, on account of its great mass. We must also take into account the possibility that it is no longer permissible to take over from ordinary mechanics the statement for the potential energy, if *both* "point charges" are really extended states of vibration which penetrate each other. (p. 57)

It seems probable that in the case of the electron interaction with the potential of a proton, the classical potential of the proton is somehow related to the wave function of the proton and we should be dealing with one potential interacting with another or equivalently, one wave function interacting with another or one field interacting with another.

After laying the foundations of the theory of quantum mechanics, Schrödinger devoted his research to the quest for a unified field theory. In the Introduction to *Space-Time Structure*, Schrödinger [161] described the quest:

> The ideal aspiration, the ultimate aim is not more and not less that this: A four-dimensional continuum endowed with a certain intrinsic geometric structure, a structure that is subject to certain inherent purely geometrical laws, is to be an adequate model or picture of the 'real world around us in space and time' with all that it contains and including its total behaviour, the display of all events going on in it.

Rueger [154] assessed Schrödinger's goals:

> His ambitious aim was no less than a unification of wave mechanics and GTR; atomic physics and cosmology together would provide an explanation of the discrete structure of matter and elucidate the nature of matter waves. This may sound too grandiose to be take seriously.

We have reached the time when such an undertaking must be taken seriously. Some scholars studying General Relativity have argued that GR really doesn't require quantizing while scholars pursuing Quantum Field Theory have ignored gravitation because it is too weak to be of importance. Glashow [70] claims "Julian regards the quest for Unification Now as an act of unbridled arrogance. I can only concur." I must totally disagree. The arrogance comes about when we believe that real progress can be made by ignoring anything in nature. Nature is a unified whole, we must study her as a whole, not in parts. We can never expect to find a unified field theory by ignoring any of the parts. Progress is never made through ignorance, even when that ignorance is intentional.

Max Born could not accept Schrödinger's undulatory interpretation and preferred to interpret the wave function as a probability distribution. Jammer [98] provides a succinct statement of Born's interpretation:

> Summarizing Born's original probabilistic interpretation of the function, we may say that $|\psi|^2 \, dx$ measures the probability density of finding the particle within the elementary volume dx, *the particle being conceived in the classical sense as a point mass possessing at each instant both a definite position and a definite momentum*. Contrary to Schrödinger's view, ψ does not represent the physical system, nor any of its physical attributes, but only our *knowledge* concerning the latter.(p. 42)

Somewhere between de Broglie and Born, the wave attributes of matter shifted from being a property of individual electrons, protons, et cetera, to being a statistical concept of a large number of identical particles. Since it developed to maturity under the influence of Bohr, who worked in Copenhagen, the "standard" interpretation of quantum mechanics goes by the title of the "Copenhagen Interpretation" of quantum mechanics. Although the interpretation is thought to be standard, there seem to be as many versions of the "Copenhagen Interpretation" as there are articles by that title. In any case, the foundation of the "Copenhagen Interpretation" is the interpretation of the wave function as a probability distribution.

It is important to keep in mind that the Copenhagen Interpretation is an attempt to find meaning for the solution of the Schrödinger equation: a scalar function. The study of other equations including the Klein-Gordon equation, the Dirac equation and the Yang-Mills equation has led to the realization that a scalar description of elementary particles is incomplete. A more complete description of matter (elementary particles) requires the use of "internal degrees of freedom" represented by quantities which are multiplied by the wave function [140] (p. 13). Thus, the solutions of the Schrödinger equation cannot provide a complete description of nature. The Schrödinger equation is non-relativistic and provides only an approximation to reality. The study of the relativistic equations led to the necessity that the "probability" be negative, a problem which has perplexed even the most ardent supporters of the Copenhagen Interpretation, for example, Hanson:

> There are *genuine* improprieties within quantum theory, e.g. 'renormalization' and the unintelligible negative probabilities [79].

Quantum mechanics is the currently accepted theory of the very small, general relativity is the currently accepted theory of the very large. These theories were two great steps forward in mankind's attempt to understanding the universe. One of the major goals of theoretical physics has been to combine these theories and "Quantize gravity." The seemingly insurmountable problem in quantizing gravity is that the axioms of quantum theory and the axioms of general relativity are incompatible [157]. In order to make any progress in the unification of the two theoretical revolutions of the last century, we must question the fundamental concepts of both fields, and we must be prepared to sacrifice sacred cows on the altar of truth.

Mendel Sachs opined:

> ... any real progress in our understanding of matter can only come if we agree to reject some of the currently held notions of *either* the quantum theory *or* the theory of relativity.[157]

More likely *both* Quantum theory and General Relativity will have to sacrifice some sacred principles. It seems that we must backtrack from both QM and GR to find a new foundation for future progress.

Einstein criticized Eddington's attempt at a unified field theory:

> The theory supplies us, in a natural manner, with the hitherto known laws of the gravitational field and of the electromagnetic field, as well as with a connection as regards their nature of the two kinds of fields; but it brings us no enlightenment on the structure of electrons. (quoted in [167])

This is also a valid criticism of probabilistic quantum mechanics: "it brings us no enlightenment on the structure of electrons." Probabilistic Quantum mechanics may be capable of telling us what an electron will do, but it will never be able to tell us what an electron is, nor can quantum mechanics tell us why electrons do what they do. Any probabilistic theory is then fatally flawed as a description of nature.

David Finkelstein [64] noted:

> There have been many attepts to bring order to the array of obseved particles by calling some of them excited states of others. To do this is to attribute to the basic particles certain internal degrees of freedom which, being capable of excitation, can account for the existence of various states with different properties.

A.O. Barut [12] expressed a similar opinion:

> We have to study the structure of the electron, and if possible, the single electron, if we want to understand physics at short distances.

So, where did Einstein get the idea that this was an important part of an acceptable theory? Probably from studying Mie's Theory [123]. Weyl [175] introduces Mie's theory with these words:

> The theory of Maxwell and Lorentz cannot hold for the interior of the electron; therefore, from the point of view of the ordinary theory of electrons we must treat the electron as something given priori, as a foreign body in the field. A more general theory of electrodynamics has been proposed by Mie, by which it seems possible to derive the matter from the field. (page 206)

Near the end of his book Weyl continues:

> If Mie's view were correct, we could recognise the field as objective reality, and physics would no longer be far from the goal of giving so complete a grasp of the nature of the physical world, of matter, and of natural forces, that logical necessity would extract from this insight the unique laws that underlie the occurrence of physical events. (page 311)

There are then problems with interpreting the wave function as a probability distribution and it is the Principle of Superposition which forces the probability interpretation upon us. This was noticed by Einstein:

> I think it is not possible to get rid of the statistical character of the present quantum theory by merely adding something to the latter without changing the fundamental concepts about the whole structure. Superposition principle and statistical interpretation are inseparably bound together. If one believes that the statistical interpretation should be avoided and replaced, it seems one cannot conserve a linear Schrödinger equation which implies by its linearity the principle of superposition.(quoted in [157], p. 279)

The problem has not disappeared, according to Cushing [32]:

> Thus, superposition, with the attendant riddles of entanglement and reduction, remains the central and generic interpretative problem of quantum theory.

The two fundamental principles of quantum theory are the superposition principle and the observable as eigenvalue principle. The principle of superposition is valid if and only if the equation is linear, that is the *definition* of linear. An equation is linear if and only if a linear combination of solutions is again a solution to the equation.

The problem is that quantum mechanics is mathematically inconsistent. The source of the problem is that there are two versions of the Schrödinger Equation:

S(td)

$$H\psi = i\frac{\partial\psi}{\partial t}$$

the time dependent Schrödinger equation and

S(ti)

$$H\psi = E\psi$$

$$i\frac{\partial\psi}{\partial t} = E\psi$$

the time independent Schrödinger equations.

Any solution of S(ti) is also a solution of S(td), but not conversely. A system evolving according to S(ti) is always in an eigenstate of energy (which is just another way of saying that energy is conserved). A system evolving according to S(td) need not be in an eigenstate between interactions. The quantum collapse occurs when we model the wave moving according to $S(td)$ and then, suddenly at the time of interaction we require it to be in an eigenstate and hence to also be a solution of $S(ti)$. The collapse of the wave function is due to a discontinuity in the equations used to model the physics, it is not inherent in the physics. The difficulty lies in the fact that $S(td)$ is linear while $S(ti)$ is not. The two equations are incompatible. The time independent Schrödinger equations, $S(ti)$, are the equations of bound systems which yield the spectra of atoms and which have been verified in many settings.

Although H is a linear operator, the eigenvalue equation is not a linear equation. Thus, if

$$H\Psi_1 = E_1\Psi_1$$

and

$$H\Psi_2 = E_2\Psi_2$$

Then adding, we obtain:

$$H(\Psi_1 + \Psi_2) = H\Psi_1 + H\Psi_2 = E_1\Psi_1 + E_2\Psi_2$$

Thus, the sum or superposition of two wave functions is not an eigenfunction of H unless $E_1 = E_2$. The same observation is true for any quantum number. Thus the fundamental equation for quantum mechanics is not linear and the superposition principle is not valid. But the entire structure of quantum mechanics is based on the supposition that the equations *are* linear and that the superposition principle holds.

Because of the principle of superposition is forced upon an inherently nonlinear equation, the standard approach must demand that the wave function of the system collapse into an eigenstate as the result of the measurement.

Although the operator involved is a linear operator, the eigenvalue problem is not a linear equation unless the eigenvalues are equal, and even then, the eigenvalue of the sum of two wave functions is the same as that of each of its summands while the energy of two waves interacting should be the *sum* of the energies.

Standard quantum theory allows states to evolve via a linear equation and then an observation forces superposition upon an eigenvalue equation, an inherently nonlinear equation. In order to undo the *assumption* of superposition standard quantum mechnics then states that each of the summands has a certain probability of being observed. I have to conclude that standard quantum mechanics is a mathematical fraud.

Cramer [31] notes that

> ... most of the efforts to revise or replace the Copenhagen interpretation have focused on the problem of collapse, which remains the most puzzling and counterintuitive aspect of the interpretation of quantum mechanics.

Again, the only way in which the probability interpretation and collapse of the wave function are forced upon us is by the assumption that a linear combination of the eigenfunctions can represent a physical state. This assumption has several shortcomings. If during a measurement, or an interaction, the "arbitrary continuous function" is forced into an eigenstate, is it not reasonable to assume that it was in that eigenstate all along? Since the measurement which forces the system into an eigenstate is just an interaction, what kind of interactions cause the system to collapse? Since particles are continuously interacting with the outside world, shouldn't they continuously be in an eigenstate? Or does the system know that a particular interaction is going to be observed by a human, and knows to hop into an eigenstate?

There are no "arbitrary continuous functions" in nature. Nature allows only definite, well defined states to exist. We observe only certain particles: electron, pion, proton, Hydrogen atom, etc. We do not observe "mixed-breed" particles as would be the case if "arbitrary continuous functions" described physical states. There is no need to assume that the eigenfunctions of the total energy operator constitute a mathematically complete set of functions. Rather, we have the prediction that the eigenstates of certain operators provide a physically complete set: only those functions are necessary in the description of the particles which occur in nature.

According to the formalism introduced here, the wave function must be an eigenfunction of the evolution operator at all times. Not so in the standard theory where the eigenvalue equation is the abstract Schrödinger equation: $H\psi = E\psi$.

The program according to Schiff [159], (p.50) is as follows:

> ... we make the mathematical assumption that all the eigenfunctions $u_E(r)$ of the total energy operator constitute a complete set of functions in the sense that an arbitrary continuous function can be expanded in terms of them. Then, if we have any wave function $\psi(r)$ at a particular instant of time that is normalized in the box L^3 and obeys periodic boundary conditions at the walls, the assumed existence of the expansion
>
> $$\psi(r) = \sum_E A_E u_E(r)$$
>
> makes it possible to find unique coefficients A_E that do not depend on r... the energy eigenstates are the only possible results of precise measurement of the total energy and that the probability of finding a particular value E when the particle is described by the wave function $\psi(r)$ is proportional to $|A_E|^2$.

The eigenfunctions $u_E(r)$ are wave functions describing a bound state. It is absurd to expect that the wave functions of a free state can be expressed as a sum of the wave functions of bound states.

If energy is an observable, then the energy of a wave function is an eigenvalue of some operator. If an expansion in terms of eigenfunctions is allowed, then ψ is not an eigenfunction of the energy operator and the energy of the wave is not well defined. In fact, the assumption of the expansion in terms of eigenfunctions gives a probability of $1 - |A_E|^2$ that energy is not conserved.

If the superposition principle holds, the principle of conservation of energy does not hold. If the superposition principle holds, no principle of conservation is valid. The approach developed in [114] and here is based on group theory. The conserved quantities are the invariants of the group: the mathematics demands that all conservation laws be exact. Since the conservation of energy is exact the principle of superposition cannot hold.

3 Einstein's criteria

> Is it conceivable that a field theory permits one to understand the atomistic and quantum structure of reality? Almost everybody will answer this question with "no." But I believe that at the present time nobody knows anything reliable about it.. . . One can give good reasons why reality cannot at all be represented by a continuous field. From the quantum phenomena it appears to follow with certainty that a finite system of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accord with a continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality. But nobody knows how to obtain the basis of such a theory.
> ––—Albert Einstein [53](pp. 165-66)

There is a theory which leads naturally from the algebraic to the continuum, the theory of Lie algebras, Lie groups and Homogeneous spaces. The Lie algebra can be used to generate the quantum numbers while the Lie group/homogeneous space setting provides the continuous geometric background. The trick then is to find the correct algebra and group.

> I do not believe in micro- and macro- laws, but only in (structure) laws of general rigorous validity. And I believe that these laws are logically simple, and that reliance on this logical simplicity is our best guide. Thus, it would not be necessary to start with more than a relatively small number of empirical facts. If nature is not arranged in correspondence with this belief, then we have altogether very little hope of understanding it more deeply.
> —Albert Einstein [52]

In the quest for a unified field theory, the basic question is where to start? Clearly, some empirical facts are required as input, others will come out of the theory. When observing a physical object, different observers may disagree on its location, its momentum or a hundred other details, but if one observer says the object is a proton, then all other observers must agree. If one observer says the object is an electron, all other observers must agree otherwise the world would be totally incomprehensible. So that is where we begin.

4 The Algebra of Elementary Particles

First set of empirical facts: The following elementary particles have been observed: the electron, e^-; the proton p^+; the neutron n; the neutrino ν and their corresponding antiparticles: the antielectron, e^+; the antiproton p^-; the antineutron $\bar{n}$; the antineutrino $\bar{\nu}$. Hundreds of other particles have been observed, but starting with too much data can only lead to confusion. We start with some data and hope to be able to explain the rest.

Second set of empirical facts: The following decays of elementary particles have been observed: $n \to p^+ e^- \bar{\nu}$; $\pi^- \to e^- \bar{\nu}$.

These must appear the same to all observers. The incoming particles and the daughter particles will have different energies and momenta in different frames of reference, but their identities must be the same for all observers.

Third empirical fact: the electron and the proton interact via the electric force. The electron has a charge of -1 and the proton has a charge of +1.

First theoretical input: The elementary particles interact via Lie bracket.

Justification of this hypothesis will be our major objective.

This means that the hydrogen atom, $H = [e^-, p^+]$, or would it be $H = -[e^-, p^+]$? We need another approach to find the right signs.

Second theoretical input: Denote the carrier of the electric force by γ_E, the charge is then the eigenvalue of the γ_E:

$$[\gamma_E, p^+] = p^+$$

$$[\gamma_E, p^-] = -p^-$$

$$[\gamma_E, e^+] = e^+$$

$$[\gamma_E, e^-] = -e^-$$

In terms of matrices, with T = transpose, $(AB)^T = B^T A^T$. Thus,

$$[A, B]^T = (AB - BA)^T = B^T A^T - A^T B^T = -[A^T, B^T] \tag{1}$$

If γ_E is in the Cartan subalgebra and is diagonal, then $\gamma_E^T = \gamma_E$ and thus:

$$\begin{aligned} &B^T = [\gamma_E, B]^T = (\gamma_E B - B\gamma_E)^T = B^T \gamma_E^T - \gamma_E^T B^T \\ &= B^T \gamma_E - \gamma_E B^T = -[\gamma_E, B^T] \end{aligned} \tag{2}$$

We end up with:

$$[\gamma_E, B^T] = -B^T$$

The eigenvalue switch of the transposes then behaves just like the quantum number switch between elementary particles. So if B represents a particle, then B^T represents the antiparticle. These correspond to the creation and annihilation operators of QFT.

The discussion so far requires that we have a matrix:

$$\begin{pmatrix} & & & e^- \\ & & & \pi^- \\ & & & p^- \\ e^+ & \pi^+ & p^+ & \gamma_E \end{pmatrix} \tag{3}$$

The matrix is taken as four by four since space-time is four dimensional. It includes the elementary particles we must have, with the assumption that other particles will prove to be either excited states of the basic ones or combinations or combinations of excited states of the basic ones.

So a photon is represented by the matrix:

$$\gamma_E = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$$

The factor of i is required to make the matrix compact.

An electron is represented by the matrix:

$$e^- = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and the interaction of a photon and an electron is given by the Lie bracket:

$$[\gamma_E, e^-] = \left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]$$
$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= -ie^-$$

The proton is represented by:

$$p^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The interaction of the photon and the proton is:

$$[\gamma_E, p^+] = \left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right]$$
$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$= ip^+$$

The interaction of a proton and an electron is given by:

$$[e^-, p^+] = \left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right]$$
$$= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

An electron bonding to a proton is a hydrogen atom, thus we have filled two more slots:

$$\begin{pmatrix} & & H & e^- \\ & & & \pi^- \\ \bar{H} & & & p^- \\ e^+ & \pi^+ & p^+ & \gamma_E \end{pmatrix}$$

Since the H is a hydrogen atom, and not an elementary particle, it seems out of place. It is necessary, for when adding the quantum numbers of a proton and an electron the result is the quantum numbers of a hydrogen atom. The presence of the hydrogen atom in the array does not mean that whenever a proton and an electon collide the result is a hydrogen atom. It means that whenever a proton and an electon collide the result will have the quantum numbers of a hydrogen atom. The same could be said of any of the interactions we consider.

In order to fill the next slot, we calculate how the particle (call it X) interacts with an electron:

$$[e^-, X] = \left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Thus,

$$[e^-, X] = -\pi^-$$

Since we know that the pion decays, $\pi^- \to e^-\bar{\nu}$ we conclude that X is the $\bar{\nu}$ and we have filled two more slots:

$$\begin{pmatrix} & \nu & H & e^- \\ \bar{\nu} & & & \pi^- \\ \bar{H} & & & p^- \\ e^+ & \pi^+ & p^+ & \gamma_E \end{pmatrix}$$

The interaction of a proton and an antiproton is given by:

$$[p^-, p^+] = \left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Since the interaction of a particle and its antiparticle yields pure energy, we conclude that the other slots on the diagonal consist of photon like objects (neutral currents), which we will label γ_1, γ_2, γ_3 and to be consistent, we will change the label on γ_E to γ_4:

$$\begin{pmatrix} \gamma_1 & \nu & H & e^- \\ \bar{\nu} & \gamma_2 & & \pi^- \\ \bar{H} & & \gamma_3 & p^- \\ e^+ & \pi^+ & p^+ & \gamma_4 \end{pmatrix}$$

The negative pion is represented by the matrix:

$$\pi^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and the interaction of a proton and a pion is given by the Lie bracket:

$$[\pi^-, p^+] = \left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right]$$
$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The neutron is the only possible outcome of this interaction. We are forced to the conclusion that a proton and a pion combine to make a neutron.

We conclude that the neutron is represented by the matrix:

$$n = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

This allows us to fill the last two slots in the matter matrix, since the anti-neutron is the transpose of the neutron:

$$\begin{pmatrix} \gamma_1 & \nu & H & e^- \\ \bar{\nu} & \gamma_2 & n & \pi^- \\ \bar{H} & \bar{n} & \gamma_3 & p^- \\ e^+ & \pi^+ & p^+ & \gamma_4 \end{pmatrix}$$

We have arrived at a tentative representation of the matter matrix. Each of the γ_I will generate quantum numbers which act like the charges of γ_4 to give a quantum number. These four quantum numbers are conserved in all interactions. Each particle has four quantum numbers, one is 1, another is -1 and the other two are zero, Since γ_1 generates the second quantum number of the electron, we will call its eigenvalue the lepton number. Since γ_3 generates the second quantum number of the proton, we will call its eigenvalue the antibaryon number (this is for technical reasons, since the particle has one positive and one negative eigenvalue or quantum number). Since γ_2 generates the second quantum number of the pion, we will call its eigenvalue the meson number.

The four γ_I generate quantum numbers which define superselection rules and which partition the elementary particles into 16 superselection sectors. They also define four forces: γ_1 mediates the weak force; γ_2 is related to the spin-spin interaction; γ_3 mediates the strong force and γ_4, the electromagnetic force. For two particles, A and B, the same quantum numbers are obtained from the bracket $[A, B]$ as from the tensor product $A \otimes B$; the bracket is used when a change in particle type is involved and the tensor product is used when there is no change in particle type. The tensor force: $A \otimes B$ is an exchange force: $A \otimes B = [[A, \gamma_I], B] = [A, [\gamma_I, B]]$. Describing the interaction of elementary particles on the discrete level of Lie algebras makes the forces look like exchange forces, exponentiating to obtain the geometry of the Lie group will make the γ_I interaction look like a continuous field.

These numbers are also related to the statistics of the particle: the eigenvalue of $\frac{1}{2}(\gamma_1 - \gamma_2 + \gamma_3 - \gamma_4)$ is 1 for Bosons and -1 for Fermions.

Since the Lie bracket determines the interaction, this is a realization of Yang's dictum: 'symmetry dictates interaction' [184]. We might equally invert the statement and say that 'interaction dictates symmetry'. But if both statements are true, then symmetry is the interaction.

It seems that the electron, e^-; the proton p^+; the neutrino ν and their corresponding antiparticles: the antielectron, e^+; the antiproton p^-; the antineutrino $\bar{\nu}$ are the only truly elementary particles.

Beginning with the well known six charged particles and the photon, we arrived at a matrix which models 12 particles and their interactions via four forces. This is one more than the standard model, but the fourth is the well known spin-spin or Pauli force.

The above particles account for the $u(3,1)$ factor in $u(3,1) \times u(1)$. The $u(1)$ factor is the graviton. The Lie algebra $u(3,1) \times u(1)$ consists of those elements of $u(3,2)$ which commute with the $u(1)$ graviton, which is denoted by γ_5. The use of the symbol γ_I is meant to connote something similar to the photon, they are not to be confused with the Dirac matrices.

The fifth γ_5 plays a dual role as the graviton and defining a complex structure on the base space. Thus there are five fundamental forces identified, four of these correspond to the conserved quantities: lepton number, baryon number electric charge and meson number (spin is also included in the mix). The fifth is gravity and gravity alone has no corresponding conserved quantity (unless it is energy and it it not the case that matter attracts matter via gravitation rather energy attracts energy). Gravitation has other peculiar features: it is universal and always attractive. This means that the graviton γ_5 should play a prominent role in the geometry of unification, and indeed, it does.

In the standard model, the Lie algebra acts on the elementary particles as states in a Hilbert space. In the model of matter introduced by the author [113, 114], the elementary particles are modeled as operators (vertical vector fields on a principle fiber bundle over a complex space-time) which form the Lie algebra and their interactions are modeled by the Lie bracket (commutator) when a change in particle type is involved or as a tensor product when there is no change in particle type.

Each γ_I is the generator of a $U(1)$ circle bundle over the complex space-time.

5 The Structure of U(3,2)

According to Helgason [82], the structure of the Lie algebra $u(p,q)$ consists of the $p+q$ by $p+q$ matrices with complex coefficients of the form:

$$\begin{pmatrix} Z_1 & Z_2 \\ {}^t\bar{Z}_2 & Z_3 \end{pmatrix}$$

with $Z_1 = u(p)$ and $Z_3 = u(q)$. Thus $u(3,1)$ consists of the matrices:

$$u(3,1) = \begin{pmatrix} u(3) & Z_2 \\ {}^t\bar{Z}_2 & u(1) \end{pmatrix}$$

Where Z_2 is an arbitrary 3 by 1 complex matrix.
While $u(3,2)$ consists of the matrices:

$$u(3,2) = \begin{pmatrix} u(3) & Z_4 \\ {}^t\bar{Z}_4 & u(2) \end{pmatrix}$$

Where Z_4 is an arbitrary 3 by 2 complex matrix.
The $u(3)$ and $u(2)$ sectors are compact while the Z_4 sector is non-compact.
The complete parameterization of $u(3,2)$ is then:

$$\begin{pmatrix} iy_{11} & x_{12}+iy_{12} & x_{13}+iy_{13} & x_{14}+iy_{14} & x_{15}+iy_{15} \\ -x_{12}+iy_{12} & iy_{22} & x_{23}+iy_{23} & x_{24}+iy_{24} & x_{25}+iy_{25} \\ -x_{13}+iy_{13} & -x_{23}+iy_{23} & iy_{33} & x_{34}+iy_{34} & x_{35}+iy_{35} \\ x_{14}-iy_{14} & x_{24}-iy_{24} & x_{34}-iy_{34} & iy_{44} & x_{45}+iy_{45} \\ x_{15}-iy_{15} & x_{25}-iy_{25} & x_{35}-iy_{35} & -x_{45}+iy_{45} & iy_{55} \end{pmatrix}$$

Another way to decompose $u(3,2)$ is relevant to our discussion:

$$u(3,2) = \begin{pmatrix} u(3,1) & V \\ V^\dagger & u(1) \end{pmatrix}$$

Where V is a 4 by 1 complex matrix (a complex four vector) and $V^\dagger$ denotes the adjoint of V. Let us denote the components of V as

$$\begin{matrix} x_{15}+iy_{15} \\ x_{25}+iy_{25} \\ x_{35}+iy_{35} \\ x_{45}+iy_{45} \end{matrix}$$

Physically the components of V represent a translation vector in the tangent space of $QAdS$.
The components of $V^\dagger$ are then:

$$x_{15}-iy_{15} \quad x_{25}-iy_{25} \quad x_{35}-iy_{35} \quad -x_{45}+iy_{45}$$

are the complex conjugates for the first three and the negative of the complex conjugate for the fourth. The difference in sign occurs because the $x_{45}+iy_{45}$ component is in the Z_4 sector and compact while the others are noncompact.

While $x_{15}+iy_{15}, x_{25}+iy_{25}$, and $x_{35}+iy_{35}$ represent spatial coordinates, $x_{45}+iy_{45}$ represents time. Thus the transformation

$$\begin{aligned} x_{15}+iy_{15} &\Rightarrow x_{15}-iy_{15} \\ x_{25}+iy_{25} &\Rightarrow x_{25}-iy_{25} \\ x_{35}+iy_{35} &\Rightarrow x_{35}-iy_{35} \end{aligned}$$

is P (parity). While

$$x_{45}+iy_{45} \Rightarrow -x_{45}-iy_{45}$$

is T (Time reversal).

The interpretation of the parameterization of $u(3,2)$:

$$\begin{pmatrix} \gamma_1 & \nu & H & e^- & x_{15}+iy_{15} \\ \bar{\nu} & \gamma_2 & n & \pi^- & x_{25}+iy_{25} \\ \bar{H} & \bar{n} & \gamma_3 & p^- & x_{35}+iy_{35} \\ e^+ & \pi^+ & p^+ & \gamma_4 & x_{45}+iy_{45} \\ x_{15}-iy_{15} & x_{25}-iy_{25} & x_{35}-iy_{35} & -x_{45}+iy_{45} & \gamma_5 \end{pmatrix}$$

Recall that C (charge conjugation) is defined by the transpose of the $u(3,1)$ component.

None of the discrete mappings T, C, P are symmetries of $u(3,2)$ but the product $T \times C \times P$ maps $U(3,2)$ onto itself.

As Greenberg [74] points out,

> CPT is fundamental because it is intimately related to Lorentz invariance.

The above calculations in terms of the discrete symmetries of $u(3,2)$ show why CPT is related to Lorentz invariance and why "CPT violation implies Lorentz Violation."

They also explain what Fleming [65] calls "The Dependence of Lorentz Boost generators on The Presence and Nature of Interactions"; or perhaps one should reverse that dependence.

6 36 interactions

The interactions which can be computed directly using the Lie bracket lead to a graphic way of describing the interaction. First, pick a γ_I, then chose a particle on the same row as the γ_I and another on the same column as the γ_I, this determines a rectangle with the two particles on opposite vertices. The result of these two particles interacting is the γ_I and the particle opposite the γ_I. Particles on the same row or column can interact via the γ_I in their row or column.

	γ_i	γ_2	γ_3	γ_4
ν	i	$-i$	0	0
$\bar{\nu}$	$-i$	i	0	0
H	i	0	$-i$	0
$\bar{H}$	$-i$	0	i	0
e^-	i	0	0	$-i$
e^+	$-i$	0	0	i
n	0	i	$-i$	0
$\bar{n}$	0	$-i$	i	0
π^-	0	i	0	$-i$
π^+	0	$-i$	0	i
p^+	0	0	$-i$	i
p^-	0	0	i	$-i$

This table is a correction of the corresponding table in [114] where the factors of i were omitted. They were not necessary at the level of discussion there, but they are necessary for further progress.

The first six interations are the particle-antiparticle interaction where the quantum numbers on the left and right are zero.

Interaction 1

$$\begin{pmatrix} \gamma_1 & \nu \\ \bar{\nu} & \gamma_2 \end{pmatrix}$$

Physically, this means:

$$\nu\bar{\nu} \leftrightarrow \gamma_2\gamma_1$$

Mathematically, we have:

$$[\nu, \bar{\nu}] = \gamma_2 - \gamma_1$$

Interaction 2

$$\begin{pmatrix} \gamma_1 & H \\ \bar{H} & \gamma_3 \end{pmatrix}$$

$$H\bar{H} \leftrightarrow \gamma_3\gamma_1$$

$$[H, \bar{H}] = \gamma_3 - \gamma_1$$

Interaction 3

$$\begin{pmatrix} \gamma_2 & n \\ \bar{n} & \gamma_3 \end{pmatrix}$$

$$n\bar{n} \leftrightarrow \gamma_3\gamma_2$$

$$n\bar{n} = \gamma_3 - \gamma_2$$

Interaction 4

$$\begin{pmatrix} \gamma_1 & e^- \\ e^+ & \gamma_4 \end{pmatrix}$$

$$e^-e^+ \leftrightarrow \gamma_4\gamma_1$$

$$[e^-, e^+] = \gamma_4 - \gamma_1$$

Interaction 5

$$\begin{pmatrix} \gamma_2 & \pi^- \\ \pi^+ & \gamma_4 \end{pmatrix}$$

$$\pi^-\pi^+ \leftrightarrow \gamma_4\gamma_2$$

$$\pi^-\pi^+ = \gamma_4 - \gamma_2$$

Interaction 6

$$\begin{pmatrix} \gamma_3 & p^- \\ p^+ & \gamma_4 \end{pmatrix}$$

$$p^-p^+ \leftrightarrow \gamma_4\gamma_3$$

Interaction 7

$$\begin{pmatrix} \gamma_1 & H \\ \bar{\nu} & n \end{pmatrix}$$

$$H\bar{\nu} \leftrightarrow n\gamma_1$$

There is another way to compute the results of an interaction, that is to add the quantum numbers of the incoming particles to produce the quantum numbers of the outgoing particles:

$$\begin{array}{ccccc} \bar{\nu} & -i & i & 0 & 0 \\ H & i & 0 & -i & 0 \end{array}$$

Adding, we obtain the quantum numbers of the neutron:

$$n \quad 0 \quad i \quad -i \quad 0$$

There is a problem with this method, particles whose quantum numbers are all zero will not be detected.

Interaction 8

$$\begin{pmatrix} \gamma_1 & \nu \\ \bar{H} & \bar{n} \end{pmatrix}$$

$$\bar{H}\nu \leftrightarrow \bar{n}\gamma_1$$

Interaction 9

$$\begin{pmatrix} \gamma_1 & e^- \\ \bar{\nu} & \pi^- \end{pmatrix}$$

$$e^-\bar{\nu} \leftrightarrow \pi^-\gamma_1$$

Interaction 10

$$\begin{pmatrix} \gamma_1 & \nu \\ e^+ & \pi^+ \end{pmatrix}$$

$$\nu e^+ \leftrightarrow \pi^+\gamma_1$$

Interaction 11

$$\begin{pmatrix} \gamma_1 & e^- \\ \bar{H} & p^- \end{pmatrix}$$

$$e^-\bar{H} \leftrightarrow p^-\gamma_1$$

Interaction 12

$$\begin{pmatrix} \gamma_1 & H \\ e^+ & p^+ \end{pmatrix}$$

$$He^+ \leftrightarrow p^+\gamma_1$$

Interaction 13

$$\begin{pmatrix} \nu & H \\ \gamma_2 & n \end{pmatrix}$$

$$H\gamma_2 \leftrightarrow n\nu$$

Interaction 14

$$\begin{pmatrix} \bar{\nu} & \gamma_2 \\ \bar{H} & \bar{n} \end{pmatrix}$$

$$\gamma_2\bar{H} \leftrightarrow \bar{n}\bar{\nu}$$

Interaction 15

$$\begin{pmatrix} \nu & e^- \\ \gamma_2 & \pi^- \end{pmatrix}$$

$$e^-\gamma_2 \leftrightarrow \pi^-\nu$$

Taking the neutrino to the other side, where it becomes an antineutrino, we have the familiar:

$$e^-\bar{\nu}\gamma_2 \leftrightarrow \pi^-$$

Interaction 16

$$\begin{pmatrix} \bar{\nu} & \gamma_2 \\ e^+ & \pi^+ \end{pmatrix}$$

$$\gamma_2 e^+ \leftrightarrow \pi^+\bar{\nu}$$

Interaction 17

$$\begin{pmatrix} \nu & H \\ \bar{n} & \gamma_3 \end{pmatrix}$$

$$H\bar{n} \leftrightarrow \gamma_3\nu$$

Interaction 18

$$\begin{pmatrix} \bar{\nu} & n \\ \bar{H} & \gamma_3 \end{pmatrix}$$

$$n\bar{H} \leftrightarrow \gamma_3\bar{\nu}$$

Interaction 19

$$\begin{pmatrix} H & e^- \\ \gamma_3 & p^- \end{pmatrix}$$

$$e^-\gamma_3 \leftrightarrow p^-H$$

Interaction 20

$$\begin{pmatrix} \bar{H} & \gamma_3 \\ e^+ & p^+ \end{pmatrix}$$

$$\gamma_3 e^+ \leftrightarrow p^+\bar{H}$$

Interaction 21

$$\begin{pmatrix} \bar{H} & p^- \\ e^+ & \gamma_4 \end{pmatrix}$$

$$p^-e^+ \leftrightarrow \gamma_4\bar{H}$$

Interaction 22

$$\begin{pmatrix} H & e^- \\ p^+ & \gamma_4 \end{pmatrix}$$

$$e^- p^+ \leftrightarrow \gamma_4 H$$

Interaction 23

$$\begin{pmatrix} n & \pi^- \\ p^+ & \gamma_4 \end{pmatrix}$$

$$\pi^- p^+ \leftrightarrow \gamma_4 n$$

Interaction 24

$$\begin{pmatrix} \bar{n} & p^- \\ \pi^+ & \gamma_4 \end{pmatrix}$$

$$p^- \pi^+ \leftrightarrow \gamma_4 \bar{n}$$

Interaction 25

$$\begin{pmatrix} n & \pi^- \\ \gamma_3 & p^- \end{pmatrix}$$

$$\pi^- \gamma_3 \leftrightarrow p^- n$$

Interaction 26

$$\begin{pmatrix} \bar{n} & \gamma_3 \\ \pi^+ & p^+ \end{pmatrix}$$

$$\gamma_3 \pi^+ \leftrightarrow p^+ \bar{n}$$

Interaction 27

$$\begin{pmatrix} \nu & e^- \\ \pi^+ & \gamma_4 \end{pmatrix}$$

$$e^- \pi^+ \leftrightarrow \gamma_4 \nu$$

$$\pi^- \bar{n} \leftrightarrow p^- \gamma_2$$

Interaction 28

$$\begin{pmatrix} \bar{\nu} & \pi^- \\ e^+ & \gamma_4 \end{pmatrix}$$

Interaction 29

$$\begin{pmatrix} \gamma_2 & \pi^- \\ \bar{n} & p^- \end{pmatrix}$$

Interaction 30

$$\begin{pmatrix} \gamma_2 & n \\ \pi^+ & p^+ \end{pmatrix}$$

$$n\pi^+ \leftrightarrow p^+\gamma_2$$

$$n \leftrightarrow p^+\gamma_2\pi^-$$

Once the above rules for interactions are understood, there is an obvious generalization possible:

1. Take any two particles not in the same row or column.
2. Form the rectangle determined by these particles.
3. The results of the interaction of the given particles are the particle on the opposite corners. This interaction can go either way, depending on the relative energies.

The rest of the interactions have to be verified by showing that the sum of quantum numbers of the particles on the left equals sum of the quantum numbers of the particles on the right. There are only six possibilities generated by the generalized rules not listed in Table 1; these will be called secondary interactions and are:

Interaction 31

$$\begin{pmatrix} \bar{\nu} & n \\ e^+ & p^+ \end{pmatrix}$$

$$ne^+ \leftrightarrow p^+\bar{\nu}$$

On the left we have:

$$\begin{array}{ccccc} e^+ & -i & 0 & 0 & i \\ n & 0 & i & -i & 0 \\ \\ Sum & -i & i & -i & i \end{array}$$

On the right we have:

$$\begin{array}{ccccc} \bar{\nu} & -i & i & 0 & 0 \\ p^+ & 0 & 0 & -i & i \\ \\ Sum & -i & i & -i & i \end{array}$$

Thus the sum of the quantum numbers on the left is equal to the sum of the quantum numbers on the right.

Taking the positron to the other side, where it becomes an electron, we have the familiar:

$$n \leftrightarrow p^+\bar{\nu}e^-$$

Interaction 32

$$\begin{pmatrix} \nu & e^- \\ \bar{n} & p^- \end{pmatrix}$$

$$e^-\bar{n} \leftrightarrow p^-\nu$$

On the left we have:

$$\begin{array}{ccccc} e^- & i & 0 & 0 & -i \\ \bar{n} & 0 & -i & i & 0 \end{array}$$

On the right:

$$\begin{array}{ccccc} Sum & i & -i & i & -i \end{array}$$

$$\begin{array}{ccccc} \nu & i & -i & 0 & 0 \\ p^- & 0 & 0 & i & -i \end{array}$$

$$\begin{array}{ccccc} Sum & i & -i & i & -i \end{array}$$

Since Interaction 33 is the antiparticle conjugate of Interaction 32, the quantum numbers are the opposite, as expected.
Interaction 33

$$\begin{pmatrix} \nu & H \\ \pi^+ & p^+ \end{pmatrix}$$

$$H\pi^+ \leftrightarrow p^+\nu$$

For the quantum numbers on the left we have:

$$\begin{array}{ccccc} H & i & 0 & -i & 0 \\ \pi^+ & 0 & -i & 0 & i \end{array}$$

$$\begin{array}{ccccc} Sum & i & -i & -i & i \end{array}$$

$$\begin{array}{ccccc} \nu & i & -i & 0 & 0 \\ p^+ & 0 & 0 & -i & i \end{array}$$

$$\begin{array}{ccccc} Sum & i & -i & -i & i \end{array}$$

Interaction 34

$$\begin{pmatrix} \bar{\nu} & \pi^- \\ \bar{H} & p^- \end{pmatrix}$$

$$\pi^-\bar{H} \leftrightarrow p^-\bar{\nu}$$

$$\begin{array}{ccccc} \bar{H} & -i & 0 & i & 0 \\ \pi^- & 0 & i & 0 & -i \end{array}$$

$$\begin{array}{ccccc} Sum & -i & i & i & -i \end{array}$$

$$\begin{array}{ccccc} \bar{\nu} & -i & i & 0 & 0 \\ p^- & 0 & 0 & i & -i \end{array}$$

$$\begin{array}{ccccc} Sum & -i & i & i & -i \end{array}$$

Interaction 35

$$\begin{pmatrix} H & e^- \\ n & \pi^- \end{pmatrix}$$

$$e^- n \leftrightarrow \pi^- H$$

$$\begin{array}{ccccc} e^- & i & 0 & 0 & -i \\ n & 0 & i & -i & 0 \end{array}$$

$$\begin{array}{ccccc} Sum & i & i & -i & -i \end{array}$$

$$\begin{array}{ccccc} H & i & 0 & -i & 0 \\ \pi^- & 0 & i & 0 & -i \end{array}$$

$$\begin{array}{ccccc} Sum & i & i & -i & -i \end{array}$$

Interaction 36

$$\begin{pmatrix} \bar{H} & \bar{n} \\ e^+ & \pi^+ \end{pmatrix}$$

$$\bar{n} e^+ \leftrightarrow \pi^+ \bar{H}$$

$$\begin{array}{ccccc} e^+ & -i & 0 & 0 & i \\ \bar{n} & 0 & -i & i & 0 \end{array}$$

$$\begin{array}{ccccc} Sum & -i & -i & i & i \end{array}$$

$$\begin{array}{ccccc} \bar{H} & -i & 0 & i & 0 \\ \pi^+ & 0 & -i & 0 & i \end{array}$$

$$\begin{array}{ccccc} Sum & -i & -i & i & i \end{array}$$

We have used the quantum numbers to identify the possible interactions. Later we will see two other ways of interpreting the interactions.

These secondary interactions cannot be described in terms of the bracket in the matrix representation.

Now we are in a quandary, if we want to work in a Lagrangian Field Theory, which is standard in Quantum Field Theory, these elements of the Lie Algebra cannot generate the symmetries, what would it mean to exponentiate a proton? There are two avenues open to us, we can renounce the Lagrangian approach or we can look for another meaning to the Lie algebra. Here we recall the work of Peter Higgs, who suggested that elementary particles are best modeled by *broken* symmetries.

To construct the symmetries from the broken symmetries, we look for appropriate linear combinations of the generators. We need to form linear combinations of all the generators in such a way that the whole set fits together to form a Lie algebra. Unfortunately, the solution is not unique, there are several candidates. Working with just the π^- and π^+ for instance, we could construct

$$\tau_1 = (\pi^+ + \pi^-)$$

$$\tau_2 = -i(\pi^+ - \pi^-).$$

$$\tau_3 = [\tau_1, \tau_2]$$

This linear combination leads to the physicist's version of $su(2)$. Recall that physicists multiply the mathematicians' generators by an extra factor of i. That tradition will not be followed here.

The mathematician's version of $su(2)$ is obtained with:

$$\sigma_1 = (\pi^- - \pi^+)$$

$$\sigma_2 = i(\pi^- + \pi^+)$$

$$\sigma_3 = [\sigma_1, \sigma_2]$$

While another linear combination:

$$\rho_1 = (\pi^- + \pi^+);$$

$$\rho_2 = i(\pi^- - \pi^+);$$

$$\rho_3 = [\rho_1, \rho_2];$$

gives the mathematician's version of $su(1,1)$.

Third theoretical input: A little experience comes into play here. We know that the Lorentz group is necessary for both electromagnetism and relativity, so we require that the Lie algebra include $so(3,1)$. Given the presence of $su(2)$ and $su(1,1)$ it seems that $u(3,1)$ is the appropriate choice. We obtain $u(3,1)$ by taking the $su(2)$ (compact) linear combinations for the electrically neutral particles and the $su(1,1)$ (noncompact)linear combinations for the electrically charged particles. This distinction is necessary since the electric force is long range and the other forces are not.

Fourth theoretical input: The $u(3,1)$ Lie algebra exponentiates to the Lie Group $U(3,1)$. Now it appears clear that this $U(3,1)$ is in the bundle of some fiber bundle with spacetime as the base. This was the starting point in [114]. There the Lie group $SU(3,2)$ was taken as the fundamental symmetry of nature. This leads to the principle fiber bundle:

$$SU(3,2) \rightarrow SU(3,2)/SU(3,1) \times U(1)$$

In order to obtain all the quantum numbers, this had to be slightly modified to include one more generator:

$$U(3,2) \rightarrow U(3,2)/U(3,1) \times U(1)$$

In either case, the base space provides a complex spacetime $U(3,2)/U(3,1) \times U(1)$ which I dubbed Quantum Anti-de Sitter Space ($QAdS$) because it provides a geometric setting which allows us to quantize the field theory and to give a geometric meaning to the structure of the elementary particles. $QAdS$ is a complexification of de Sitter Space (AdS) which is the base space in the principle fiber bundle:

$$SO(3,2) \rightarrow SO(3,2)/SO(3,1)$$

Einstein [50] considered "A Generalized Theory of Gravitation" in which "...the total field is represented by a Hermitian tensor." The natural setting for a Hermitian tensor is a complex space-time. Einstein goes on to say:

> From a group theoretical point of view the introduction of a Hermitian tensor is somewhat arbitrary...

In our setting, with the complex space-time $QAdS = U(3,2)/U(3,1) \times U(1)$ defined in terms of group theory, the introduction of a Hermitian tensor is quite natural.

7 The Vortex-Spin Networks

Each elementary particle has only two nonzero quantum numbers, one and negative one. These numbers can be displayed in a directed graph with two vertices with the direction of the graph going from the positive quantum number to the negative. For instance, we represent e^- by the directed line segment:

$$1 \;\rightarrow\; 4$$

This allows us to think of vertex 1 as a source and vertex 4 as a sink. The anti-particle is then represented by the same edge traversed in the opposite direction.

$$4 \;\rightarrow\; 1$$

Thus, we have a vortex representation of the elementary particles. But there is no representation of the γ_i. To this end, we need to introduce a fifth vertex, representing the fact that the elementary particles interact via gravitation. Then we have a graphical representation of the elementary particles.

$$\begin{array}{rccc} & 1 & \rightarrow & 4 \\ e^- & \uparrow & \swarrow & \\ & 5 & & \end{array}$$

$$\begin{array}{rccc} & 2 & \rightarrow & 1 \\ \bar{\nu} & \uparrow & \swarrow & \\ & 5 & & \end{array}$$

$$\begin{array}{rccc} & 2 & \rightarrow & 3 \\ n & \uparrow & \swarrow & \\ & 5 & & \end{array}$$

$$\begin{array}{rccc} & 1 & \rightarrow & 3 \\ H & \uparrow & \swarrow & \\ & 5 & & \end{array}$$

$$\begin{array}{rccc} & 2 & \rightarrow & 4 \\ \pi^- & \uparrow & \swarrow & \\ & 5 & & \end{array}$$

$$\begin{array}{rccc} & 4 & \rightarrow & 3 \\ p^+ & \uparrow & \swarrow & \\ & 5 & & \end{array}$$

The antiparticles have the arrows going in the opposite direction. Thus, it seems that the edge from vertex 4 to vertex 5 represents the photon, the generator of the electric charge. However, since that edge is traversed in one direction for the electron and the opposite direction for the positron, and the photon is its own antiparticle, this cannot be the final picture. We still need to analyze the particle interactions.

The above graphs are similar to the spin-networks:

> A spin network in S is a triple $\psi = (\gamma, \rho, \iota)$ consisting of: 1. a graph γ in S. 2. for each edge e of γ, an irreducible representation ρ_e of G, 3. for each vertex v of γ an intertwining operator
>
> $$\iota_v : \rho_1 \otimes ... \otimes \rho_n \rightarrow \rho_{1'} \otimes ... \otimes \rho_{m'}$$
>
> from [8].

The difference between spin-networks and the vortex interactions analyzed above is that for each edge e of γ, instead of an irreducible representation ρ_e of G, we have an *element* of the Lie algebra. But also our interactions include the strong interaction, the weak interaction and the electric charge interaction while the standard spin-network just models spin. Spin is included in our picture, it is just not a fundamental interaction.

Since the index 5 is that of space-time, these diagrams suggest that a γ_I, i.e. a field, is an excitation of space-time while elementary particles are bridges between excited states of space-time. This picture will be pursued later.

8 The Interactions

The first interaction we will analyze will be the electron interacting with the proton. To find the result of the interaction, we first align the edge the particles share:

$$\begin{array}{ccccccc} e^- & & & & p^+ & & \\ 1 & \leftarrow & 5 & & 5 & \leftarrow & 3 \\ & \searrow & \uparrow & & \downarrow & \nearrow & \\ & & 4 & & 4 & & \end{array}$$

The 4-5 edge is aligned and we adjoin the two, obtaining:

$$\begin{array}{ccccc} e^- & & & p^+ & \\ 1 & \leftarrow & 5 & \leftarrow & 3 \\ & \searrow & \uparrow\downarrow & \nearrow & \\ & & 4 & & \end{array}$$

The photon, γ_4 is then represented by

$$\begin{array}{ccc} \gamma_4 & & \\ 4 & \leftrightarrow & 5 \end{array}$$

To complete the analysis of the interaction, we cut out the photon and obtain:

$$\begin{array}{ccccc} e^- & & & p^+ & \\ 1 & \leftarrow & 5 & \leftarrow & 3 \\ & \searrow & & \nearrow & \end{array}$$

The two arrows rotate to merge and we end up with:

$$\begin{array}{ccccc} & & 5 & & \\ & \swarrow & & \nwarrow & \\ 1 & & \rightarrow & & 3 \end{array}$$

Which is H, hydrogen.

We anticipate that the other gammas will have similar representations:

$$\begin{array}{ccc} \gamma_1 & & \\ 1 & \leftrightarrow & 5 \end{array}$$

$$\begin{array}{ccc} \gamma_2 & & \\ 2 & \leftrightarrow & 5 \end{array}$$

$$\begin{array}{ccc} \gamma_3 & & \\ 3 & \leftrightarrow & 5 \end{array}$$

Next, we look at the interaction of the proton and the pion. We align the two matching edges:

$$\begin{array}{ccccccc} \pi^- & & & & p^+ & & \\ 2 & \leftarrow & 5 & & 5 & \leftarrow & 3 \\ & \searrow & \uparrow & & \downarrow & \nearrow & \\ & & 4 & & 4 & & \end{array}$$

Again, we merge the two aligned edges:

$$\begin{array}{ccccc} \pi^- & & & p^+ & \\ 2 & \leftarrow & 5 & \leftarrow & 3 \\ & \searrow & \uparrow\downarrow & \nearrow & \\ & & 4 & & \end{array}$$

The γ_4 vertex is removed:

$$\begin{array}{ccccc} \pi^- & & & p^+ & \\ 2 & \leftarrow & 5 & \leftarrow & 3 \\ & \searrow & & \nearrow & \end{array}$$

And, finally the two arrows are rotated to merge obtaining:

$$\begin{array}{ccccc} & & 5 & & \\ & \swarrow & & \nwarrow & \\ 2 & & \rightarrow & & 3 \end{array}$$

Which we recognize as the neutron.

In order to analyze a particle-antiparticle pair, a diagram of the type above is not sufficient since the pair share two vertices in common. Instead, we will overlap the two graphs. Look at the electron-positron pairing, with the e^- arrows drawn going counter clockwise on the outside:

$$\begin{array}{cccc} 5 & & e^- & \nwarrow \\ & \searrow\nwarrow & & \\ \downarrow\uparrow & & 4 & \\ & \swarrow\nearrow & & \\ 1 & & e^+ & \swarrow \end{array}$$

The flows on the 1-4 edge cancel, leaving:

$$\begin{array}{ccc} 5 & & \\ & \searrow\nwarrow & \\ \downarrow\uparrow & & 4 \\ & & \\ 1 & & \end{array}$$

Which are γ_1 and γ_4.

9 Particle Creation

Let's look at particle creation, beginning with γ_3 and γ_4, shown merging:

$$\begin{array}{ccc} 5 & & \\ & \searrow\nwarrow & \\ \downarrow\uparrow & & 4 \\ & & \\ 3 & & \end{array}$$

Note at this point that the γ_3 has arrows in both directions (i.e. it is a circle and hence generates a $u(1)$ subgroup.

The 3-4 gap is bridged and there are three circles, 5-4, 5-3 and 3-4:

$$\begin{array}{cccc} 5 & & & p^- \nwarrow \\ & \searrow\nwarrow & & \\ \downarrow\uparrow & & 4 & \\ & \swarrow\nearrow & & \\ 3 & & & p^+ \swarrow \end{array}$$

Then the three circles bifurcate into two bigger circles:

$$\begin{array}{ccc} 5 & & \\ & \searrow & \\ \uparrow & & 4 \\ & \swarrow & \\ 3 & & p^+ \swarrow \\ 5 & & p^- \nwarrow \\ & \nwarrow & \\ \downarrow & & 4 \\ & \nearrow & \\ 3 & & \end{array}$$

At this point, the $u(1)$ circle has broken and one direction is identified as the p^+ and the other direction identified as the p^-.

This bifurcation is a sign that we must be dealing with nonlinear field equations since linear equations do not manifest bifurcations.

10 Three Particle Interactions

There are many diagrams which could be drawn involving the interaction of three or more particles. We will give just four:

$$\begin{array}{ccccccccc}
 & & 2 & & \rightarrow & & 1 & & \\
 & \nearrow & \bar{n} & \searrow\nwarrow & \bar{\nu} & \swarrow\nearrow & e^- & \searrow & \\
3 & & \leftarrow & & 5 & & \leftarrow & & 4
\end{array}$$

$$\begin{array}{ccccccccc}
 & & 3 & & \rightarrow & & 2 & & \\
 & \nearrow & p^+ & \searrow\nwarrow & \bar{n} & \swarrow\nearrow & \bar{\nu} & \searrow & \\
4 & & \leftarrow & & 5 & & \leftarrow & & 1
\end{array}$$

$$\begin{array}{ccccccccc}
 & & 4 & & \rightarrow & & 3 & & \\
 & \nearrow & \pi^- & \searrow\nwarrow & p^+ & \swarrow\nearrow & \bar{H} & \searrow & \\
2 & & \leftarrow & & 5 & & \leftarrow & & 1
\end{array}$$

$$\begin{array}{ccccccccc}
 & & 1 & & \rightarrow & & 4 & & \\
 & \nearrow & \bar{\nu} & \searrow\nwarrow & e^- & \swarrow\nearrow & p^+ & \searrow & \\
2 & & \leftarrow & & 5 & & \leftarrow & & 3
\end{array}$$

11 Interpretation

There are many questions left unanswered: how do the different particles exhibit such different masses when the diagrams are all the same? Should this be represented in terms of sides of different tensions or different lengths or both? The interactions are of very different strengths, yet the the diagrams are all the same. How are these questions related? There is clearly a relationship between the mass of an elementary particle and the interactions in which it participates. However, the nature of that relationship remains a puzzle.

Yet there are some lessons to be learned from this simplistic analysis. The basic fields are those of the elementary particles: proton field, electron field, pion field, etc. There is no pure electromagnetic field. The photons are separate objects, the electric field of the proton is separate from the photons. Thus, the photons are not carriers of the electric field. Photons are elementary particles. This must be so, since excited states of photons have been identified as elementary partcles.

If we take these diagrams we have drawn as flow-lines then each particle is represented by the flow on a 3-torus rather than a 2-torus which is so common. This makes sense in terms of gyroscopes. Three rotations yield a stability which two don't. Thus it seems that inertia is essentially the conservation of angular momentum in a five complex dimensional space.

Einstein's general theory of relativity models the gravitational field as the curvature of space-time without saying how matter manages to curve space-time. I suggested that matter itself is the curvature of the space-time $\mathrm{U}(3,2)/\mathrm{U}(3,1)\times\mathrm{U}(1)$ but now the present work implies that the curvature of space-time is cause by the rotation of something, whether we call that something "space-time", a "background field", an aether, a plenum, an apeiron or what-ever is irrelevant.

12 Excited States

Continuing with the analysis of other particle decays, we will first work with excited states of the fundamental particles. To explain the similarities between the muon and the electron, these particles must have the same algebraic factor and differ only in the function factor. Since the Lie algebra factor accounts for all the particle interactions except gravity, the data confirm this observation as Morita stated:

> Today we know that the muon behaves like a heavy electron and the hypothesis of muon-electron universality introduces the same interaction with the coupling constants for both muon and electron.[124]

and affirmed by Jauch and Rohrlich:

> ... the muon is known from its magnetic moment to be correctly described as a 'heavy electron' [100] (pp. 536-537)

For consistency, the two neutrinos ν_e and ν_μ also must have the same algebraic factor. Then in the decay

$$\mu^- \to e^- \nu_e \nu_\mu \tag{4}$$

the function factor of the μ^- decays into a lower energy function, the function factor of the e^-, while the excess energy goes to create what is essentially a particle-antiparticle pair. Since it is not exactly a particle-antiparticle pair, the correctness of this description of the decay is questionable, so further analysis is required. But this analysis can only be accomplished once we have the function factors in hand. Since the neutrinos have a mass, we would expect the heavier neutrinos (tau and mu) to decay into the electron neutrino. However, we should not expect neutrino oscillations in that the electron-neutrino will not turn spontaneously into a mu-neutrino.

Barut [10] argues that the muon cannot be an excited electron since we do not observe the decay $\mu \to e^-\gamma$. According to the present picture, in (4) the neutrino-antineutrino pair is essentially a photon. This decay may occur with the γ in turn forming a neutrino-antineutrino pair, but in too short a time span for present-day technology to detect it.

The next decay up for analysis is

$$\pi^- \to \mu^- \bar{\nu} \tag{5}$$

The algebraic factors of all of the particles have been identified:

$$\begin{array}{ccccc} \bar{\nu} & -i & i & 0 & 0 \\ \mu^- & i & 0 & 0 & -i \end{array} \tag{6}$$

Adding,

$$\pi^- \quad 0 \quad i \quad 0 \quad -i \tag{7}$$

Thus, the identification of the algebraic factor of the pion is confirmed. Continuing with the analysis of new decays, let us examine

$$K^- \to \mu^- \bar{\nu} \tag{8}$$

Modulo functions, we just saw that $\mu^- \bar{\nu} = \pi^-$, so (8) implies that

$$K^- =_F \pi^- \tag{9}$$

We analyze other decay routes of the K in the Appendix. The next decay leads to some interesting consequences and provides our first internal check for consistency:

$$\Lambda \to p^+ \pi^-$$

$$\begin{array}{ccccc} \pi^- & 0 & i & 0 & -i \\ p^+ & 0 & 0 & -i & i \end{array}$$

Adding:

$$\Lambda \quad 0 \quad i \quad -i \quad 0 \tag{10}$$

Thus, has the same Lie algebra factor as n. So from the decay

$$\Lambda \to n\pi^0 \tag{11}$$

we conclude that π^0 has all-zero spectrum, i.e., the Lie algebra factor of the is diagonal. Since γ is known to be diagonal, this observation is consistent with the known decay

$$\pi^0 \to 2\gamma \tag{12}$$

At this point it is not clear experimentally which γ_I are involved.

Since the strangeness number of the Λ is not zero, an interesting conclusion is that strangeness, like the muon number, is not an internal quantum number. This does not rule out the possibility that the strangeness could be an eigenvalue of one of the generalized Casimir operators of $u(3,2)$. We limit the present classification of the particles to an analysis of the spectra of the four basis elements of the Cartan subalgebra, the roots of the Lie algebra. This is the easy part of the classification because the eigenvalues are those of matrices. To complete the classification will require that we know the generalized Casimir operators of $u(3,2)$ and their eigenfunctions. Consequently, the task is highly nontrivial. We expect these Casimir operators to yield invariants corresponding to mass, momenta, magnetic moment, plus new quantum numbers. In this work, only the first steps are taken as justification for pursuing the entire program.

We now turn to the task of finding the Lie algebra factor of the particles in the Stable Particle Table of the Particle Data Group's Tables of Particle Properies. We tabulate this information in the Appendix with an analysis of other particle decay schemes. Let us now analyze:

$$\Sigma^+ \to n\pi^+ \tag{13}$$

$$\begin{array}{ccccc} n & 0 & i & -i & 0 \\ \pi^+ & 0 & -i & 0 & i \end{array}$$

Adding:

$$\begin{array}{ccccc} \Sigma^+ & 0 & 0 & -i & i \end{array}$$

which is the same algebraic factor as the proton. Given this observation, decays of the Σ into a proton plus diagonal decay products-a set of particles whose net product is diagonal (the sum of the roots is zero) should occur. This may be a particle whose algebraic factor is diagonal or a particle-antiparticle pair, or something more complicated. Since the π^0 is diagonal, such a decay is

$$\Sigma^+ \to p^+\pi^0 \tag{14}$$

Since γ is diagonal, the observed decay

$$\Sigma^0 \to \Lambda\gamma \tag{15}$$

implies

$$\Sigma^0 =_F \Lambda =_F n \tag{16}$$

To find the Lie algebra factor of the particles with many decay routes, I will select the decay that is the easiest to analyze. The results tabulated in the Appendix show the consistency of this analysis.

The three decay modes of the τ,

$$\begin{array}{ccc} \tau^- & \to & \pi^-\nu \\ \tau^- & \to & \rho^-\nu \\ \tau^- & \to & K^-\nu \end{array} \tag{17}$$

allow us to conclude that the π^-, ρ^- and K^- all have the same algebraic factor:

$$\pi^- \quad =_F \rho^- \quad =_F K^- \tag{18}$$

For the daughter particles in (17), the roots are:

$$\begin{array}{ccccc} \nu & i & -i & 0 & 0 \\ \pi^- & 0 & i & 0 & -i \end{array}$$

Adding, we obtain the roots of the parent:

$$\tau^- \quad i \quad 0 \quad 0 \quad -i \tag{19}$$

which, as expected are the same as e^-.

The decays

$$\eta \to e^+e^-$$

$$K^0 \to e^+e^-$$

involve a particle-antiparticle pair and calculation of the bracket reveals that

$$\eta =_F K^0 = \gamma_1 - \gamma_2$$

Since K^0 is diagonal, the decay

$$D^+ \to K^0\pi^+ \tag{20}$$

allows us to conclude that

$$D^+ =_F \pi^+$$

Since D^0 decays into a particle-antiparticle pair,

$$D^0 \to \pi^+\pi^-$$

we calculate the bracket to obtain

$$D^0 =_F \gamma_2 - \gamma_4$$

Because η is diagonal, the decay

$$F^+ \to \eta\pi^+ \tag{21}$$

implies that

$$F^+ =_F \pi^+$$

Since D^0 is diagonal, from the decay

$$B^+ \to D^0 \pi^+ \tag{22}$$

we conclude

$$B^+ =_F \pi^+$$

The particle-antiparticle pair appearing in the decay

$$B^0 \to D^0 \pi^- \pi^+$$

implies that $B^0 =_F D^0 =_F \gamma_2 - \gamma_4$.

A diagonal daughter assists us again:

$$\Lambda_c^+ \to p^+ K^0 \tag{23}$$

Since K^0 is diagonal, we must have

$$\Lambda_c^+ =_F p^+$$

Since π^0 is diagonal, the decay

$$\Xi^0 \to \pi^0 \Lambda$$

implies

$$\Xi^0 =_F \Lambda$$

13 SOME COMPLICATIONS

All of the particles analyzed thus far were excited states of one of the fundamental particles. The particles analyzed from here on are not of this simple form; instead they are composite particles, some being excited states of nuclei. Since γ is diagonal, the decays

$$\Xi^- \to \Sigma^- \gamma \tag{24}$$

$$\Omega^- \to \Xi^- \gamma \tag{25}$$

imply

$$\Xi^- =_F \Sigma^- =_F \Omega^-$$

which shows that the Ξ^-, Σ^- and Ω^- all have the same algebraic factor. Identification of this factor would be easy if Σ^- were the antiparticle of Σ^+ but it is not. The analysis to this point has been effortless; at this stage we encounter our first subtlety. Analysis of the decay routes will reveal the mystery:

$$\Sigma^- \rightarrow n\pi^- \tag{26}$$

The algebraic factors and roots of the right-hand side of (26) are

$$\begin{array}{ccccc} n & 0 & i & -i & 0 \\ \pi^- & 0 & i & 0 & -i \end{array}$$

Adding, we obtain

$$\begin{array}{ccccc} \Sigma^- & 0 & 2i & -i & -i \end{array}$$

which is not an entry in Table 1.

(27)

The Ξ^- decays yield the same algebraic quantum numbers:

$$\Xi^- \rightarrow \Lambda\pi^- \tag{28}$$

The algebraic factors and roots of the right-hand side of (28) are

$$\begin{array}{ccccc} \Lambda & 0 & i & -i & 0 \\ \pi^- & 0 & i & 0 & -i \end{array}$$

Adding:

$$0 \quad 2i \quad -i \quad -i$$

which is again the same set of numbers, but not an entry in Table I.

$$\Xi^- \rightarrow p^+\pi^-\pi^- \tag{29}$$

The algebraic factors and roots of the right-hand side of (29) are

$$\begin{array}{ccccc} p^+ & 0 & 0 & -i & i \\ \pi^- & 0 & i & 0 & -i \\ \pi^- & 0 & i & 0 & -i \end{array}$$

Summing

$$0 \quad 2i \quad -i \quad i$$

which again is the same set of numbers, but not an entry in Table 1.

The numbers obtained from these various decays are consistent, but not an entry in Table I. Evidently we have the correct numbers for these three particles. The decays of the Ω^- serve to reinforce this conclusion:

$$\Omega^- \to \Lambda K^- \tag{30}$$

The algebraic factors and roots of the right-hand side are

$$\begin{array}{ccccc} \Lambda & 0 & i & -i & 0 \\ K^- & 0 & i & 0 & -i \end{array}$$

Summing

$$0 \quad 2i \quad -i \quad i$$

Next, we analyze the decay $\Omega^- \to \Xi^0\pi^-$:

$$\begin{array}{ccccc} \Xi^0 & 0 & i & -i & 0 \\ K^- & 0 & i & 0 & -i \end{array} \tag{31}$$

Summing

$$0 \quad 2i \quad -i \quad i$$

So we have confirmed these numbers as the algebraic quantum numbers of these three particles. But they are not entries in Table I. Every set of numbers encountered heretofore was in that table. The numbers in Table I are those numbers arising when two particles interact via the Lie bracket. Obviously some other model for the interaction is necessary at this stage. Earlier we saw another type of interaction which preserves the quantum numbers: the tensor product. The numbers for the Σ^- are consistent with

$$\Sigma^- =_F n \otimes \pi^- =_F \pi^- \otimes p^+ \otimes \pi^- \tag{32}$$

These numbers are again consistent with the interaction:

$$K^- p^+ \to \Omega^- K^+ K^0 \tag{33}$$

The roots of the left-hand side are

$$\begin{array}{ccccc} p^+ & 0 & 0 & -i & i \\ K^- & 0 & i & 0 & -i \\ & 0 & i & -i & 0 \end{array}$$

while the algebraic quantum numbers of the right-hand side are

$$\begin{array}{lcccc} K^+ & 0 & -i & 0 & i \\ \Omega^- & 0 & i & 0 & -i \\ & 0 & i & -i & 0 \end{array}$$

showing that the numbers agree before and after the interaction. This analysis raises another question: Which other particles require the tensor product for their description? The decay

$$\Lambda_c^+ \to \Delta^{++} K^- \tag{34}$$

with

$$\begin{array}{lcccc} \Lambda_c^+ & 0 & 0 & -i & i \\ K^- & 0 & i & 0 & -i \end{array}$$

implies that Δ^{++} has the algebraic quantum numbers

$$\begin{array}{lcccc} \Delta^{++} & 0 & -i & -i & 2i \end{array}$$

Again, this is not an entry in Table I. The double charge also would indicate that Δ^{++} is different from anything so far encountered. Again, these quantum numbers can be obtained from a tensor product:

$$\Delta^{++} =_F \pi^+ \otimes n \otimes \pi^+ \tag{35}$$

$$\begin{array}{lcccc} n & 0 & i & -i & 0 \\ \pi^+ & 0 & -i & 0 & i \\ \pi^+ & 0 & -i & 0 & i \end{array}$$

Adding:

$$\begin{array}{lcccc} \Delta^{++} & 0 & -i & -i & 2i \end{array}$$

The following conjecture is obvious: particles interact via the "tensor force" via an "exchange" of one factor. Thus, a particle of type $A =_F B \otimes C \otimes D$ is possible iff C can interact via bracket with both B and D. Then we have three ways to obtain the same algebraic quantum numbers:

$$A =_F B \otimes C \otimes D$$

$$A =_F B \otimes [C, D]$$

$$A =_F [B, C] \otimes D$$

This leads to the further conjecture that nuclei bond together by the tensor interaction with protons interchanging particles with the same algebraic factor as the pion such as the W. Since $n =_F [p^+, \pi^-]$, the nucleus of the deuteron is

$$n \otimes p^+ =_F [p^+, \pi^-] \otimes p^+ =_F p^+ \otimes [\pi^-, p^+]$$

We see that protons in the nucleus react by exchanging real (not virtual) pions. Thus, the new model of matter has tremendous implications for nuclear physics. We see that some of the "particles" now thought to be elementary are composite. Thus, there is no clear line between nuclear and particle physics. In the late 1930's there were several papers along this line. Many physicists visualized the nucleus as protons exchanging various particles. This model fell into disfavor with most physicists who, believing in particle democracy, felt that the neutron was as fundamental as the proton and hence the nucleus consisted of protons and neutrons held together by the exchange of some other (virtual) particles. I am thus advocating the return to the older viewpoint. Kursunoglu [101] and Barut [10] [11] have also advocated this return and the interested reader should refer to their papers for the history and further consequences of these ideas.

Specifying the Lie algebra factor of the particles provides a classification based on half of the information that will ultimately be available. Classification based on the Lie algebra factor has accounted for the four superselection rules: spin, baryon number, lepton number, and electric charge [60]. Further classification based on the generalized Casimir operators of $u(3, 2)$ is required. We expect that analysis to lead to further relations between the function factor and the algebraic factor of the particle. Once that is done, we will be able to calculate the masses and the transition probabilities. This problem was treated in a similar setting by Barut and Kleinert [13][14] [15] and by Herrick and Sinanoglu [87]. In the models based on compact groups, the Wigner-Eckhart theorem shows that analysis of the transition probabilities requires the Clebsch-Gordan coefficients for the group. But the Wigner-Eckhart theorem is not valid for noncompact groups, and consequently the analysis is neither routine nor straightforward.

From work done by other researchers, it is clear that the second-order Casimir operator will account for the mass-energy relationship. The role of the higher-order Casimir operators is not clear simply because there is no precedent theory with so many differential operators of higher order.

APPENDIX goes here

14 The Quantum Cult

> I shall argue presently that nobody yet understands the quantum theory
> —Howard Stein

Einstein had nothing but scorn for the development of quantum mechanics based on probability.

> I find the idea that there should not be laws for being but only laws for probabilities simply disgusting. —Albert Einstein in [167](p. 91)

In a letter to Schrödinger in May 1928, Einstein wrote:

> The Heisenberg-Bohr tranquilizing philosophy –or religion?– is so delicately contrived that, for the time being, it provides a gentle pillow for the true believer from which he cannot easily be aroused. So let him lie there.

Is quantum mechanics science "–or religion?–" A statement by one of Bohr's disciples clearly shows the religious undertones of the Copenhagen movement:

> Young physicists are raising doubts about the correctness of the basic ideas of quantum mechanics, and try to do better. These efforts are, I am afraid, rather futile, because they rest on a complete misunderstanding of the problem. . . there is nothing more accidental in the emergence of complementarity logic than in the emergence of man himself as a product of organic evolution. . . I may therefore conclude on an optimistic note by assuring the younger generation that the instruments of rational analysis we are handing over to them contribute to a further increase of our power of understanding. [152](p. 384)

The "emergence of complementarity logic" likened unto the "emergence of man" and thus of divine origin? If our efforts are to be futile, let us find that out for ourselves! All that any generation of scientists can say legitimately to the next generation is "We did our best, hopefully what we have done will allow you to do better". Bohr and his followers tried to cut off free inquiry and say that they had discovered ultimate truth—at that point their efforts stopped being science and became a revealed religion with Bohr as its prophet.

According to Larry Lauden [107], the efforts to cut off debate about the faults of quantum mechanics reaches even into philosophy:

> Until the 1920s, it was usually maintained that science was one form of knowledge, to be sure, but that there were also other disciplines–such as philosophy and theology–which also embodied claims to genuine knowledge. Where earlier thinkers had stressed the interdependence of various forms of knowledge, the positivists insisted that science–as the only genuine form of knowledge–must be hermetically sealed off from other activities. On their account, it is neither necessary nor desirable that scientific theories should be criticized from "outside," as it were; that is, in evaluating whether a scientific theory is acceptable, it is not appropriate to commend or criticize a theory in terms of its compatibility (or incompatibility) with anything "non-scientific."

Any field of knowledge which wants to cut off debate and outside criticism does not deserve to be called a science. Only a revealed religion does not allow its followers to question the teachings of the masters, in this sense, quantum theory is a cult. The problem of course is that the theory was firmly established before all the data was in.

15 The Reality of Matter Waves

In 1840, Michael Faraday explicitly introduced the idea of a field into physics. The idea of field was inherent in Newton's work on "centripetal force" and Descartes' concept of vortices in the ether. Faraday found that the phenomena of electricity and magnetism were best understood in terms of "lines of force." Faraday was convinced of the reality of these fields and Clerk Maxwell gave us Maxwell's equations which model these physical fields, but again modern quantum theory takes these fields to be to be merely probability distributions.

In 1927, de Broglie suggested that electrons were waves, with an associated momentum and energy defined in terms of the frequency of the wave. He envisioned the electron as a wave, but in the course of the evolution of quantum mechanics, the evidence for the reality of the matter wave was swept under the rug and modern quantum theory takes the wave to be to be a probability distribution.

Einstein eschewed the quantum mechanical viewpoint in which everything was expressed in terms of probabilities:

> I still believe in the possibility of a model of reality—that is to say, of a theory which represents things themselves and not merely the probability of their occurrence.[49] (page 20)

In order to find a new quantum mechanics which would satisfy Einstein's demands, we must show that quantum events have causes and we must show that the probabilistic interpretation of matter waves is not tenable. Preferably, the new interpretation of matter waves would be in terms of the geometric properties of space-time.

If we take this hint seriously, we would be following the path indicated by Schrödinger in a letter to Einstein dated 19 July, 1939:

> Dear Einstein,
>
> A few months ago, a Dutch newspaper carried a report which sounded comparatively intelligent that you have discovered something important about the connection between gravitation and matter waves. I would be terribly interested in that because I have really believed for a long time that the Ψ waves are to be identified with waves representing disturbances of the gravitational potential; not of course with those you studied first, but

> rather with ones that transport real mass, i.e. a non-vanishing T_{ij}. That is, I believe that one has to introduce matter into the general theory of relativity, which contains the T_{ij} only as "asylum ignorantiae" (to use your expression), not as mass points or something like that, but rather, shall we say, as quantized gravitational waves.[59] , p. 33)

There is a real problem with the interpretation of matter waves as waves of probability. The matter wave carries energy and momentum defined in terms of the wave length and frequency of the wave. How can a "nonphysical probability distribution" carry energy and momentum? The electromagnetic field carries energy and momentum defined in terms of the field strength. The field strength in modern quantum theory is a probability distribution. Again, we must ask "How can a nonphysical probability distribution carry energy and momentum?"

I believe that the solution to the problem can be found in the physics developed between Faraday and de Broglie: the vortex theory of matter.

16 Aethereal Matter

In 1884, Charles Howard Hinton described antimatter in terms of the vortex theory of matter. He didn't get the credit he deserved because antimatter wasn't discovered until 1932, by which time the vortex theory of matter had been abandoned and his prediction long forgotten. But still his description is clear:

> Take a pencil, and round it twist a strip of paper–a flat spill will do. Now, having fastened the ends on to the pencil by two pins, so that it will not untwist, hold the paper thus twisted on the pencil at right angles to the surface of a looking glass; and in the looking-glass you will see its image. Now take another pencil and another piece of paper, and make a model of what you see in the glass. You will be able to twist this second piece of paper in a spiral round this second pencil so that it is and exact copy of what you see in the glass. Now put the two pencils together end to end, as they would be if the first pencil were to approach the glass until it touched it, meeting its image: you have the real copy of the image instead of the image itself. Now pin together the two ends of the pieces of paper, which are near together... hold firmly and pull the other ends...,so as to let each twist exercise its nature on the other.
>
> You will see that the two twists mutually annihilate each other. Without your unwrapping the paper, the twists both go, and nothing is left of them.
>
> This is the mechanical conception I wish you to adopt—there are such things as twists. Suppose by some means to every twist there is produced its image twist. These two, the twist and its image, may exist separately; but suppose that whenever a twist is produced, its image twist is also produced, and that these two when put together annihilate each other.
>
> If we consider a twist and its image, they are but the simplest and most rudimentary type of an organism. What holds good of a twist and its image twist would hold good of a more complicated arrangement also. If a bit of structure apparently very unlike a twist, and with manifold parts and differences in it—if such a structure were to meet its image structure, each of them would

> instantly unwind the other, and what was before a complex and compound whole, opposite to an image of itself, would at once be resolved into a string of formless particles. A flash, a blaze and all would be over.
>
> To realize what this would mean we must conceive that in our world there were to be for each man somewhere a counter-man, a presentment of himself, a real counterfeit, outwardly fashioned like himself, but with his right hand opposite his original's right hand. Exactly like the image of the man in a mirror.
>
> And then when the man and his counterfeit met a sudden whirl, a blaze, a little steam, and the two human beings, having mutually unwound each other, leave nothing but a residuum of formless particles.

Hinton was not quite right. At the end, instead of "a residuum of formless particles" there would be a great flash of energy, but I could not improve on the rest of Hinton's description of antimatter. Which is rather strange, wonderful and most marvelous in that Hinton's words were published in 1884. Charles Howard Hinton was a mathematician who wrote science fiction and the above description of antimatter appeared in Scientific Romances, Volume 1 (1884).

I didn't read it there. Because the story of Charles Hinton gets even stranger. Not only did he predict antimatter forty two years before Dirac received the Nobel prize for predicting it, he talked about topics of current interest: higher dimensions. I read Hinton's description of antimatter in *Speculations on the Fourth Dimension; Selected Writings of Charles H. Hinton*, [153]. Just a few pages after this remarkable prediction, Hinton described the role of higher dimensions in determining the structure of matter as a vortex in four dimensions.

Paul Dirac won a Nobel Prize for his prediction of anti-matter. Dirac concluded his Nobel acceptance speech with a conjecture:

> If we accept the view of complete symmetry between positive and negative electric charge so far as concerns the fundamental laws of Nature, we must regard it rather as an accident the Earth (and presumably the whole solar system), contains a preponderance of negative electrons and positive protons. It is quite possible that for some of the stars it is the other way about, these

> stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind. The two kinds of stars would show exactly the same spectra, and there would be no way of distinguishing them by present astronomical methods.

This model of the universe was developed by Hannes Alfven and Oskar Klein [4] and discussed in the books by Alfven [2, 3] and Lerner [111]. Klein's mechanism for separating matter and antimatter is described in terms of plasmas, hence the name, the cosmic plasma model of the universe [139]. In the plasma universe, there are equal amounts of matter and antimatter, but they are very far apart implying that the universe is much larger than implied by the current big bang model.

17 Wave or Particle?

If light were a particle, it would behave in a certain manner according to certain equations which govern the behavior of particles. Sometimes it does, and sometimes it doesn't. If light were a wave, it would behave in a certain manner according to certain equations which describe the behavior of waves.

Sometimes it does, and sometimes it doesn't. Sometimes one model works best, sometimes the other. Bohr's way out of this impasse was to introduce a "new logical instrument." He called it "complementarity," denoting thereby the logical relation between two descriptions or sets of concepts which, though mutually exclusive, are never-the-less both necessary for an exhaustive description of the situation." [96].

Bohr wanted to retain both the particle and the wave aspects of light. These are classical concepts which have limited applicability in a relativistic quantum world. Bohr's problem has been encountered in other settings by other authors. Mathematician Lars Garding [68] discussed the interplay between models and reality:

> I believe that man has a theoretical drive. His brain is a sorting machine where outside impulses are stored, ordered and reworked into models of the world. Overwhelmed by a confusing and complicated reality, he sometimes takes refuge in the simple and safe world of the models... For many people, the model that they believe in takes the place of the real world. Metaphorically speaking, they live in the model.

In terms of mathematical physics, one often encounters physicists who have confused the mathematical model with reality. To ask whether light is a particle or a wave is an example of such confusion. This was Bohr's fundamental philosophical error.

According to Beller [18]:

> The central pillars of the complementarity principle are wave-particle duality, the definition of concepts versus observational possibilities and the indispensability of classical concepts.

"Wave-particle duality" is just a statement of the problem, it does not resolve anything! To say that light sometime behaves like a particle and sometimes like a wave is to admit that we don't have a model of light which covers all experimental situations. We should be looking for such a model of light instead of formalizing our ignorance as a profound principle of nature. We will never progress in our understanding of nature as long as we sit in a state of self-congratulatory compliancy.

Classical concepts have been found inadequate in many areas of modern physics, why elevate them to such an exalted position? Particles and waves are classical concepts which have proven themselves inadequate for the description of the subatomic world. If light were a particle, it would behave in a certain manner according to certain equations which govern the behavior of particles. Sometimes it doesn't, therefore light is not a particle. If light were a wave, it would behave in a certain manner according to certain equations which describe the behavior of waves. Sometimes it doesn't, therefore light is not a wave. Consequently, light is *neither* a wave nor a particle. These classical concepts are inadequate for the description of phenomena at the atomic and subatomic levels.

The inertia of classical concepts is a deep problem as Weinberg [174] points out:

> Bohr's version of quantum mechanics was deeply flawed, but not for the reason Einstein thought. The Copenhagen interpretation describes what happens when an observer makes a measurement but the observer and the act of measurement are themselves treated classically This is surely wrong: Physicists and their apparatus must be governed by the same quantum mechanical rules that govern everything else in the universe.

The job of science is to describe reality, not just our sense perception of reality. We know that our senses deceive us. Even when our senses are amplified by magnificent gadgets, our sense perception of reality is not reality.

The complementarity principle is an affirmation of the delusion that what we perceive is reality. The photon is a particle or it is a wave. The difference is in the experimental set-up. That's rubbish. Not since the popularity of Ptolemaic epicycles has so large a fraction of the scientific community been taken in by a faulty philosophy.

Quantum Mechanics is a theory of observation. But observations lie several levels above subatomic reality. Once we understand that subatomic reality and how our senses work we will be able to explain what we observe. The reason for the confusion in quantum theory is that it tries to predict what will happen without any understanding of the mechanisms underlying the phenomena. That is the reason quantum mechanics relies so heavily on probability.

Another problem as pointed out by Sachs is that quantum mechanics

> ... tacitly assumes that what it is that is real is exhausted by what one can measure.[157]

18 The Setting for Quantization via Unification

> ...since G_c is mathematically more intelligible than G_∞, it looks as though the thought might have struck some mathematician, fancy-free, that after all, as a matter of fact, natural phenomena do not possess an invariance with the group G_∞, but rather with a group G_c, c being finite and determinate, but in ordinary units of measure, extremely great. Such a premonition would have been an extraordinary triumph for pure mathematics.
>
> —Hermann Minkowski

The search for a method of quantization that is consistent with the mathematical requirements of general relativity has been unsuccessful. There is no theory of gravity consistent with the requirements of quantum mechanics. There are enough clues from quantum mechanics and from general relativity to clearly indicate that a new theory of quantum gravity must be based on group theory via an appropriate Lie Group G, differential geometry in the form of a homogeneous space G/H and operator theory in the form of the Casimir operators and the Cartan subalgebra of the Lie algebra of G.

Relativistic quantum theory requires the Lorentz group. In standard quantum field theory, the Lorentz group is then extended to include translations which leads to the *Poincaré* Group.

Other theories of elementary particles require "internal symmetries". For several years, there were attempts to combine internal and space-time symmetries into one large group. O'Raifeartaigh [134] and others declared these attempts dead by with the so-called "no-go" theorems. These no-go theorems were supposed to have shown that internal and space-time symmetries could not be combined. However, a closer look reveals that they only proved that the *Poincaré* group could not be extended in an appropriate way. Those theorems should have been called "The no way, *Poincaré* Theorems." There are other viable space-time groups, notably the de Sitter groups $SO(4,1)$ and $SO(3,2)$ which are not ruled out by the no-go theorems. This led the author to question if there were other possibilities for combining space-time and internal symmetries [112, 113, 114]. The ideal group for the unification of physics, G, would be a simple group containing the Lorentz group and $H = SU(3) \times SU(2) \times U(1)$.

Since G contains the Lorentz group, it is noncompact and a reasonable requirement is that H is the maximal compact subgroup of G. These requirements lead uniquely to $G = SU(3,2)$. It is a major point that the use of $SU(3,2)$ can be justified from both the requirements of elementary particle physics and from the requirements of general relativity. Viewing G as an extension of the de Sitter group $SO(3,2)$ led to the consideration of the complex-space-time $SU(3,2)/SU(3,1) \times U(1)$ as a complexification of $SO(3,2)/SO(3,1)$, the well studied Anti-de Sitter space. The vertical bundle represents the 'internal symmetries' of elementary particles.

In order to obtain additive quantum numbers, we take the observables including the dynamic operator D to be a first order (later we will remove this restriction) differential operator acting on a vertical vector field:

$$[D, fX] = (Df)X + f[D, X]$$

The operator should be hermitian, or skew hermitian: but this is automatically satisfied if the inner product is the Killing form and the wave functions are in the Lie algebra of vector fields: $< D\phi, \psi >= - < \phi, D\psi >$. The dynamical operator governs the time evolution of the system and should not change particle types. If the dynamical operator is not to change particle types, we must demand that $[D, X] = 0$ for all X in $su(3,1) \times u(1)$. The dynamical operator should also be translation invariant and hence must commute with all the generators of $su(3,2)$ and forces us to extend the group. But simultaneously, we must demand that the evolution under D stay in the base space. Fortunately, these demands are met if we take D to be the generator of $u(3,2)$ which is not in $su(3,2)$. The space $U(3,2)/U(3,1) \times U(1)$ is diffeomorphic to $SU(3,2)/SU(3,1) \times U(1)$ so the underlying geometry is unchanged by the introduction of D.

19 Noncanonical commutation rules

The standard treatment of nonrelativistic Quantum mechanics was developed using generalized coordinates q_i, the conjugate momenta p_j, $(i, j = 1, 2, 3, 4)$ and the "canonical commutation relations" (CCR) :

$$CCR1 \qquad [q_i, q_j] = 0$$

$$CCR2 \qquad [p_i, p_j] = 0$$

$$CCR3 \qquad [q_i, p_j] = \delta_i^j$$

Where δ_i^j is the Kronecker delta function defined by:

$$\delta_i^j = \begin{cases} 1 & if \quad i=j \\ 0 & if \quad i\neq j \end{cases} \tag{36}$$

Since the trace of $[q_i, p_j]$ is zero and the trace of the n dimensional identity is n, we see that CCR3 cannot be satisfied in finite dimensions if we assume that δ_i^j is the identity matrix.

From the formal theory of Lie algebras. we recall the Theorem of Ado: Any finite dimensional Lie algebra has a faithful representation by finite matrices (cf. Varadarajan [173]).

We must conclude that the "CCR" are not the commutation rules for a Lie Algebra. If the "CCR" are not legitimate commutation relations defining a Lie algebra, how are we to interpret the "CCR" ? From differential geometry we recall the theorem that for any Lie group G, any $A \in L(G)$, the Lie algebra of G, there is a unique maximal geodesic $\gamma(t)$ in G with $\gamma(0) = e$ (the identity of G) and $\gamma'(0) = A$. The mapping $A \to \gamma(t)$ is called the exponential mapping, $exp(tA)$.

The basic geometric setting for the CCR is R^3 which is a manifold of a Lie group under addition and the Lie algebra which is the tangent space of the manifold which is the Lie algebra of partial differential operators $\partial_x, \partial_y, \partial_z$, then the geodesics are merely the mappings $t \to tx$. etc. and the exponential mapping: Exp: $L(G) \to G$ is:

$$Exp(\partial_x) = x$$

$$Exp(\partial_y) = y$$

$$Exp(\partial_z) = z$$

Then in the Canonical Commutation relations. $CCR2$ is the relation between the partial derivatives. $CCR3$ is the relationship between the variables and the corresponding partial derivatives. But $CCR1$ is an attempt to define an additional algebraic structure on the underlying manifold R^3, considered as a Lie Group, beyond the vector addition which defines the group operation (and not the Lie Algebra), evidently intended to imply that the measurement of coordinates is abelian. The problem with this expression is that the only operation on the group is vector addition while $CCR1$ requires both multiplication and subtraction. $CCR1$ is then mathematical nonsense because the multiplication is coordinate dependent.

The momenta are operators in the tangent space but there can be no operators which correspond to positions since positions (coordinates) relate to elements of the manifold. Beyond the mathematical inconsistencies, there are real physical problems with the CCR. Since the uncertainty principle does not apply to commuting variables, $[q_1, p_2] = 0$ means that we can set up an experiment which will measure the first component of the position q_1 of a particle and simultaneously measure the second component p_2 of its momentum to arbitrary accuracy! Three of the CCR:

$$[q_1, p_2] = 0$$
$$[q_1, p_3] = 0$$
$$[p_2, p_3] = 0$$

imply that we could run an experiment in which the position q_1, and the momenta p_2 and p_3 were measured simultaneously with arbitrary accuracy! This is physically absurd, no such experiment exists. Essentially, this result means that if the p_i were to represent the momenta, the q_i could not be the position.

There are further problems with the standard mathematical treatment of the CCR's. These problems stem from notational confusion with the Kronecker delta function. When dealing with operators $q = x$ and $p = \partial_x$ then δ_i^j is a scalar. If we deal with quantum mechanics as matrix mechanics, the standard treatment takes the Kronecker delta δ_i^j = identity matrix.

In four dimensions, for example:

$$\delta_i^j = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{37}$$

Each component of the identity matrix satisfies the scalar definition of the delta function, but there-in lies the rub. The delta function in CCR3 is a scalar function. which takes only the values 1 or 0. Since equation (37) is a matrix equation, (36) and (37) are inconsistent and many problems are created when they are equated.

For instance. look at $CCR3$ in a four dimensional matrix formulation:

$$CCR3 \qquad [q_i, p_j] = q_i p_j - p_j q_i = \delta_i^j$$

Let us write out the nonzero $CCR3$ in 4 dimensions:

$$[q_1, p_1] = q_1 p_1 - p_1 q_1 = \delta_1^1 \tag{38}$$

$$[q_2, p_2] = q_2 p_2 - p_2 q_2 = \delta_2^2$$

$$[q_3, p_3] = q_3 p_3 - p_3 q_3 = \delta_3^3$$

$$[q_4, p_4] = q_4 p_4 - p_4 q_4 = \delta_4^4$$

Since we are dealing with matrices, there is no reason to believe that $\delta_2^2 = \delta_3^3$. The notation forces us to conclude that:

$$\delta_1^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{39}$$

$$\delta_2^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\delta_3^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\delta_4^4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

But the observation about the inequality of the traces still applies. Now we need to take Kaluza and Klein's suggestion of 5 dimensions seriously. Suppose we go to 5 dimensions and use the following New Commutation rules:

NCR1

$$[q_i, q_j] = 0$$

NCR2

$$[p_i, p_j] = 0$$

NCR3

$$[q_1, p_1] = \begin{pmatrix} i\hbar & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i\hbar \end{pmatrix}$$

$$[q_2, p_2] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & i\hbar & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i\hbar \end{pmatrix}$$

$$[q_3, p_3] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i\hbar & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i\hbar \end{pmatrix}$$

$$[q_4, p_4] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i\hbar & 0 \\ 0 & 0 & 0 & 0 & -i\hbar \end{pmatrix}$$

Now the trace of both sides is zero and therefore consistent. So we can ask if there are matrices satisfying these conditions. If we add the requirements that the map $q_i \rightarrow p_i$, is the matrix transpose followed by multiplication by $i\hbar$ we can take

$$q_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$q_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$q_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$q_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$p_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i\hbar & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$p_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i\hbar & 0 & 0 & 0 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i\hbar & 0 & 0 \end{pmatrix}$$

$$p_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i\hbar & 0 \end{pmatrix}$$

With this presentation the off diagonal CR are not zero. we have for instance:

$$[q_1, p_2] = \begin{pmatrix} 0 & i\hbar & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This seems to spoil the usefulness of the new formalism vis $\grave{a}$ vis quantum theory, until we realize that the standard use of spinors with:

$$Q_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$P_2 = \begin{pmatrix} 0 & i\hbar & 0 & 0 & 0 \end{pmatrix}$$

The spinor

$$Q_1 \otimes P_2 = \begin{pmatrix} 0 & i\hbar & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

looks just like the Lie bracket.

Thus the bracket in the new formalism is essentially interchangeable with the tensor product in the old formalism and spinors are the bracket of a q_i with a p_j .

A subtle change has occurred which is usually overlooked in standard treatments of the CCR. If the q_i are to be coordinates, they are in the manifold. The above discussion representing the q_i as matrices and in the same space as the p_i puts the q_i in the tangent space. Thus the q_i cannot represent coordinates and cannot be position operators.

From our vantage point then it seems that the CCR are mathematically unsound. Within differential geometry, the Lie bracket is an operation defined between two elements of the Lie algebra, i.e. the tangent space. The CCR are not so defined, CCR1 defines a bracket between two elements of the base space while CCR3 defines a bracket between a tangent vector and the base space. Only CCR2 is consistant with the requirements of differential geometry but it is not true in our homogeneous space setting.

A note on the history of the CCR seems in order. In the words of Max Born [22] :

> Repeating Heisenberg's calculation in matrix notation, I soon convinced myself that the only reasonable value of the non-diagonal elements should be zero, and I wrote down the strange equation
>
> $$\mathbf{pq} - \mathbf{qp} = \frac{h}{2\pi i}\mathbf{1}$$
>
> where **1** is the unit matrix. But this was only a guess, and my attempts to prove it failed.

Thus, the foundations of quantum mechanics rest on “only a guess.” While this guess was good enough to keep physicists busy on the development of quantum mechanics for almost 80 years, it is not good enough to survive into the era of the unified field theory.

20 Quantum Mechanics without uncertainty

The Heisenberg uncertainty principle comes in several forms:

Energy- time:

$$\Delta E \Delta t \geq \hbar$$

Momentum-position:

$$\Delta x \Delta p_x \geq \hbar$$

$$\Delta y \Delta p_y \geq \hbar$$

$$\Delta z \Delta p_z \geq \hbar$$

Angular momentum-angle:

$$\Delta j_1 \Delta \theta_1 \geq \hbar$$

$$\Delta j_2 \Delta \theta_2 \geq \hbar$$

$$\Delta j_3 \Delta \theta_3 \geq \hbar$$

In fact, an uncertainty principle can be formulated for any two noncommuting Hermitean operators:

$$(\Delta A)(\Delta B) \geq |[A, B]/2|$$

In each pair, there is a conserved quantity: Energy, Momentum and Angular momentum are conserved while time, position and angle are coordinate dependent and hence nonobservable.

There is an obvious way around the uncertainty principles then: formulate the theory in terms of mutually commuting variables only. We want to keep only the operators corresponding to the conserved quantities. Fortunately, the theory of Lie algebras comes to the rescue again, there are operators which can be constructed from the generators of some Lie algebras which commute with all the other elements of the Lie algebra and hence with the corresponding group actions, these are the (generalized) Casimir operators, together with the Cartan subalgebra, which in this case are the diagonal operators. So we must begin with a study of the relevant Lie algebras and their Casimir operators. Unfortunately, the standard theory of Casimir operators is ‘not even wrong.’ [115]

21 Quantization as an Eigenvalue Problem

Dirac's method of quantization utilized the formal analogy between the Poisson brackets of classical mechanics and the Lie algebra brackets of the quantum operators. This "quantization rule" assigns a quantum operators to each classical observable so that the Lie bracket corresponds to the Poisson bracket. Since the quantum operators do not commute, this quantization recipe is ambiguous. The theorems of van Hove [172] and Groenwald [73] show "that a quantization of all the classical observables is in general impossible" (Abraham and Marsden [1], p.433). Any attempt to extend this recipe beyond the simplest cases will encounter severe problems. This was the motivation for the program of geometric quantization developed by Kostant and Souriau. Hurt [92] details the difficulties with the Dirac program and contains a good bibliography. However well motivated, this program of geometric quantization itself ran into problems again due to the desire to quantize too broad a spectrum of classical dynamical systems. There are two separate but related questions for any new system of quantization to answer: "Which classical systems can be quantized" and "How is the mapping of classical observable to quantum operator defined?"

The classical systems which are the easiest to quantize are those with maximal symmetry. Symmetries in a classical system lead to conserved quantities. The symmetry implies the existence of an infinitesimal generator: a first order differential operator. This suggests that the quantum operators we are looking for are constructed from the infinitesimal operators of the symmetry group, i.e. a Lie algebra. The observables should commute with each other which leads us to consider the Cartan subalgebra together with the Casimir operators of the Lie algebra. Assuming we have the correct group, this quantization scheme is a realization of Klein's [102] program:

> The operators to be used in quantum field theory should have a simple connection to a transformation group (so far insufficiently known) which contains the general coordinate transformations in spacetime as a subgroup. The quantum conditions ought to characterize the group in question.

Mensky [121] attempted to find a "reformulation of the relativistic quantum theory of particle interactions in terms of elementary particle states with no appeal to the concept of quantized field." The goal here is the same and like Mensky, our attempt is based on group theory, for the group dictates symmetry and “symmetry dictates interactions ” (Yang [184]) . But there is a slight refinement, for we note that actually symmetry dictates conservation laws, while the conservation laws dictate the possible interactions. In turn, the interactions dictate the symmetries.

With the observable = eigenvalue hypothesis as a starting point, our first goal must be to find those operators whose spectrum yields the observable quantum numbers. In the standard approach to quantum theory, the observables are self-adjoint operators on some Hilbert Space. If the Lie algebra of $u(3,2)$ is to play an essential role, we must insist that the observables be the Cartan subalgebra of $u(3,2)$ together with the generalized Casimir operators of $u(3,2)$. However, while the eigenvalues of the Cartan subalgebra will be additive, those of the Casimir operators may not be.

We will look upon $u(3,2)$ as a spectrum generating algebra [128]. However, this concept requires some modification since we are modeling particles and fields in terms of the geometry of $U(3,2)$. Before becoming too involved in a technical discussion of the quantum operators, let us briefly examine the reasons for quantization. What phenomena is quantization expected to explain? What is a quantized theory? There are several places in nature where the changes in the state of a system appear discontinuous. These phenomena are often explained by proving the existence of discrete energy levels of the system. The discontinuous transitions are the result of the system “jumping” from one quantum level to another. In other experiments, objects which are classically modeled as particles behave as waves and vice-versa. All charged particles have a charge which is an integral multiple of the charge of the electron. An acceptable theory of quantization should explain these observations. The necessity of a wave description of nature should come from the theory and not be imposed upon it. The foundations for a new quantum theory should be geometric as Einstein suggested, for a theory based upon a probabilistic foundation could ever explain why charges are quantized or why particles have the masses they do.

In order to quantize charge, Dirac [35] had to postulate the existence of magnetic monopoles. The existence of magnetic monopoles implies that electric charges and magnetic charges are quantized (Yang [183]). The converse is not true. The quantization of charge does not imply the existence of magnetic monopoles and neither does experiment. Despite 60 years of searching, there is no experimental evidence to support the existence of magnetic monopoles. We need a method of quantization via which charges can be quantized without introducing magnetic monopoles. This method must also explain why the charge of the proton is exactly equal to the charge of the positron. I introduced such a method in [114].

A possible objection to this approach is that only classical systems with symmetries can be quantized. But the classical systems with symmetries are exactly those classical systems with conserved quantities and thus in the present scheme, it is only the conserved quantities which can be quantized. This seems very reasonable since quantization of a system cannot be consistent if that quantization can change in time. Thus. while there could be operators corresponding to momentum and energy which are conserved, there could not be operators corresponding to position or to time which are not conserved.

The internal quantum numbers (electric charge, baryon number, lepton number, meson number) are integers: a finite and discrete spectrum. This suggests that the operators we should study are matrices. In particle physics the usefulness of the adjoint representation of a Lie Algebra for generating spectra is well known:

$$ad(X)Y = [X, Y] \tag{40}$$

This was the method I used to identify the internal quantum numbers as eigenvalues of the Cartan subalgebra of $u(3, 2)$ in [114]. Our particles are modeled as section of the vertical bundle. How are the two pictures related? Fortunately, the identical formula occurs, albeit with a different interpretation.

Let $F(t)$ be the flow of the vector field X and F^* the pullback map under the diffeomorphism induced by that flow, then the Lie derivative of a tensor field K with respect to the vector field X is defined by

$$L_X K(p) = \lim_{t \to 0}(K(p) - F * (t)K(p))/t$$

It is a standard exercise in differential geometry to prove that the Lie derivative of a vector field Y with respect to another vector field X is given by:

$$L_X Y = [X, Y] \tag{41}$$

(Kobayashi and Nomizu [103], p.29).

In (40), both X and Y must be elements of the Lie algebra (left invariant vector fields), while in (41), the vector fields are arbitrary. Comparison of (40) and (41) indicates that the map $X \rightarrow L_X$ is an extension of the adjoint representation.

Thus, all the numbers in the spectrum of the adjoint representation are also in the spectrum of the corresponding Lie derivative. The Lie derivative however acts on other geometric objects besides the left invariant vector fields and thus there will be other spectra. But is there any physical meaning to these other spectra?

Let X be a vector field on a manifold M with flow F(t) through p, i.e. $F : R \rightarrow M;$, $F(0) = p$ and

$$\frac{dF(t)}{dt} = X(F(t)) \tag{42}$$

Suppose $h : M \rightarrow R$ is an eigenfunction of L_X (the Lie derivative with respect to X):

$$L_X h = ah \tag{43}$$

Then

$$L_X h = Xh = (dh, X). \tag{44}$$

Consider the composition $h(F(t)) : R \rightarrow R$.

$$\frac{dh(F(t))}{dt} = (dh(F(t)), \frac{dF(t)}{dt}) = (dh(F(t), XF(t)) \tag{45}$$
$$= X(h(F(t))) = ah(F(t))$$

so by the theory of ordinary differential equations,

$$h(f(t)) = kexp(at) \tag{46}$$

Consequently, h grows exponentially along the orbits of X. This seems to be unphysical since there are no known physical quantities associated with particle motion which behave this way, unless h, X and a are allowed to be complex valued. If a is pure imaginary, say $a = i\alpha$, the same calculation shows that

$$h(F(t)) = kexp(i\alpha t) = k(cos\alpha t + isin\alpha t) \tag{47}$$

and thus, h exhibits a wave type behavior on the orbits of X. This is exactly what is desired! By imposing the imaginary eigenvalue condition, wave functions and quantum numbers are automatically obtained. Evidently, for functions. the only eigenvalues of importance are the pure imaginary ones. Once again, the complex exponentials have been rediscovered as the eigenfunctions of the translation operators (Hamming [77]).

If $U(3,2)$ is the correct group, and if we use the geometry of the group correctly, the physics of the model will agree with the observed physics! Some well known properties of the Lie derivative have direct bearing on the eigenvalue problem. For ease of reference, these properties will be listed as a series of Lemmas.

Lemma

If the tensors S_1 and S_2 are eigenvectors of L_X with eigenvalues a_1 and a_2, then their tensor product $S_1 \otimes S_2$ is also an eigenvector with eigenvalue $a_1 + a_2$.

Lemma

If the vectors Y_1 and Y_2 are eigenvectors of L_X with eigenvalues a_1 and a_2, then their bracket $[Y_1, Y_2]$ is either zero or is also an eigenvector with eigenvalue $a_1 + a_2$.

These two lemmas show that taking the tensor product or Lie bracket of vector fields "preserves" the roots, i. e. the root of the product is the sum of the roots of the factors. This is exactly what happens when two particles interact, the quantum numbers are additive. Thus, with the roots as the internal quantum numbers, either of these products is a candidate for an acceptable way to model particle interactions.

$$\begin{aligned} L_{fX}(gY) = [fX, gY] &= fg[X,Y] + f(Xg)Y - g(Yf)X \\ &= -[gY, fX] = -L_{gY}(fX) \end{aligned} \tag{48}$$

With particles modeled as vertical vector fields, then the natural geometric model for their interaction is the Lie Bracket, which is the Lie Derivative of one vector field with respect to another. If the bracket in turn is the force, then (48) states that the force on the particle fX due to the particle gY is equal in magnitude to and opposite in direct to the force on particle gY due the presence of particle fX. This is the one property which distinguishes the Lie derivative from the covariant derivative (Thirring [170], p. 152). The covariant derivative with respect to X is the Lie derivative with respect to X^*, the vertical lift of X (Kobayashi and Nomizu [103], p.116).

The covariant derivative satisfies the relation: $D_{fX}(gY) = fD_x(gY)$ and consequently, the symmetry of the interaction is lost: $DfX(gY) \neq DgY(fX)$

In the standard treatment of nonabelian gauge theories, (Itzykson and Zuber [94]) , one starts with “gauge fields” :

$$A_m(x) = A^a_m(x)t_a \tag{49}$$

where the t_a are the elements of the Lie algebra of the “gauge group” and the $A^a_m(x)$ are functions on space-time. In the geometric setting of principal fiber bundles, these gauge fields are vertical vector fields. Thus, there is no new mathematics being introduced in the description of the fields of the particles. what is different here is the use of the Lie derivative instead of the covariant derivative and the modeling of the way two fields interact: via Lie bracket instead of via superposition.

To implement the conclusions following (47), the wave function must satisfy the equation:

$$L_{X_I} f = ia_I f \tag{50}$$

for certain generators X_I of the group $U(3,2)$. This equation is very similar to the formulation of the Dirac equation by Cognola, Toller, Slodati, Vanzo and Zerbini [29] and thus should replace the Dirac equation in the present picture. Since $u(3,2)$ contains $su(2,2)$, the Lie Algebra generated by the Dirac matrices, the $u(3,2)$ commutation relations subsume the Dirac matrices and (50) is not unexpected.

In standard quantum theory, the observables are required to commute with the Hamiltonian, the operator which determines the time evolution of the physical system. From the general theory of Lie Algebras, we know that there is a standard construction which yields operators which commute with all the generators of the symmetry group: the Casimir operators. Thus, it might seems natural to take one of the Casimir operators as the Hamiltonian, however, taking the group as the fundamental object, there should be a Lie group generator which provides the dynamics. There are places in quantum theory where first order differential operators are useful, notably to obtain the additivity of quantum numbers. Thus, we cannot take a purely Hamiltonian viewpoint.

We must require that the function factor be an eigenfunction of all of the commuting operators.

In the present picture, $U(3,2)$ is what Barut [9] called a "dynamical group". Thus we hope that U(3,2) will prove to be a group which "above and beyond the space-time symmetry gives the actual quantum numbers and degeneracy of the system".

22 Interpretation of the Quantization

> I am dubious that the standard quantum field theory-perturbation theory-renormalization theory is anything more than a rickety crutch which. we can only dream of throwing away. . . How do we quantize?. . . I am convinced that we just have not thought of the right way to do it or what it means—Robert Hermann

Quantum-Anti-de Sitter space, $QAdS = U(3,2)/U(3,1) \times U(1)$ is a homogeneous complex manifold. This is the ideal situation in which to carry out the program of geometric quantization. But the existence of a mathematical procedure does not mean it is physically correct and must be utilized. Arnol'd [6] was the first to note that cohomological considerations are important in the quantization process, Since then, several researchers have introduced other cohomologies in the settings of geometric quantization, field theory and twistor theory, N. Hurt [92] goes so far as to claim that ". . . for almost all the standard examples presented in elementary quantum theory- the harmonic oscillator, the hydrogen atom, or Kepler's problem, the spinning particle, etc.- the geometric underpinnings are contained in classical results on cohomology of bundles over complex homogeneous spaces". This agrees with the present model for matter as vertical vector fields over the complex homogeneous space $QAdS = U(3,2)/U(3,1) \times U(1)$.

The work on the cohomology of homogeneous spaces is the modern version of the earlier work connecting group theory with quantum mechanics. Wigner [180] recalls von Laue's comment about the importance of group theory: the fact that ". . . almost all rules of spectroscopy follow from the symmetry of the problem is the most remarkable result". Wigner spoke of the atomic spectra, but in a vast generalization of his work, it seems that soon it will be possible to derive the spectra of molecules, nuclei and elementary particles from their respective symmetries. In the standard approach to quantum theory, two things are needed: a Hilbert space and a family of commuting hermitian operators on the space whose spectrum yields the observables (quantum numbers). In the last section, we saw that the observables are a maximal commuting set of Lie derivative operators constructed from the generators of the Lie algebra u(3,2). What spaces should these operators work in? We can compute the Lie derivative of any geometric object but which spectra are physically relevant? From the work done so far, it is clear that we must look at the spectrum of these operators acting on vertical vector fields, but which other spectra will be important is not yet clear.

Particle physicists are accustomed to dealing with symmetry groups of particles. These groups necessarily contain the group $SU(3) \times SU(2) \times U(1)$. Because particles are represented by elements of the vertical bundle, the bundle indices play the role of spinor indices familiar from quantum field theory. Since $U(3, 2)$ acts on this space, the space of particles is a representation space of the underlying symmetry group. Quarks were introduced as basis elements in such a representation space. thus quarks are a passive representation space of the algebra. The group acts on the quarks, but the quarks themselves don't have any means by which to interact. Particles are active, so the representation space should carry some additional algebraic structure to allow for the interaction of the particles. In the present model, this interaction is allowed via the Lie bracket so we have a new algebra of fields (Lee, Weinberg and Zumino [110]).

In the theory of quarks, one has a Lie algebra acting on a vector space:

$$AV$$

In the present theory, as we saw when discussing the structure of $U(3, 2)$, we have instead:

$$\begin{pmatrix} u(3,1) & V \\ V^\dagger & u(1) \end{pmatrix}$$

Thus the quarks are seen to be the dimensions of space-time. Quarks are not particles.

Once the space of vertical vector fields has been chosen as the proper space on which to model an individual particle, it seems proper to take the tensor product of vertical vectors as the model of several particles interacting but not changing particle type.

In their foundational paper on axiomatic quantum field theory, Wightman and Garding [179] showed that relativistic quantum fields should be viewed as "operator valued distributions" In the present work, no distinction can be made between a particle and its fields. An element of the Lie algebra can be viewed as an operator, the corresponding Lie derivative. Thus tangent vectors are simply a geometric interpretation of "operator valued distributions".

Dirac [38] discussed the tetrad (a.k.a veirbein) formulation of General Relativity:

> For dealing with spinors in a Riemann space one must introduce a fourleg at each point described by field functions $h_{\mu a}$ satisfying
>
> $$h^{\mu}_{a} h_{\mu b} = \eta_{ab}, \qquad \eta^{ab} h_{\mu a} h_{\nu b} = g_{\mu\nu}$$
>
> where η_{ab} is the fundamental tensor of special relativity. The $h_{\mu a}$ become the fundamental field quantities of the gravitational field, instead of the $g_{\mu\nu}$.

The 'fourleg', a.k.a. tetrad or vierbein formalism finds its perfect fulfillment in the homogeneous space setting, where the vertical bundle is already present and does not need to be tacked on. The homogeneous space carries a metric, the Killing form of the Lie algebra, then as Dirac pointed out, the vectors become the fundamental objects, not the metric. But to push the point one step further, it is not just the vectors on spacetime which are important, rather the vectors on the entire group manifold of $U(3, 2.)$

The vertical vectors can be thought of as a supplement to the tangent space of space-time in the sense that Einstein and Mayer [56] and Rosen and Tauber [151] considered bundles of 4 + n dimensional vector spaces over space-time. The number of extra dimensions is arbitrary in their approach but is fixed geometrically here.

This choice of function with values (i.e. a section) in the space of vertical vectors is to be compared with the standard space of scalar valued functions. If one chooses to work on the space of scalar valued functions, then to obtain a Lie Algebra structure on this space one must impose an additional structure such as the Poisson bracket. Dirac [34] treated this Poisson bracket as the fundamental object of study in quantum theory and used the correspondence of classical Poisson bracket goes to quantum bracket as the recipe for the quantization of a classical dynamical system. As previously discussed, his quantization scheme is beset with difficulties. In contrast, the Lie Algebra structure of our model is present from the outset and need not be tacked on.

In spite of these advantages. however, there are some nontrivial problems with this choice of vector space. Because the bilinear form inherited from the Killing form of $su(3, 2)$, it is not positive definite. Recall that the Killing form of a simple Lie Algebra is negative definite iff the group is compact (Helgason [82], proposition 11.6.6). Thus noncompact groups and indefinite metrics in state space of necessity go together. Since the Lorentz group (which is noncompact) is essential to physics then the indefinite Killing metric is also essential.

Dirac [36] was the first to note the necessity of using an indefinite metric in a relativistic quantum field theory. His ideas were developed by several researchers including Heisenberg [81]. The research of the first 25 years is summarized in the book by Nagy [126]. Other work includes Gupta-Bleuler indefinite metric quantization, Sudarashan's [168] Indefinite Metric Nonlocal Field Theory, T. D. Lee's [108] Indefinite Complex Field Theory, Gleeson and Sudarshan's [71] description of "good theories" with an indefinite metric. These authors started with an indefinite metric. Sometimes the indefiniteness of the metric is imposed for other reasons. Lovelace [116] studied "dilation operators that are hermitian only in an indefinite metric space." Rawnsley, Schmid and Wolf [145] give an excellent history of the development and importance of the subject. The greatest problem with the indefinite metric is the physical interpretation. It has been known for some time that the axioms of quantum theory are incompatible with the axioms of general relativity.

If the possibility of negative states exists, the *interpretation* of the wave function as a probability distribution is in jeopardy. But it seems unreasonable to demand that the *interpretation* of the wave function as a probability distribution in the nonrelativistic theory will survive the transition to the relativistic setting when it is clear that the theory itself cannot survive intact.

Some other opinions on indefinite metrics read as follows:

> ...the requirement of positive definiteness for quantum mechanical Hibert space ...will exclude noncompact groups from consideration, Indeed it appears that the only noncompact group of significance to physics is the Lorentz group, which enters the theory in a slightly different way from the groups associated with the Yang-Mills field.

> ...consistency of the theory can be maintained only by renouncing either the existence of a state of lowest energy or the positive

> definiteness of Hilbert space, both of which are undesirable from the viewpoint of physical interpretation.
>
> ...if we accept an indefinite metric for Hilbert space then no other types of counter terms beyond those already considered are needed... It is not difficult to see that the theory... requires only slight modification to take into account an indefinite metric.

Clearly these are conflicting opinions as to the necessity and usefulness of noncompact groups in gauge theories. Yet these contradictory statements are typical of the diversity of opinions present in the literature. Even more impressive is the fact that these quotations are all from the same article by B.S. deWitt [33]. Statement (a) is found on page 675, (b) is on page 646 and (c) is on page 819.

It seems evident that if all the forces of nature are to be truely unified, all the groups should enter the theory in the same way, i.e. thru a simple group which contains all the other necessary groups, except the group of diffeomorphisms of the underlying spacetime- this infinite dimensional group of necessity "enters the theory in a slightly different way."

Jauch and Rohrlich [100] discuss "Locality, Covariance and Indefinite Metric" (section 5). They state "The local formulation of quantum electrodynamics thus necessarily involves an indefinite metric for the Hilbert space H". The possibility of a nonlocal field theory is their next topic. Nonlocal fields must be discussed here also.

Wu and Yang [182] raised the question:

> Is there a possible physical meaning to a gauge theory with a group that is semisimple and noncompact, since the energy for such a system is necessarily not positive definite according to theorem 5? We do not know the answer to this question. A simple negative answer suggests itself, but we do not believe such an answer is necessarily right.

Hsu and Xin [91] addressed this question:

> We find that within the present perturbation framework, unitarity and gauge symmetry are incompatible for the SL(2,C) fields... Therefore it seems unlikely that one can formulate a satisfactory quantum theory based on the Yang-Mills field theory

> with the $SL(2, C)$ group or the Lorentz group. We believe that such a dilemina exists in any other Yang-Mills field theory with a noncompact group.

Cahill and Ozenli [26] suggested a way out of the dilemma by pointing out that unitary gauge theories of noncompact groups are possible but renormalization is still a problem within perturbation theory.

Now we have enough information to see exactly what the problems are. As Hsu and Xin note, the problems occur "within the present perturbation framework". So the problems are all caused by the presently accepted quantization methods and with the application of these methods to gauge theories. Not only is a new approach to the quantization required but since the gauge formalism is incompatible with noncompact groups and a noncompact group is essential, we need to find an alternative method of obtaining the physics from the symmetry, from the geometry of the group.

In his investigation of unified field theories, Radford (1984) studies the Dirac equation characterized by the noncompact global invariance groups $U(p, q)$ and shows that the second quantization program singles out the maximal compact subgroup as representing the particles. But there are difficulties with extending the standard nonrelativistic methods of quantization to the relativistic setting.

The greatest problem with the indefinite metric is the physical interpretation. It has been known for some time that the axioms of quantum theory are incompatible with the axioms of general relativity. If the possibility of negative states exists, the interpretation of the wave function as a probability distribution is in Jeopardy. This interpretation is weak in another way. Many of the infinities which occur in quantum field theory are there because the particles are localized by the use of delta functions. The appearance of these infinities seems to be telling us that the particles are not points but some sort of "blur". If this is the case, the localization of the particle is impossible and the probabilistic interpretation of the wave function is untenable. As Landau and Lifshitz [106] state the problem:

> The concept of such a probability clearly requires that the coordinate can in principle be measured with any specified accuracy and rapidity, since otherwise this concept would be purposeless and devoid of physical significance.

Thus, if the particle is not a point, it seems impossible to interpret the wave function as a probability distibution. It seems unreasonable to demand that the interpretation of the wave function as a probability distribution in the nonrelativistic theory will survive the transition to the relativistic setting when it is clear that the theory itself cannot survive intact. The probability interpretation of the wave function will be discussed further after the introduction of another geometric interpretation of the wave function.

23 Sharpening the Axioms

> Axioms are principally useful in providing efficient guides to clear thinking and should be changed for good and sufficient reasons.–R. F. Streater and A.S. Wightman [166]

> Axiom: A self-evident principle or one that is accepted as true without proof as the basis for argument; a postulate.

> In modern metascience 'axiom' means *initial assumption* not self-evident pronouncement. There need be nothing intuitive and there is nothing final in an axiom; so much so that axioms are often tried for the sake of argument, i.e. to see what they entail and whether what they entail is approximately true.
> —Mario Bunge [25]

Even a cursory study of the history of axiomatic systems in mathematics will reveal four fundamental observations: mathematicians have an example in hand before they axiomatize; mathematicians often modify their axioms to observe the consequences; often there is another shorter list of axioms which accomplishes the required goal and sometimes the existence of an errant list of axioms has misled generations of mathematicians. It is in this spirit that the axioms of quantum field theory will be examined.

According to Schweber [163]

> All of the investigations which have been carried out make the following assumptions about the theory:
> I. The usual postulates of quantum mechanics are valid, i.e., that the states of the systems are represented by vectors in a Hilbert space, **H**, and that the observables of the system can be represented by self-adjoint operators on **H**.
> II. The theory is invariant under inhomogeneous Lorentz transformations.

The present context of sections of vector fields on $QAdS$ as the Hilbert works only if the sections have compact support. Since the vector field is the field of an elementary particle, its support would be the "sphere" of influence. At the time of origin of the particle, its fields begin to spread and would spread at a finite speed and hence be nonzero only in a compact subset of space-time. So we do have a Hilbert space. With an evolution operator and observables satisfying $< D\phi, \psi >= - < \phi, D\psi >$ we are close to satisfying I. However, the theory being deveoped here is invariant under actions of $U(3,2)$ rather that the inhomogeneous Lorentz group (the *Poincaré* group).

24 Eigenfunctions and Observables

We have seen that wave functions do not add, so that the probability interpretation of the wave function is no longer tenable. So does the wave function have any physical meaning in the classical sense?

There are classical physical objects which do add: forces, potentials, electromagnetic fields, etc. When two particles interact, their quantum numbers add. If these observables are eigenvalues, they too must add. Eigenvalues add in a familiar situation. For the operator $H = d/dx$, with eigenfunctions $f = exp(ax)$ and $g = exp(bx)$ then $fg = exp((a + b)x)$. This generalizes immediately. If H is a first order differential operator with eigenfunctions f and g:

$$Hf = af \qquad Hg = bg.$$

Then

$$H(fg) = (Hf)g + f(Hg) = afg + fbg = (a + b)fg$$

At first glance, it seems essential that H be first order so that the product rule holds.

However, if we take

$$H = \frac{\partial}{\partial x}\frac{\partial}{\partial y}$$

Then:

$$He^{f(x)+g(y)} = He^{f(x)}e^{g(y)}$$
$$= \frac{\partial}{\partial x}\frac{\partial}{\partial y}e^{f(x)}e^{g(y)}$$
$$(f'(x) + g'(y))e^{f(x)}e^{g(y)}$$

Thus the product rule holds (and eigenvalues add) for second order operators if the two operators are independent. Clearly this result can be generalized to any number of independent operators.

In light of this example, it is not surprising that the correct mathematics is found in a generalization of the eigenvalue equation. The eigenvalues of the derivative determine the quantum numbers. But which derivatives should we use? Gauge theories use the covariant derivative. For the quantum numbers to be conserved quantities, the derivatives themselves cannot change. For if the derivative were to change, so would the conservation laws. If the covariant derivative were involved, the derivative would be changing and consequently, another derivative must be used. Since the derivative must be associated with the action of the Lie group, the relevant derivative must be the Lie derivative. Thus we must look for another way of putting physically important quantities such as the potential A into the picture using the Lie derivative.

The Dirac equation reads: (Itzykson and Zuber [94], p. 64)

$$(i\partial - eA - m)\psi = 0 \tag{51}$$

This equation is unsatisfactory because the mass m must be put in by hand. But it can be rewritten as an eigenvalue equation:

$$(i\partial - eA)\psi = m\psi \tag{52}$$

From the present viewpoint, this is unsatisfactory because the potential A is arbitrary and should not be part of the derivative. We should be able to determine A from the geometry since the potential A should be closely related to the wave equation of the particle which creates the potential. If we take A to the other side of the equation. we obtain:

$$i\partial\psi = (eA + m)\psi. \tag{53}$$

But in this form the equation is still not satisfactory because while the eigenvalue m depends on ψ, the potential A does not. We are forced to conclude that we cannot have a wave equation for one wave function, we must have an equation which interrelates both interacting wave functions.

For ease of exposition, we will treat A as though it were a scalar:

$$H\psi = E\psi. \tag{54}$$

Where the first order differential operator H is an infinitesimal generator of a Lie group. To have an eigenvalue equation would require that E be a constant. But we need to consider the case where E is a function. This case is a generalized eigenvalue problem.

Example Let $H = x\partial_x + t\partial_t$ and $\psi = exp(ikx - i\omega t)$. Then

$$H\psi = (x\partial_x + t\partial_t)exp(ikx - i\omega t) = (ikx - i\omega t)exp(ikx - i\omega t) \tag{55}$$

Here ψ is a plane wave and H is the two dimensional version of the dynamic operator of Fubini, Hansen and Jackiw [67].

Example

The Cauchy-Riemann Equations in polar coordinates, with

$$(r,\theta) \rightarrow (R,\alpha) \tag{56}$$

$$r\frac{\partial R}{\partial r} = \frac{\partial \alpha}{\partial \theta}R$$
$$\frac{\partial R}{\partial \theta} = -r\frac{\partial \alpha}{\partial r}R$$

Thus, if f is analytic, R is a generalized eigenfunction of two operators. We will need a few theorems about generalized eigenvalue problems before we can apply this formalism to physical problems. Most of these are direct generalizations of statements about ordinary eigenvalues.

Theorem If H is a first order differential operator with eigenfunction E and eigenvalue a: $HE = aE$. Then $\psi = exp(E)$ is a generalized eigenfunction of H with eigenvalue aE.

Proof:

$$H\psi = H(exp(E)) = (HE)exp(E) = aEexp(E)$$

We obtain an interesting result by differentiating again:

$$H^2\psi = H^2(aEexp(E))$$

$$= (aHE)exp(E) + aE(Hexp(E))$$
$$= (a^2E)exp(E) + (aE)^2exp(E) = a^2(E+E^2)exp(E)$$
$$= a^2E(1+E)exp(E)$$

If we call the operator L instead of H and use s instead of E and take a=h, we obtain:

$$L^2exp(s) = h^2s(1+s)exp(s)$$

Which is exactly the eigenvalue equation for spin, but now spin is a function, not a number, thus allowing spin to assume a dynamical role in the theory, as suggested by Lurcat [118].

Theorem Let H be a first order differential operator with eigenfunctions E_1 and E_2 and corresponding eigenvalues a_1 and a_2, then the product E_1E_2 is a generalized eigenfunction of H with eigenvalue $a_1 + a_2$.

Proof: Direct calculation.

Corollary Let H be a first order differential operator with eigenfunction f, eigenvalue m, $Hf = mf$ and let ψ be a generalized eigenfunction of H with eigenvalue g, then $f\psi$ is a generalized eigenfunction of H with eigenvalue (g+m).

Proof:

$$H(f\psi) = (Hf)\psi + f(H\psi) = mf\psi + fg\psi = (g+m)f\psi \tag{57}$$

If A is an eigenfunction of H with eigenvalue e and E is an eigenfunction of H with eigenvalue m, then:

$$H(Eexp(A) = (HE)(exp(A)) + E(HA)exp(A)$$

$$= mEexp(A) + EeAexp(A) = (eA+m)Eexp(A)$$

Thus $\psi = Eexp(A)$ is a solution to the equation $H\psi = (eA+m)\psi$.

The operator can be second order and obtain the same result. If we have:

$$\frac{\partial E}{\partial t} = mE$$

$$\frac{\partial E}{\partial x} = 0$$

$$\frac{\partial A}{\partial x} = eA$$

$$\frac{\partial A}{\partial t} = 0$$

Then it follows that:

$$\frac{\partial}{\partial t}\frac{\partial}{\partial x}E\exp(A) = (eA+m)\,E\exp(A)$$

We will be using this result once we have the operator representation of $u(3,2)$.

25 Superselection Rules

> ...standard non-relativistic quantum mechanics...makes absolutely no sense! —Roger Penrose

The fact that the principle of superposition is not universally applicable has been noted before, if there are *superselection rules* in the theory. *Superselection rules* were first recognized by Wick, Wightman and Wigner [178]:

> We shall say that a superselection rule operates between subspaces if there are neither spontaneous transitions between their state vectors (i.e. if a selection rule operates between them) and if, in addition to this, there are no measurable quantities with finite matrix elements between their state vectors.

These rules have been discussed and elucidated by many authors since:

> In the usual formalism of Quantum field theory a superselection rule means that there are operators in the Hilbert space which commute with all observables. Typical examples of such "superselecting operators" are the total electric charge or the total baryon number. The customary representation of the algebra of observables is reducible. It can be decomposed into irreducible ones which we shall call "sectors." Each sector corresponds to a definite numerical value of the charge.
>
> —Haag and Kastler [76]

> Pure states corresponding to different superselection sectors cannot be coherently superposed. —Roberts [147]

> If there are superselection rules in a theory, then not all hermitian operators are observables, and the superposition principle does not hold in H.
>
> — Streater and Wightman [166]

Indeed, within the present theory, the only observables are constructed from the group generators. The superselection rules here correspond to electric charge, baryon number, lepton number and spin. The superselection sectors are represented by the matter matrix.

Thus, there are four superselection rules, defined as eigenstates of the γ_i which define 16 superselection sectors. The superselection rules merely say that it is improper to add the wavefunction of a proton to that of an electron, etc. But, with observables taken to be eigenvalues, we see that any conserved quantity derived from an eigenvalue defines a superselection rule. This goes counter to the opinion of Wick, Wightman and Wigner. I believe that energy, linear momentum, angular momentum, etc. also determine superselection rules and consequently, the corresponding wave functions cannot be superposed unless all the eigenvalues are the same. But by the time we require that all the eigenvalues be identical, we have required that the wave function is the solution to several differential equations and we may have uniqueness of the solution.

26 The Need to Restore Causality

> The purpose of physics is to understand the world, not just to be able to predict (calculate) the results of the experiments. For many people the ability to predict provides a sufficient understanding, but not for me. This is the reason why I cannot accept the purely pragmatic (Copenhagen) interpretation of quantum mechanics.
>
> —Emilio Santos [158]

The purpose of science is to explain the world we live in. The first step in the scientific process is observation, the collection of data. The second step is the organization of that data into comprehensible patterns. The third step is the framing of a hypothesis which explains the observed patterns and predicts other patterns which are then, hopefully, observed.

The third step, classically, meant finding a cause for the observed phenomena. This is what I mean by causality: every event has a cause. The idea of causality is separate from the doctrine of determinism which states that if we knew the initial conditions, we can determine the future. Quantum theory, with its uncertainty principle forces us to give up the idea of even knowing the present with ultimate precision. What is not so well appreciated is that relativity theory is not deterministic, for special relativity says that we can only know those things which are in our past light cone. We cannot predict the future because something may be happening which is outside of our past light cone now but which will influence our future. Clearly, the doctrine of determinism is indefensible on either account.

Mathematically modeling the doctrine of determinism requires two inputs: data concerning the experimental set up (Initial conditions) and a set of causal equations. The doctrine of determinism is dead, because one cannot determine the present to an arbitrary degree of accuracy. However, this does not prevent the equations of a theory from being deterministic! The solution of a differential equation requires knowledge of the initial conditions. The uncertainty principle destroys the possibility that the initial conditions can be precisely known. This says nothing whatsoever about the equations of evolution.

But modern quantum mechanics has given up on the idea of causality. Norwood Russell Hanson, a Philosopher of science, summed up the role of causality in quantum theory:

> In elementary particle theory, today phenomena are 'encountered' which are neither causal, nor picturable, nor even mechanical in any classical sense. For example, the theory requires that the nucleus of every unstable isotope be identical with every other nucleus of that type—in a stronger sense of 'identical' than anything yet encountered in physics. But these nuclei decay in an unpredictable way (another part of the theory requires that); so the decay cannot be conceived as a caused event. For the nuclei of those atoms which do decay are internally identical, until the instant of decay with those which do not decay.[78](p.92)

However, Einstein [47] found this viewpoint unacceptable:

> It is the goal of theoretical physics to create a logical, conceptual system, resting upon the smallest number of mutually independent hypotheses, which allows one to comprehend causally the entire complex of physical processes.

In classical physics, everything has a cause. In quantum theory, this is not necessarily true. Radioactive decay is the prime example. Supposedly the question of why a decay happens has no answer. This decay is supposedly acausal. The decay just happens spontaneously. This theory was presented by Gurney and Condon in 1929 [75] and is now part of the foundation of quantum mechanics. But the ideas were etched in stone before all of the data was in. Before we can discuss how to return causality to quantum phenomena, we must look at some of the new data.

27 The Restoration of Causality

> Should a more flexible theory be proposed, the probability interpretation would have to be deeply modified or even given up.—Mario Bunge [25]

In the quantum world, we are told, we must give up the notion of causality. There are things which occur in the subatomic domain which do not have causes. That is one reason quantum theory deals in probabilities.

Consider the decay of uranium. A given uranium atom will decay, but when? In a sample of uranium atoms, a certain number will decay in a given time, but which ones? More importantly, why do they decay? The current paradigm holds that there is no cause. This is a prime example of the premise that many of the current ideas in science were firmly entrenched before all the data was in. Researchers stopped looking for an answer because the previous generation told them there wasn't one.

We live in a sea of neutrinos and antineutrinos. Although they interact weakly, the sheer number of them should have some visible effect on our world. I believe that effect is radioactive decay.

The sun produces energy by converting protons to alpha particles, essentially, hydrogen turning into helium. According to the standard theory, the reaction goes:

$$4p^+ \longrightarrow \alpha + 2e^+ + 2\nu$$

The hydrogen burning process in the stars produces a sea of neutrinos. Stars made of anti-matter would produce a sea of anti-neutrinos to add to this sea of neutrinos, anti-neutrinos also come from the decay of heavy atoms (I am not suggesting that the anti-neutrinos are locally as numerous as the neutrinos) The neutrino- anti-neutrino annihilation would thus occur throughout space, sending two photons in opposite directions. The photons carry the same energy and momentum as the neutrinos carried. Because the neutrinos interact weakly, are extremely light and travel at near the speed of light, the sea of neutrinos can be treated like a photon gas and hence, the spectrum observed will be that of a Black Body radiation.

We know that neutrinos can cause certain nuclei to decay, that is how their existence was proven in the first place. We know that chain reactions involving neutrons cause a collection of atoms to decay faster. It is not too unreasonable to presume that there is another agent for the chain reaction process: the neutrino. It is not too big a jump then to presume that all decays are of the chain reaction sort.

We live in a sea of neutrinos which were not known at the time acausal quantum mechanics was launched. It seems reasonable to suppose that beta decay is caused the interaction of a nucleus with a neutrino in the background neutrino sea acting as a catalyst. Likewise, it seems reasonable to suppose that the electron jump is due to the interaction of the electron with a neutrino in the background neutrino sea. Thus there is no need for the idea of “quantum tunneling”.

If it is true that the neutrino and/or antineutrino is the cause of radioactive decay, then we look to the matter matrix of the Theory Of Matter [114] to discover what sorts of interactions could be responsible.

The nucleus consists of neutrons, protons and negative pions. The neutrino (ν) could interact with a negative pion (π^-) to produce an electron, or it could interact with the neutron to produce a hydrogen atom, but no H atoms are observed exiting the nuclei of heavier atoms. The anti-neutrino could interact with a proton to produce an electron and a neutron, but that would require much more energy than the anti-neutrino carries. It seems then that the most probable interaction to look for is the neutrino-pion interaction.

However, there is another possible mechanism. In a laser, a photon passing by an excited atom causes induced emission of a photon of the same frequency. Perhaps the same mechanism with a neutrino or anti-neutrino passing through a nucleus would cause induced emission of a neutrino from the nucleus. Tourrenc, Angonin and Wolf [171] describe the process in detail.

If this is really the cause of radioactive decay, then there should be a shielding effect due to the presence of other particles close to the decaying atom. This shielding effect should cause a lengthening of the half-life of bound matter. Chemically bound radioactive atoms should have a longer half-life than free atoms of the same species. The stronger the bond, the better the shielding and the longer the half life. This is a testable prediction.

The mathematics of computing the time for a neutrino to cause the decay of an atom leads to exactly the same formula as the assumption that there is no cause for the decay. Thus the rate of decay will be the same as the rate of “interaction”, so a theory which addresses only the probability can say nothing about the cause. A theory which says nothing about causes is not a complete science. Statistical quantum mechanics is only at the second stage of the scientific process, it is not at the third stage of having explanatory power. Thus while statistical quantum mechanics does a very good job with predicting decay rates, it says nothing about the causes of those decays. Quantum mechanics got around this incompleteness by saying there was no cause, but now that a possible cause has been found, the silence of quantum mechanics is indefensible. Another type of quantum mechanical phenomena which the neutrino-antineutrino sea could explain is the decay of excited atoms. An atom in an excited state has an electron in a high energy orbit. If this electron encounters a neutrino or an anti-neutrino, it could be jarred enough to make the quantum jump. This implies the existence of a neutrino-electric effect exactly like the photoelectric effect. Again, this is a testable prediction.

If neutrinos are the cause of radioactive decay, then solar neutrinos would be the primary source. Since several interplanetary craft use nuclear power, the data could be examine to see if the radioactive decay rate decreases as a nuclear reactor gets further from the sun. Better still, if a reactor which has been at a far distance from the sun could travel back closer to the sun, the decay rate and hence the power output should increase. This is the third testable prediction.

In the inverse Compton effect, a high energy electron "spontaneously" gives off a photon. Again, there is a probable cause: the electron interacts with the neutrino-antineutrino sea.

In the matrix of the theory of matter, every off diagonal particle except the neutrino is known to be massive. This seems to imply that the neutrino is massive. In the same spirit, we observe that all of the particles in the matter matrix except the neutron and pion are stable. Perhaps it is not too great a leap of faith to suggest that the neutron and pion are actually stable and that the reason they decay is their interaction with the neutrino-antineutrino sea.

Instead of saying that the typical free neutron decays in 918 seconds, we should be saying that on average, a free neutron will interact with a neutrino in 918 seconds.

Instead of saying that the typical free pion decays in 2.60×10^{-8} seconds, we should be saying that on average, a free pion will interact with a neutrino in 2.60×10^{-8} seconds.

28 Experiment and Theory

Experimental evidence must be evaluated in the context of some theory. John Norton [130] discusses the role of experimental evidence in relation to theory. After mentioning several variations of the *underdetermination thesis*, which "asserts that a given body of evidence must fail to determine uniquely a single theory," Norton writes:

> Thus, under the accumulated weight of these theses and their variants, it would seem that the life of the scientist who chooses theories on the basis of available evidence is a precarious one indeed. His evidence cannot pick out a unique theory, for a theory is always underdetermined by evidence. Worse, alternative theories, all equally adequate to the evidence, are readily accessible to him. We might well expect that our scientist will become little more than a vagabond capriciously wandering from theory to theory or perhaps, like Buridan's ass, will be frozen into inactivity by the inability to choose among a plethora of equally viable theories.

Of course, this is not what happens in practice. In spite of the fact that many different theories could explain the evidence, as Norton goes on to say:

> In the case of a mature science, there is most commonly a single favored theory which is felt to be picked out uniquely by the evidence. Challenges to the theory from aberrant hypotheses or experiments are rarely considered seriously.

According to Pierre Duhem [43]:

> A physical theory is not an explanation. It is a system of mathematical propositions, deduced from a small number of principles, which aim to represent as simply, as completely, and as exactly as possible a set of experimental laws.

Duhem goes on to say:

> Agreement with experiment is the sole criterion of truth for a physical theory.

But if this were so, then the Ptolemaic theory of epicycles was an excellent scientific theory. There is more than agreement with experiment, there is simplicity. Further implicit in our understanding of the validity of a scientific theory, there is beauty. But as we have seen, perhaps the most important criteria for truth in a physical theory is explanatory power.

Near the end of his life, Einstein [51] restated his objections to quantum theory:

> I reject the basic idea of contemporary statistical quantum theory, insofar as I do not believe that this fundamental concept will provide a useful basis for the whole of physics... I am, in fact, firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this theory operates with an incomplete description of physical systems.

Although most physicists today believe that Einstein was just too inflexible to go along with the radical viewpoint of quantum theory, the truth is quite the contrary, as Max Jammer [98] pointed out:

> Einstein was in fact, one of the chief architects of the quantum theory; and second, contrary to widespread opinion, he rejected the theory not because he, Einstein—owing perhaps to intellectual inertia or senility–was too conservative to adapt himself to new and unconventional modes of thought, but on the contrary, because the theory was in his view too conservative to cope with the newly discovered empirical data.

Bruno Bertotti [19] summarized "Schrödinger's vision":

> We cannot speak of an objective world, but only of the common elements in the perceptions and judgements of all the individuals; and this is suggestive of a single Mind of which each of us is the temporary and fragmentary manifestation. Everyday awareness of the union with this Mind is the basis of spiritual life. The physical world, being an aspect of this Mind is rational and unique; hence one can characterize it mathematically against every other alternative. Quantum mechanics is an unsatisfactory and passing freak of our age.

Thus two of the major players in the foundation of the physics of this century had major reservations about the evolution of quantum mechanics.

Richard Feynman [63] told an audience not to worry if they don't understand quantum field theory:

> You see, my physics students don't understand it either. That is because I don't understand it. Nobody does.

And this comes from one of the founders of the theory. If the theory is so incomprehensible, can it possibly be true? I believe quantum field theory just a smoke screen masking our ignorance of nature.

Michael Redhead [146] claims: "...the conceptual foundations of QFT are genuinely obscure in a number of respects..."

Kristin Shrader-Frechette [165] argued:

> ...a new paradigm seems needed in high energy physics because elementary particle theory does not answer the important questions facing it. It does not represent a closed and internally consistent discipline. Moreover its difficulties appear to be so great that a fundamental revision of certain concepts of high energy physics seems called for.

It didn't take long for the normal scientists to reply:

> Our position is that Shrader-Frechette is entirely mistaken in her assessment of the current state of particle physics. Far from being in a state of crisis, the field has achieved a new level of predictive and explanatory success.[84]

So, who is right? Only time will tell, but my money is on a crisis.

It is interesting to note that when a philosopher of science attacked quantum field theory, the response was immediate and vicious. But when major figures from within physics, like Dirac and Schwinger spoke, the critics were silent.

Other physicists think the theory is correct:

> While Theoretical physicists often disagree about the details of the theory, and especially about the way it should be applied to practical problems, the great majority agrees that the theory in

> its main features is correct. The minority who reject the theory, although led by the great names of Albert Einstein and Paul Dirac, do not yet have any workable alternative to put in its place.[44]

One of the major difficulties in persuading physicists that there is a problem with quantum field theory is this type of response: "If there is something wrong with QFT, show me a better alternative!" There is no logic in that response, for one does not have to have an alternative to know that there is something very wrong with the present situation. If this logic were applied by a person on a sinking ship, we would hear: "So the boat is sinking, I won't leave until a rescue boat appears." It makes no sense logically, for then you go down with the ship.

In 1983, at the Second New Orleans Conference on Quantum Theory and Gravitation, Paul Dirac [42] delivered his last public talk (actually, since I was there, I know that his published paper had nothing to do with the subject of his oral presentation). He addressed the problems of quantum field theory:

> We have a theory in which infinite factors appear when we try to solve the equations. These infinite factors are swept into a renormalization procedure. The result is a theory which is not based on strict mathematics, but is rather a set of working rules.
>
> Many people are happy with this situation because it has a limited amount of success. But this is not good enough. *Physics must be based on strict mathematics.* One can conclude that the fundamental ideas of the existing theory are wrong. A new mathematical basis is needed.

I was at that meeting and delivered my first professional talk in which I presented a new mathematical basis for such a theory [112]. Unfortunately, Dirac slept through my talk.

The attitude of the "working Physicist" is represented well by a comment of N.D. Mermin [122]:

> The fact is that although the underlying quantum mechanical view of the world is extraordinarily confusing—Bohr is said to have remarked that if it doesn't make you dizzy then you don't understand it—yet quantum mechanics as a computational tool is entirely straightforward.

The problem with this viewpoint is that if you don't understand what you are calculating, then what do the calculations mean? Besides, the statement is not true. Calculations in Quantum Field theory often involve subtracting infinity from infinity and this is nonsense.

29 Mathematical aside

Pick two real numbers, a and b. Let

(1) $c = a - b$

Multiply both sides of (1) by $a - b$

(2) $c(a - b) = (a - b)(a - b)$

Now expand both sides:

(3) $ca - cb = a^2 - ba - ab + b^2$

Rearrange terms:

(4) $ca - a^2 + ab = cb - ba + b^2$

From the left hand side, factor out a. From the right hand side, factor out b.

(5) $a(c - a + b) = b(c - a + b)$

Since $(c - a + b)$ appears on both sides, we can cancel it, leaving:

$a = b$

Since a and b were two arbitrary real numbers, this is nonsense! What happened?

Going from equation (5) to (6), we divided by $(c - a + b)$. But since in equation (1) we defined $c = a - b$, then

$$c - a + b = 0$$

So in going from equation (5) to equation (6), we divided by zero. This is only an example of what can go wrong if one inadvertently divides by zero. Essentially if you allow division by zero, you can prove anything!

Recall the definition of log base $c = 0.1 : a = log_c x$ if $(0.1)^a = x$.

If $a = log_c x$ means $(0.1)^a = x$

$b = log_c y$ means $(0.1)^b = y$

then $xy = (0.1)^a(0.1)^b = (0.1)^{a+b}$

Thus $log_c(xy) = a + b$

The function log_c turns multiplication of numbers into addition of their logarithms. The log of infinity is not defined, but since, at least symbolically, $(0.1)^\infty = 0$, then $log_c 0 = \infty$. Thus multiplying both sides of an equation by zero is the same as adding infinity. Furthermore, subtracting infinity is the same as dividing by zero. In the process of "renormalization" as used in quantum field theory, infinities are subtracted from each side of the equation. As demonstrated above, this is tantamount to division by zero and is mathematical nonsense, no matter how it is dressed up.

The reason for this mathematical aside was to show that if one disregards the rules of mathematics, one can prove anything. It has been noted that it is remarkable that the calculations of Quantum Field Theory are in close agreement with experiment. Given that the rules of mathematics have been violated in QFT, it is even more remarkable that the calculations are not in perfect agreement with experiment!

30 Getting past the Impasse

In 1956, Julian Schwinger wrote in the preface to a collection of original papers on Quantum Electrodynamics:

> The post-war developments of quantum electrodynamics have been largely dominated by questions of formalism and technique, and do not contain any fundamental improvement in the physical foundations of the theory. Such a situation is not new in the history of physics; it took the labors of more than a century to develop the methods that express fully the mechanical principles laid down by Newton. But, we may ask, is there a fatal fault in the structure of field theory? Could it not be that the divergences—apparent symptoms of malignancy—are only spurious byproducts of an invalid expansion in powers of the coupling constant and that renormalization, which can change no physical implication of the theory, simply rectifies this mathematical error? This hope disappears on recognizing that the observational basis of quantum electrodynamics is self-contradictory.

In discussing the development of Quantum Electrodynamics, Alexander Rueger [154] claims:

> One is not likely to find another major episode in the history of science where scientists talked with so much emphasis and persistence about the "future correct theory" while still working on the present deficient version.

Here is the main philosophical shortcoming of modern physics. Despite the obvious flaws of Quantum Field Theory, people continued to work on it because there was no alternative theory. To continue to work on the theory is to endorse it, despite its flaws.

The present faith of many is in "M-Theory", where the M means Membrane or Mystery, or more likely Mistake.

Other physicists hold other opinions of course. In discussing Quantum Field Theory, James T. Cushing claims:

> Here, just as in the development of quantum mechanics, there seems to be no real need for competing research programs . . . Competing

> research programs are *required* only when one program must be abandoned for another (crisis), since scientists must have an alternative if they are to switch.

Cushing remark is self-contradictory. Once quantum mechanics has been shown to be faulty, if "scientists must have an alternative if they are to switch", then that alternative must have been created beore the "crisis", which means that someone should be working on alternative research programs before the crisis, which in turn implies that there should be alternative research programs at all times!

How many upset scientists does it take to cause a revolution in science? Clearly someone needs to realize that the existing paradigm is flawed. Then someone needs to come up with a new theory. Of course, these could be the same person, so we only need one person to start a revolution. But this is not the way things usually work. We would expect there to be several people pointing out the flaws in an existing paradigm before a new paradigm is introduced. Then at least the innovators would have some support within the scientific community.

I was at a meeting on the history of physics when a prominent physicist jokingly said that he has orders from his doctor not to talk to philosophers. Unfortunately, he is not alone, as Hans S. Pendl noted:

> Traditionally, most practicing physicists have held philosophical analyses of their methods and results in low esteem; and conversely, most philosophers have looked down on the thoroughly pragmatic methods of their colleagues in physics.

The physicist does not want to be told: "Look at the scrap heap of theories which have come and gone over the years. It is more likely that the work you are doing now will wind up on that scrap heap than that it will endure." But that is exactly the message from the historian and the philosopher of science.

Too often in the history of science a theory was created to explain all the known facts, but then could not adjust to the introduction of new facts, as Gerald Holton [89] notes: "As every novice is taught, the graveyard of science is littered with those who did not practice a *suspension of belief* while the data were pouring in."

It is time to add one more corpse to that graveyard: quantum mechanics.

Niels Bohr commented after a meeting of philosophers: "I have made a great discovery, a very great discovery: all that philosophers have ever written is pure drivel!" (quoted in [136], page 421). My feeling is that most of the articles written by Bohr are pure drivel, simply because he wrote before all the data were in. Bohr "did not practice a *suspension of belief* while the data were pouring in."

We forget too quickly that even the most popular theories are only approximations and a better approximation may come along at any time. Theorists live on the edge. An experiment may be run today which will contradict the predictions of your pet theory. Even if that happens, the theory will stick around for a while, because theories build up a type of intellectual momentum in the form of prejudice.

Perhaps it is characteristic of the human race that we find it difficult to change our minds in the face of evidence which contradicts our beliefs. There is a rather common bumper sticker which sums up the situation: "My mind is made up. Don't confuse me with the facts."

There have been challenges to quantum mechanics in the past, but in the process of science as usual, they were rebuffed. As Nancy J. Nersessiian [129] notes:

> Once an empirical law is well established the tendency is to ignore or try to accommodate recalcitrant experiences, rather than give up the law. The history of science is replete with examples where apparently falsifying evidence was ignored, swept under the rug, or led to something other than the law being changed.

The reason for the intellectual momentum of theories was pointed out by Leo Corry [30]:

> A scientist cannot, while under the sway of one paradigm, seriously entertain a rival paradigm.

This idea of intellectual momentum in science is just the band wagon effect and as M. J. O'Hara [131] noted: "Bandwagons have bad steering, poor brakes, and often no certificate of roadworthiness."

Joseph Schwartz addressed the problems of modern physics. Speaking of the electro-weak unification, Schwartz [162] writes:

> The result is a contrived intellectual structure, more an assembly of successful explanatory tricks and gadgets that its most

> ardent supporters call miraculous than a coherently expressed understanding of experience.

Schwartz goes on to analyze the source of the problem:

> Postwar theorists have been unable to generate new physical insight about nature because they are much too cutoff from direct contact with the physical world. Achievement at the highest levels of science is not possible without a deep relationship to nature that can permit human unconscious processes–the intuition of the artist–to begin to operate. . . The lack of originality in particle physics dates from the 1920s. It is a reflection of the structural organization of the discipline where an exceptionally sharp division of labor has produced a self-involved elite too isolated from experience and criticism to succeed in producing anything new.

In spite of these strong statements about the flaws of modern physics, Schwartz does not go far enough in rooting out the problems nor in offering new solutions. It was not originality which was lacking, there was a deficiency in scientific thinking which started then and has continued to this day. The primary difference between scientific thinking and religious thinking is immediacy. The religious mind wants an answer now. The scientific mind has the ability to wait. To the scientific mind the answer "We don't know yet" is perfectly acceptable. The physicists of the 1920's and later accepted many ideas without sufficient data or thought but with all the faith and fervor characteristic of a religion.

31 Superposition or Superselection: E.P.R. Revisited

> It is the duty of every man, as far as his ability extends, to detect and expose delusion and error. But nature has not given to everyone a talent for the purpose; and among those to whom such a talent has been given there is often a want of disposition or of courage to do it.
>
> —Thomas Paine

The criticism of quantum mechanics by Einstein, Podolsky and Rosen [57] has been discussed by many commentators, at least one article on EPR has appeared every year since then. Seventy years of debate indicates that there is a real problem with quantum mechanics which has not yet been resolved. In discussing the problem, we will follow the simplified version of Bohm [20]. The basic experimental setup is easily described: a molecule with total spin zero decays into two particles. An experiment allows us to measure the spin of one particle, which informs us of the spin of the other particle by the conservation of spin. The problem arises when we take the measurement far from the source, when the two particles are far apart, requiring that the signal travel faster than the speed of light. The paradox arises when we analyze the experiment in terms of Quantum Mechanics.

According to quantum mechanics, the particles do not have a definite value of spin until it is measured. The wave function of the system is a superposition of two wave functions representing particle one having spin up, particle two with spin down and vice-versa. The measurement of the spin of one particle then forces the other particle into a spin eigenstate, although the measurement was performed on the second particle before the information from the first experiment could have arrived. If the two systems are separated far enough apart, this would imply that the influence of the first measurement travelled faster than the speed of light.

Perhaps one more version of the experiment will reveal the problem more clearly. If the original "particle" was a photon, the daughter particles were respectively a neutrino and an anti-neutrino, and the experiment measured the lepton number instead of spin, then the analysis is the same (actually any particle-antiparticle pair and appropriate quantum number would work as well). According to quantum mechanics, the particles do not have a definite value of the lepton number until it is measured. The wave function of the system is a superposition of two wave functions representing particle one having as a neutrino, particle two as an anti-neutrino and vice-versa. Thus, according to standard quantum mechanics, nature does not decide which is a neutrino and which is an anti-neutrino until a measurement is made. But then the particles would not know how to interact! It is perhaps reasonable to believe that the act of measurement forces the particle into an eigenstate of spin, but to claim that the act of measurement would force a particle into an eigenstate of particle type is absurd. If we reject the standard formalism in this case, which we must, then we must reject it in all cases.

Implicit in the formalism of the collapse of the wave function is the belief that the observed energy is the actual energy of the particle. To require that a particle in a free state be in an eigenstate of energy at the time of observation is to require that the free particle be in a bound state at the time of interaction. This is also to require that there be only one interaction and that it be instantaneous!

Philosopher Karl Popper [141] discussed the EPR paper:

> The aim of this paper was simply to show that a particle possess both a precise position and a precise momentum. I believe that the EPR argument is valid. The argument was, unfortunately, a little confused by quite unnecessary discussions about "reality and completeness". But all this can be omitted: the argument shows, very clearly, that the Copenhagen interpretation is untenable and that a particle possesses both a precise position, and a precise momentum, even though we may be unable to prepare a particle so that its momentum does not scatter when its position is determined.

C.A. Hooker [90], analyzed the E.P.R. objection to quantum mechanics and concluded:

> Thus, the 'paradox' of Einstein, Podolsky and Rosen does not reveal any contradiction of quantum mechanics; it merely emphasizes in a most striking way the essential non-classical consequences of the quantum-mechanical super-position of states. It is this very super-position which leads to the ambiguity in the application of Einstein's criterion of "physical reality".

Not only does the super-position principle lead to the ambiguities which bothered Einstein, it also leads to other problems. Eugene Wigner [181] points out:

> ...that all the so-called paradoxes of quantum mechanics involve superpositions of classically interpretable states, the superpositions themselves being, however, not interpretable in the naive, classical fashion.

According to Jauch et al [99],

> There seems no escape from the... epistemological dilemma as long as one assumes the validity of the basic principles of quantum mechanics, in particular of the superposition principle and of the linearity of the equations of motion.

As Nancy Cartwright [28] observed: "The principle of superposition has long plagued the quantum mechanics of macroscopic bodies."

It is time we realized that the principle of superposition is a plague within the quantum mechanics of microscopic bodies as well.

Hutten [93] speculated:

> In order to describe interaction between particles, it is most likely necessary to regard them as having a finite size rather than as point particles. Within the field of classical electron theory, this leads to a non-linear extension of Maxwell's equations. A theory of elementary particles will probably require a quantization of space (and time), i.e. a fundamental length; we may therefore expect a non-linear equation in such a theory.

Sachs [155] claimed to have resolved the EPR paradox in a model with "coupled nonlinear 'classical' spinor field equations", thus recognizing that the problems lie with the superposition principle.

Roos [150] addressed the problem of the physical content of the superposition principle and found that it led to "conservation of probability, completeness of the set of states and some phase conditions in unitary matrices". Are these really physical statements? Probability is not a physical quantity which can be measured, how then can it be conserved? The requirement that the set of states be complete is not physical, it is mathematical. The physical content of the inequalities derived from the phase conditions is not clear. Thus, the physical content of the superposition principle is questionable at best, however the physical failures of the superposition principle are numerous.

In his attempts to find a finite radius of the electron, Born [23] suggested new field equations, but "The new field equations are obviously non-linear, and this has been the stumbling block in all attempts to reconcile them with the principles of quantum theory".

Enz [61] claims

> ... a nonlinear theory is certainly required for a proper description of a stable particle, because a linear theory would allow for superpositions, which is contrary to the idea of an integral stable particle.

As de Broglie [24] points out:

> *The usual theory, by limiting itself a priori to linear equations of propagation, precludes local irregularities resulting from nonlinearity (such as singular regions and wave-train boundaries). In this way it obliterates particle structure and, consequently, finally achieves a continuous picture of only statistical character.* (page 287)

Roman [148] concludes that

> *... the basic field equations must be nonlinear... since superpositions of the various solutions would always be new solutions; therefore, systems of particles would be simply a non-interfering ensemble of (non-interacting) particles.* (p. 557)

Fargue [62] discusses many of the early attempts to find a non-linear wave equation, which he believes is necessary to maintain the "permanence of the corpuscular appearance" of the particles, via a necessarily nonlinear equation with soliton solutions. In order to obtain solutions to wave equations which behave like particles the field equations must be nonlinear and Muraskin [125] obtained "Particle-like objects in a Nonlinear Field Theory".

The equations of Einstein's general relativity are perhaps the best known nonlinear equations. Just as Schrödinger tried to eliminate "particles" from quantum theory and deal only with waves, so Einstein wanted to work only with fields:

> A complete field theory knows only fields and not the concepts of particle and motion. For these must not exist independently of the field, but are to be treated as part of it. On the basis of the description of a particle without singularity one has the possibility of a logically more satisfactory treatment of the combined problem: The problem of particle and that of motion coincide.[58]

Einstein's point of view and that of Schrödinger coincide if we take the waves to be related to the potentials of the fields carried by the "elementary particles". A new mathematical basis for such a theory has been presented in a series of papers ([112],[113],[114],[115]).

Many critics of special relativity have looked for contradictions within special relativity. But there are none, for special relativity is based on hyperbolic geometry and is as contradiction free as Euclidean Geometry. If Special relativity is to be found wanting, it will have be shown to fail in the same way as Euclidian Geometry was shown to fail—in its description of nature. Einstein based his ideas about relativity on the premise that Maxwell's equations of electrodynamics are correct. But many other researchers have shown that there are serious problems with Maxwell's equations. [133]

It follows that we should expect troubles with relativity.

The special theory of relativity is based on the idea that the speed of light is independent of the speed of the observer. General relativity became popular when Eddington confirmed the prediction that light bends in a gravitational field. But light bends because its speed is changing. Thus the experimental confirmation of a prediction of General Relativity was also experimental contradiction of Special Relativity. General Relativity is based on the idea of an "inherent metric", but the only possible inherent metric is the field strength. Thus, when the Theory of General Relativity predicts that the metric is expanding, it does not mean that the universe is expanding, but rather that the field of a particle is expanding, as it must from the moment it is created.

Now let me quote Einstein on the speed of light:

> This was possible on the basis of the law of the constancy of the velocity of light. But according to Section XXI, the general theory of relativity cannot retain this law. On the contrary, we arrived at the result that according to this latter theory, the velocity of light must always depend on the coordinates when a gravitational field is present.[46] (page 111)

In [48] on page 107 Einstein gives a formula which tells how the speed of light varies in a gravitational potential: $c = c_0 \left(1 + \frac{\Phi}{c^2}\right)$, where c is the velocity of light at a place with the gravitational potential Φ and c_0 is the "velocity of light at the origin of co-ordinates." On page 114, Einstein states:

> ...the principle of the constancy of the velocity of light in vacuo must be modified, since we easily recognize that the path of a ray of light with respect to K' must in general be curvilinear, if with respect to K light is propagated in a straight line with a definite constant velocity.

(These are translations of older papers)

Einstein did say that the speed of light varies depending on the gravitational potential, indeed that is the reason light curves near a massive body. In a universe without matter and without a gravitational potential, who is to say what the speed of light would be?

Most physicists are aware of Einstein's 1905 remarks about the nonexistence of an ether in regard to the Special Theory of Relativity, but are evidently not aware that he later repudiated those remarks in the context of general relativity:

> ...we will not be able to do without the ether in theoretical physics, i.e., a continuum which is equipped with physical properties...(quoted in [104]).

Einstein called his ether g_{ij}.

32 Uncertainty Principle

Werner Heisenberg in his Nobel Lecture of December 11, 1933 discussed the Uncertainty Principle:

> ...the formalism shows that between the accuracy with which the location of a particle can be ascertained and the accuracy with which its momentum can simultaneously be known, there is a relation according to which the product of the probable errors in the measurement of the location and momentum is invariably at least as large as Planck's constant divided by 4π. In a very general form, therefore, we should have
>
> $$\Delta p \Delta q \geq h/4\pi$$
>
> where p and q are canonically conjugated variables. These uncertainty relations for the result of the measurement of classical variables form the necessary conditions for enabling the result of a measurement to be expressed in the formalism of the quantum theory.

There are several uncertainty principles, one for each pair of canonical variables. These canonical variables are position x, and momentum p; Energy E; time t; Angular momentum L and angle θ. We see from Heisenberg's comments that he (and his generation) originally understood the uncertainty principle to be an uncertainty of measurement, a limitation placed on human knowledge of reality.

Hawking [80] claims that the Heisenberg uncertainty principle is basic to quantum theory. But it is not, since the Heisenberg relations can be derived for any pair of operators which do not commute. Thus, what is basic is the existence of noncommuting operators. In quantum theory, an operator represents a measurement and the eigenvalue of that operator represents the result of the measurement. If two operators do not commute, the state is not simultaneously an eigenstate of both operators. The way around the uncertainty principle is the obvious: build a theory of observables which consists only of operators which commute.

The uncertainty principle is now taken to be a limit of reality. Whereas before one would say we can only know the location and momentum of an electron up to the limits of the uncertainty principle, current researchers take this to mean that the electron has neither a position nor a momentum. But the uncertainty principle has come to mean much more in modern physics.

According to Hawking [80] (p.106):

> The uncertainty principle also predicts that there will be similar virtual pairs of matter particles, such as electrons or quarks. In this case, however, one member of the pair will be a particle and the other an antiparticle.

As Kristin Shrader-Frechette [165](p.417) explains it, Quantum field theorists:

> . . . postulate unobserved, virtual, elementary particles in order to "balance the book of conservation laws up to the point at which the Uncertainty relations are applicable". . . According to the theory behind virtual particles, one consequence of the Uncertainty Relations is that nature is willing to "overlook a violation" of energy conservation provided it lasts a short enough time.

In the early days of science, when men could not understand a certain phenomena, they said it was God in action. This type of explanation of natural phenomena became known as the "God in the gaps" argument. Needless to say, this argument is no longer used—in such a direct manner. The modern incarnation of the God-in-the-gaps explanation is putting uncertainties-in-the-gaps. Modern physics is full of uncertainties. According to quantum field theory, all interactions between particles are due to the exchange of virtual particles, yet these fundamental "objects" are by definition the result of uncertainties.

Charles Mauguin, a member of de Broglie's examination committee spoke in 1952 about his state of mind in 1924:

> Today I have difficulty understanding my state of mind when I accepted the explanation of the facts without believing in the physical reality of the entities that provided this explanation. [120]

Unfortunately, particle physicists have no qualms about introducing any number of new particles (many virtual) to explain new phenomena. Calling upon unobserved and, in principle, unobservable particles is not science! Instead of having a theory of virtual particles, we might as well reintroduce the old idea of fairies floating around holding the world together. Replace "virtual particles" by "fairies" in many sentences in the physics literature and not only will you come up with a sentence which makes sense, but you will also have a statement which would have been believed by people in the Dark Ages! Instead of looking for a unified theory of four forces, we could look for a theory of fairies, elves, gnomes and pixies! This is the ultimate in the confusion of a mathematical model with reality! Somewhere along the line, there entered into quantum physics a great confusion between a human measurement of a quantity with the actual value of that quantity. The problem has become so acute that some researchers deny there is an external reality! Obviously modern physics theory has lost touch with reality.

Supposedly, virtual particles exist only as long as allowed by the uncertainty principle. This causes only minor problems when we are considering the interaction of two particles in close proximity, but as soon as we think about a large number of particles or particles interaction at large distances, there are real problems with the concepts.

Consider an electron approaching a charged plate of metal.

The electric field of the approaching electron is interacting with the field of every electron in the plate. This could easily be 10^{20} electrons. Now, if the approaching electron is sending out a virtual photon to interact with each of the plate electrons, the uncertainty principle is being strained to its breaking point. Is the uncertainty principle to be applied to each virtual photon individually or en masse? If physics makes any sense whatsoever, we must look at the whole system. Then, not only is the approaching electron violating the conservation of energy by sending out a photon to a single plate electron, rather it is emitting 10^{20} virtual photons simultaneously and it has virtual photons out continuously. Not only that, but each of the plate photons is emitting virtual photons at the oncoming electron. Furthermore, the plate electrons photons are emitting virtual photons at each other which would amount to having 10^{20} virtual photons for each electron or then 10^{40} virtual photons in existence at the same time. This would surely be observable and would definitely violate the conservation of energy.

Likewise, if we consider two electrons a light minute apart, the density of virtual photons is small and so the probability of an interaction is small, the motion of such electrons should be probabilistic versus the classical theory of fields where the interaction is continuous.

If the electromagnetic interaction were due to the exchange of virtual photons then the field strength would exhibit statistical fluctuations. If the field strength exhibits statistical fluctuations, then the electron's energy level would exhibit statistical fluctuations and hence the spectral lines would exhibit statistical fluctuations. They do not. Spectral lines are sharp. We conclude that the electromagnetic interaction is not due to virtual photons.

Normal photons are created as a quantized system changes energy levels, the energy of the photon is given by:

$$E = h\nu = E_2 - E_1$$

But what is the source of the virtual photon? You will search the literature in vain for an explanation of the mechanism of the creation of virtual photons. So many questions go begging: how does an electron know when to create virtual photons? How does it know at what frequency to produce these virtual photons? If electrons (and other charges particles) interact via an exchange of photons, then how do the photons and the electrons interact?

In the Quantum Field Theory Paradigm, particles interact by interchanging other particles, the quanta of the field. The model comes from the well known and well studied chemical bond where an exchange of electrons between atoms produces the force between them. Inside the atom, the electron interacts with protons via the electromagnetic field. But QED begs the question: how do the quanta and the particles interact?

Put an electron in a beam of light. It will interact with the photons and pick up momentum in the direction of the beam. Replace the electron with a positron, or even a neutral particle. Same thing happens. But supposedly if a virtual photon from an electron hits an electron, the second electron moves away from the first electron. But if the virtual photon from an electron hits a positron, the positron moves toward the electron. But a virtual photon, being the carrier of the electromagnetic interaction, will not interact with a neutral particle. Why should a "virtual photon" behave so differently from a real photon? In the QED paradigm, there are virtual photons exchanged but what happens to unused virtual photons? What force causes the photon to curve, or to return to the source?

Consider two charged particles, A and B. When they interact, according to QED, particle A emits a virtual particle in the direction of particle B. Now, we must ask, how does particle A know that particle B is approaching and that it should emit a virtual photon? Clearly it cannot know, so particle

A is constantly sending out virtual photons. Now, when particle A sends out a virtual photon, that photon carries energy and momentum. Since the photon carries momentum away from particle A, particle A must move in the opposite direction in order to conserve momentum. But now, what happens when the virtual photon reaches particle B? Particle B absorbs the photon, through some unknown mechanism, along with its momentum and energy. Thus particle B must move away from particle A again in order to conserve momentum.

Now, that observation is true if the two particle have the same charge, but what happens if they are oppositely charged? Physically, oppositely charged particles accelerate toward each other. How does this happen in terms of the exchange of virtual particles? It cannot, without violating the conservation of energy and momentum. You can read in some popular level expositions that the photon behaves like a boomerang and comes back at the second particle. But that still doesn't explain what happened when the virtual photon was created at its source. When particle A emits the photon towards particle B, particle A must react differently depending on the charge of B. That is, when the virtual photon is emitted, particle A must somehow know the charge of the particle with which the photon will ultimately interact!

As Robert S. Fritzius [66] points out, this is essentially a violation of the conservation of momentum and is "a very important unresolved problem." This conflict within QED is usually swept under the rug, but even more astonishing is the fact that QED is supposed to be a theory of electromagnetism and when the magnetic field of a charged particle acts on another charge particle, the direction of the force can be perpendicular to the line between the two particles. Try to explain that in terms of the exchange of virtual photons! The QED picture of interchanging virtual photons reduces to gibberish! If the magnetic field is due to the exchange of virtual particles, how do refigerator magnets hold the shopping list to the refrigerator? How could the virtual photons pass through the paper when real photons cannot?

The only possible conclusion is that the photon is not the "electromagnetic quantum." A photon will interact with any particle, charged or not, by bouncing off of it. The E-M field acts only on charged particles or particles with a magnetic field.

Perhaps we should go one step further and suggest that photons are not electromagnetic radiation at all. Photons are produced when a quantized system makes a transition to a lower energy level. Electromagnetic waves can be produced by shaking a magnet or a vibrating charged particle. Perhaps part of the problem with the debate over the nature of light as particle or wave was the existence of several different phenomena which were put into the same basket. Light was identified as the electromagnetic radiation when it was noticed that the experimental value for the speed of light matched the theoretical value for the speed of electromagnetic radiation. The argument is not valid now that we know that the speed of light is a limit for the speed of all material objects.

When an electron meets an anti-electron, the result is pure energy. Since the electron participates in the electromagnetic interaction, it is possible that this energy is a photon and related to the electric interaction. However, the neutrino does not interact via the electric force and so when a neutrino meets an anti-neutrino, the energy released is not the same and has nothing to do with the electric field.

The problems with explaining the electric interaction via virtual particles are great, but I have searched in vain for an explanation of the magnetic forces in terms of virtual particles. It is hard to visualize how virtual particle interactions can account for both the attractive and the repulsive force of electric charge, but trying to explain how virtual particles could cause a force at right angles as in the magnetic interaction seems to be beyond the explanatory powers of the theory.

In the standard model, the electron and the neutrino interact via the weak force by interchanging a Z. But think about the masses of these objects. The mass of an electron is 0.511 Mev, the mass of the neutrino is less than $6 \times 10^{-4} ev$. The mass of the Z is about 91 GeV. Imagine a neutrino of such small mass giving birth to a pair of virtual particles twenty orders of magnitude heavier! The uncertainty in the mass-energy is many orders of magnitude greater than the mass of the neutrino, or the electron. The idea is so preposterous that it is hard to see how anyone could believe it. Absurdity rules!

In a very incisive critique of quantum theory, W.F.G. Swann [169] wrote:

> ...the uncertainty in the principle of uncertainty is inherent in the language of expression rather than in any deep-seated characteristic of nature...And so if the experimentalist of old was a hopeless materialist, the modern theorist is apt to become a mathematical spiritualist withal, for his ghosts are of his own creation and serve but the purpose of giving the semblance of life to the picture of nature which he possesses and which, in the last

analysis is a creation of his own mind.

33 A Most singular Theory

How are we to extend Einstein's work in the direction he wanted to go? If that is what we are to do, we must turn to Einstein for guidance:

> Since according to our present day notions the primary particles of matter are also, at bottom, nothing but condensations of the electromagnetic field, our modern schema of the cosmos recognizes two realities which are conceptually quite independent of each other even though they may be causally connected, namely the gravitational ether and the electromagnetic field, or–as one might call them–space and matter.
>
> It would, of course, be a great step forward if we succeeded in combining the gravitational field and the electromagnetic field into a single structure. Only so could the era in theoretical physics inaugurated by Faraday and Clerk Maxwell be brought to a satisfactory close.
>
> The antithesis of ether and matter would then fade away, and the whole of physics would become a completely enclosed intellectual system, like geometry, kinematics and the theory of gravitation, through the general theory of relativity.[49](pages 110-111)

Before we can draw conclusions about the origins of the universe, before we can speculate about the origins of matter, we have to know what matter is! We need to have a unified field theory. Observational evidence is necessary to eliminate candidate theories, but observation is not the final authority. Since any observation must be interpreted in terms of some theory, we cannot pretend to understand what we see until we have a theory.

Einstein expressed strong opinions about what the "final" theory should look like:

> . . . there still remains outstanding and important problem of the same kind, which has often been proposed but has so far found no satisfactory solution—namely the explanation of atomic structure in terms of field theory. All of these endeavors are based on the belief that existence should have a completely harmonious structure. Today we have less ground than ever before for allowing ourselves to be forced away from this wonderful belief.[49] (page 114)

In Einstein's ideal theory, there is no "matter", there are only fields. Others held the same belief, witness Arthur Eddington's statement that the

> ... particle of matter is not fundamental; it has no meaning in itself; what you are really concerned with is its 'field' ... matter cannot be thought of apart from its field.[45](pp. 165-166)

Einstein eschewed the quantum mechanical viewpoint in which everything was expressed in terms of probabilities:

> I still believe in the possibility of a model of reality—that is to say, of a theory which represents things themselves and not merely the probability of their occurrence. [49]

But of course, something of the classical viewpoint has to be abandoned:

> On the other hand, it seems to me certain that we must give up the idea of a complete localization of the particles in a theoretical model. [49] (page 20)

There are many singularity theorems within general relativity. According to Hawking, [80](p.46) a singularity is

> ... a point in the universe where the theory itself breaks down.

Now if the theory predicts the existence of a point where the theory is wrong, then the theory itself must be wrong in other places too.

Newton's theory of gravity with its inverse square law seems to predict a singularity when $r = 0$. However, the inverse square law is not true within the planet's interior. From the surface to the center, the gravitational force decreases linearly to zero at the center.

As Einstein and Rosen observed, [58]

> ... a singularity brings so much arbitrariness into the theory that it actually nullifies its laws.

This point of view was again stated by Wheeler [176]:

> If singularities are admitted, the properties of the sources cannot be discussed adequately entirely within the framework of the theory; or in other words, the theory is no longer complete. If singularities are tolerated in general relativity, then the completeness of this theory is even more thoroughly shattered.

Despite this internal evidence that the theory of general relativity is wrong, or at least incomplete, Hawking persisted and

> The final result was a joint paper by Penrose and myself in 1970, which at last proved that there must have been a big bang singularity provided only that general relativity is correct and the universe contains as much matter as we observe. [80] (p. 50)

Is general relativity correct? Possibly, but the odds are against it. Even if it is correct in some sense, it is not complete. There are five forces in nature, the weak nuclear force, the strong nuclear force, electromagnetism, the spin-spin interaction and gravitation. General relativity is a theory of gravitation alone, but gravitation never acts alone. The singularity theorems arise from an analysis of the interaction of particles via gravity and thus ignore their interactions via other forces which are much stronger. Moreover, the theory of general relativity cannot be correct since it is incompatible with quantum theory. In dealing with singularities, where matter is compressed, quantum effects cannot be ignored, neither can the other forces which are much stronger than gravity.

Some present day scientists have more faith in general relativity than Einstein himself had. Einstein wrote in a letter to Felix Klein, dated March 4, 1917:

> However we select from nature a complex of phenomena using the criterion of simplicity, in no case will its theoretical treatment turn out to be forever sufficient. Newton's theory, for example, represents the gravitational field in a seemly complete way by means of the potential. This description proves to be wanting; the functions g_{ij} take its place. But I do not doubt that the day will come when that description, too, will have to yield to another one, for reasons which at present we do not yet surmise. I believe that this process of deepening the theory has no limits. (quoted in [135], p. 325)

The existence of singularities is a mathematical theorem, following directly from the Einstein equations of General Relativity. Hawking claims "One cannot really argue with a mathematical theorem" [80](p. 50) While we cannot argue with a mathematical theorem within the context of pure mathematics, we can, and we must, argue with the applicability of that theorem to physical reality. There are two stages to deriving conclusions about the physical world from mathematical models. First, we must construct a model and derive conclusions about the model. We construct a mathematical model to reflect certain properties of nature. The model will always reflect the information we put into it, but, if the theory is viable, we will be able to draw more information from the mathematical model than we put into it. The big question is do the additional predictions of the theory match what we observe in nature? We use the information from the model to derive conclusions about nature. Then we must compare the behavior of the model with observations of nature. Too often, people confuse the mathematical model with the physical reality it models.

According to Leon Lederman [109]:

> ...the evolution of the universe is pretty much all contained in Einstein's equations of general relativity."

What he meant to say was "Our guesses about the evolution of the universe are pretty much all derived from Einstein's equations of general relativity."

There are problems with all of the equations of Physics. Consider the problem of a falling rock. If we are close enough to the surface of the earth and we ignore air friction, the rotation of the earth, the shape of the rock and a few other things, then if we drop a rock from a height h, the distance that the rock has fallen after t seconds is given by $d = 16t^2$ where d is measured in feet. The speed after t seconds is $s = 32t$. Now in a standard physics course, I might ask my students some questions:

1) How far has the rock fallen in 1 second? What is its speed?
2) How far has the rock fallen in 10 seconds? What is its speed?
3) How far has the rock fallen in 100 seconds? What is its speed?
4) How far has the rock fallen in 1000 seconds? What is its speed?

Most students will blindly plug the numbers into the equations without thinking. If you just plug in, the answer in part 4 is 16 million feet or about 3030 miles and 32,000 feet per second! If you were close enough to the earth that the formula were valid, the rock would be lying on the ground with zero speed. From an altitude over a few miles, the formula would not be valid, the rock would either go into orbit or be burned up by air friction on entering the atmosphere. But again, most students will just plug numbers into the formula and then get angry when I don't give them credit for their work, because I want to teach them how to think, not just plug numbers into equations. The formula works, the numbers just are not relevant to reality.

The same thing has happened in general relativity theory and in quantum theory. The equations lead to nonsense, but instead of saying that the equations have been applied outside their domain of validity, the theoreticians claim they have predicted something strange and wonderful.

Alfred North Whitehead addressed the problem:

> There is no more common error than to assume that, because prolonged and accurate mathematical calculations have been made, the application of the result to some fact of nature is absolutely certain.

As Einstein put it:

> As far as the laws of mathematics refer to reality, they are not certain and as far as they are certain, they do not refer to reality.[54]

Mathematics is just one of the languages people use to communicate and to think. It is both a tool of description and prediction. We can paraphrase Einstein's comment to read:

> As far as descriptions refer to reality, they are not certain and as far as they are certain, they do not refer to reality.

You may understand my description of a natural phenomena perfectly and still have no understanding of the phenomena itself. You may understand my words perfectly, but if you have no experience to attach the words to, we may fail to communicate.

In the same way, when modeling nature mathematically, you may understand the mathematics perfectly, and still have no comprehension of the natural phenomena. On the other hand, it is possible to understand the physical phenomena on a qualitative level, yet be unable to produce a mathematical model. That is where physical mathematics comes in. Physical mathematics is an experimental science. We learn new mathematics and we try applying it to physical problems. Sometimes it works and sometimes it doesn't. Sometimes it comes so close we think it has worked and only much later do we see that the model is wrong. Then we learn some more math and try again. We keep experimenting with new mathematics and new physics. This is why interdisciplinary mathematical studies are so important to Physical Mathematics. We are never sure which pieces of mathematics will be useful until we try it. If one type of mathematics does not work, we must know enough to recognize what went wrong and to propose a new sort of mathematics to use.

As with any experimental science, there is a degree of interpretation required—how does a particular mathematical theorem fit into a physical theory? How are we to interpret the mathematical result? In the problem at hand, the equations of General Relativity, Einstein's equations, to lead to singularities. That is a mathematical theorem. What does it mean physically? These are singularities of the model, not necessarily singularities in nature. The modern exotic interpretation is that these singularities are physically black holes or a big bang. However, an earlier interpretation was that these singularities represent matter. As Arthur Eddington [45] wrote in 1921:

> The electron, which seems to be the smallest particle of matter, is a singularity in the gravitational field and also a singularity in the electrical field. (p.167)

Following the early relativists, should we conclude that the modern singularity theorems only predict that where there is a gravitational field, there is matter?

Einstein's thinking matured and he realized later that the existence of singularities implied that the General Theory of Relativity is an incomplete theory. Einstein and Rosen opined [58]

> Every field theory, in our opinion, must therefore adhere to the fundamental principle that singularities of the field are to be excluded.

This is one reason why Einstein looked for a unified field theory which would incorporate electromagnetism and would eliminate the singularities. [72]

Einstein asked:

> Is there a theory of the continuum in which a new structural element appears side by side with the metric such that it forms a

> single whole together with the metric? If so, what are the simplest field laws to which such a continuum can be made subject? And finally, are these field laws well fitted to represent the properties of the gravitational field and the electromagnetic field? Then there is the further question whether the corpuscles (electrons and protons) can be regarded as positions of particularly dense fields, whose movements are determined by the field equations. [49] (page 74)

Einstein saw the need to eliminate singularities:

> To what extent can physical fields and primary entities be represented by solutions, free from singularities, of the equations which answered the former question?[49] (page 77)

Einstein's program was later summarized by philosopher of science Michael L. G. Redhead[146]:

> There emerged for Einstein and his collaborators the vision of a unified field theory, in which the electromagnetic as well as the gravitational could be given geometrical significance, and matter, instead of being associated with singularities in this generalized field, points at which the field equations did not apply, would again be identified with local concentrations of this field.

Wolfgang Pauli [138] wrote:

> On the other hand Einstein, after he had revolutionized the way of thinking in physics with general methods which are also fundamental for quantum mechanics and its interpretation, maintained until his death the hope that even the quantum-features of atomic phenomena could in principle be explained on the lines of the classical physics of fields. . . If he speaks of a "unified field theory", he therefore has in mind this ambitious programme of a theory which solves all problems regarding the elementary particles of matter with the help of classical fields which are everywhere regular (free of singularities). (pages 224-225)

Einstein reaffirmed this program in a paper written just a few years before his death:

> Maxwell and Hertz have shown that the idea of forces at a distance has to be relinquished and that one cannot manage without the idea of continuous "fields." The opinion that continuous fields are to be viewed as the only acceptable basic concepts, which must also underlie the theory of the material particles, soon won out... These remarks presuppose it as self-evident that a field theory may not contain any singularities, i.e., any positions or parts in space in which the field laws are not valid... Consequently, there is, strictly speaking, today no such thing as a classical field theory; one can, therefore, also not rigidly adhere to it. Nevertheless, field-theory does exist as a program: "Continuous functions in the four-dimensional continuum as basic concepts of the theory." Rigid adherence to this program can rightfully be asserted of me. [51]

In the present theory, we replace the "Continuous functions in the four-dimensional continuum as basic concepts of the theory" by "Differentiable vector fields in the four-complex-dimensional continuum as basic concepts of the theory."

In Einstein's last lecture, he again spoke of singularities:

> It is pedagogic to insist that if one has a field theory, one must demand solutions without singularity. If a singularity is allowed, there are too many arbitrary assumptions, and too much arbitrariness.

(Mercer Street and other Memories, John A. Wheeler, p. 209).

If Einstein's vision to eliminate singularities was so clearly understood, how could his heritage have been so warped that the present day general relativity industry is preoccupied with theories of singularities: Black Holes, Time Warps, The Big Bang, The Big Crunch are all examples of the singularities which Einstein said marred his theory of relativity, yet they continue to be the focus of an entire community of "scholars" and appear in numerous popular works on science. But they are best thought of as examples of science fiction rather than science! I believe the problem is really deeper, the unavoidability of singularites in general relativity proves that it cannot be correct.

If a "particle" is nothing but a local concentration of field lines, then a particle is really a smear across space-time and it has no precise location in either space or time. That a particle is extended across space is obvious, but what does extension in time mean? I'm afraid that for now, can only pose the question, I have no answer. However, remember that asking the right question is the first step. We often deal with vibrations in space, but in the present theory we will need to deal with vibrations in time.

34 Discussion and Conclusions

> The moral of this story is that one should not try to accomplish too much in one go. One should separate the difficulties in physics one from another as far as possible. and then dispose of them one by one.-=-P.A.M. Dirac

> Therefore I feel that it is perhaps not only a deep truth to *say* "You can only solve one difficulty at a time" I but it may also be a deep truth to say . "You can never solve one difficulty at a time, you have to solve always quite a lot of difficulties at the same time ... " —Werner Heisenberg

So who is right? If one is working within an established theory, Dirac is correct. If one is working to overthrow an established theory, Heisenberg is correct.

Analyzing the historical development of physics. Dirac [39] wrote:

> When one looks back over the development of physics, one-sees that it can be pictured as a rather steady development with many small steps and superposed on that a number of big jumps ... These big jumps usually consist in overcoming a prejudice ... And then a physicist ... has to replace this prejudice by something more precise, and leading to some entirely new conception of nature.

The origins of the prejudices discussed here go back to the foundations of nonrelativistic quantum mechanics. Lubkin [117] presented

> ... a broad proof of the validity of superselection rules for all additively conserved quantities

But then, not believing in the mathematics, Lubkin went on to refute his own argument. This is a rare case where the prejudices are clearly displayed and Lubkin himself called this refutation "the dodge of section V" .

In the development of field theory, the Lagrangian has played such a major role that when the existence of a new theory is announced, instead of asking for the field equations, physicists routinely ask "What is the Lagrangian?" implying that any acceptable field theory must be a Lagrangian Field theory. This prejudice has hindered the development of the field.

Bryce DeWitt [33] claimed that:

> The very first and most fundamental assumption of the quantum theory is that every isolated dynamical system is describable by a characteristic action functional S.(page 587)

If DeWitt is correct then the entire foundation of quantum theory rests on the assumption that any quantum field theory is a Lagrangian Field Theory. In my 1993 article, I proved that a totally unified field theory which includes the particles

$$\begin{pmatrix} \gamma_1 & \nu & H & e^- \\ \bar{\nu} & \gamma_2 & n & \pi^- \\ \bar{H} & \bar{n} & \gamma_3 & p^- \\ e^+ & \pi^+ & p^+ & \gamma_4 \end{pmatrix}$$

and their interactions cannot be a Lagrangian field theory.

Thus in DeWitt's analysis, Quantum Field theory is inconsistent with a unified field theory. The special theory of relativity was developed long before quantum mechanics, so why has nonrelativistic quantum theory played such a dominant role in the development of other quantum theories?

Bell and Nauenberg [17] predicted

> ...the quantum mechanical description will be superseded. In this it is like all theories made by man. But to an unusual extent its ultimate fate is apparent in its internal structure. It carries in itself the seeds of Its own destruction.

Those seeds have sprouted and the resulting weeds are choking theoretical physics. One garden of weeds has evolved over the interpretation of Bell's inequalities. These inequalities are derived from nonrelativistic probabilities which must assume values between 0 and 1. However, when relativistic effects are taken into account, the "probabilities" can be negative and " ...Bell's inequality no longer has any validity" (Selleri [164]). Just before his untimely death, Bell [16] was headed "Towards An Exact Quantum Mechanics", which he argued must be nonlinear and nonlocal.

No one is surprised to discover that the nonrelativistic Newtonian theory of gravity requires that signals travel faster than light, but no one takes that seriously. Why then should be we surprised that nonrelativistic quantum mechanics also requires superliminal signals? Why should this be taken seriously? The common problem is that both deal with unchanging immutable potentials. The linearity of quantum mechanics was bought at a very high price. Only when the electron moves in a potential due to a proton which itself is unmoved by the electron, could the approximation of linearity be considered valid.

The time has arrived when we must recognize that non-relativistic quantum mechanics is a toy theory and must be discarded. Physicists are not prone to throw away toys until they have a new one. In this paper I have pointed out the flaws in the theory of nonrelativistic quantum mechanics and laid the foundations of a new program for quantization. The analysis so far has shown that the interaction of two particles must be considered as a whole, analyzing the entire interaction rather than break the problem into pieces. If wave functions are merely the exponentials of the classical potentials, then to add potentials we must multiply wave functions and it becomes obvious why we cannot add wavefunctions. This property of wave functions was observed by Caianiello [27]:

> Spinor space is not obtained, as customary, by taking the sum $+$ of ψ_1 and ψ_2 , but their product $\psi_1 \times \psi_2$.

This remark is relevant although we are modeling physical particles by "vector fields on the complex spacetime *QAdS*" and not spinors because Nash [127] has shown "There exists an exceptional equivalence of a complex Dirac spinor and a complex Minkowski space-time vector".

As I discussed in [114] our model of matter requires the use of two vector fields on the complex space-time *QAdS* since the Lie bracket of two vectors in *QAdS* is a vertical vector, i.e. a particle. By Nash's result, this is equivalent to using two spinors. Two spinors in turn form a twistor. In a way then, the geometric model of matter proposed here is related to Penrose's program to describe matter in terms of twistors.

Roman, Aghassi and Huddleston [149] characterized the state of a relativistic particle by a pair (x, ς) of four vectors. This led them to a group which includes the *Poincaré* group and dilations. The present program reversed the flow of ideas: beginning with the group $U(3,2)$, which contains the Lorentz group, translations on *QAdS* and the dilations, we arrived at the description of relativistic particles by a pair of four-vectors. Going one step farther, we take the bracket of these two four vectors and arrive at a vertical vector. The vertical vector does not describe just the "particle", it also describes the fields of the particle. Taking the correspondence principle one step better, we have shown that the quantum wave function is no more than the exponential of the classical potential.

Since superposition is not valid for functions which satisfy the Schrödinger equation S(ti), several results which depend heavily on superposition are called into question. As Sachs [157] notes, Von Neumann's proof of the impossibility of hidden variables "relies on the validity of the principle of superposition" and is not valid. The proof of the violation of CP violation in the decay of the kaon is invalid since it relies completely on superposition. As Roos [150] noted:

> CP-violation 18 years after its discovery has not been satisfactorily explained, and that one obvious but unattractive explanation may still be a non-linear quantum mechanics.

For the many reasons discussed, a non-linear quantum mechanics seems not only attractive but necessary. But as we have seen, the standard eigenvalue equation is nonlinear.

We will equate "Quantum Theory" with "harmonic analysis" where we use Helgason's [83] definition of harmonic analysis as a quest for the joint eigenspace of $D(U(3,2)/U(3,1)\times U(1))$, those functions which are eigenfunctions of all the differential operators which are invariant under translations. In other words, we need to identify the simultaneous eigenfunctions of all the Casimir operators of $U(3,2)$. Thus, although we cannot use a Lagrangian [114] we can obtain a consistent quantization scheme, in spite of the claim to the contrary by Hojman and Shepley [88]. Of course, they based their claim on the validity of the CCR which we showed are not valid.

In order to implement this program, we must identify the Casimir operators of $U(3,2)$ and their eigenfunctions. Progress toward that goal is made in [115].

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